OVERHEAD ELECTRIC POWER TRANSMISSION

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PRINCIPLES AND CALCULATIONS

BY

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PREFACE

Although this book treats mainly of the fundamental principles and scientific laws which determine the correct design of overhead electric transmission lines, it has been written primarily to satisfy the needs of the practical engineer. An attempt has been made to give the reasons of things—to explain the derivation of practical methods and formulas—in the simplest possible terms: the use of higher mathematics has been avoided; but vector diagrams, supplemented where necessary with trigonometrical formulas, have been freely used for the solution of alternating current problems. It is therefore hoped that the book may prove useful, not only to the practical designer of transmission lines, but also to those engineering students who may wish to specialize in the direction of Power Generation and Transmission, for these will find herein a practical application of the main theoretical principles underlying all Electrical Engineering.

The subject is treated less from the standpoint of the construction engineer in charge of the erection work, as of the office engineer whose duty it is to make the necessary calculations and draw up the specifications. The considerations and practical details of special interest to the engineer in charge of the work in the field have already been presented in admirable form by Mr. R. A. Lundquist in his book on Transmission Line Construction.

Much of what appears in these pages is reprinted with but little alteration from articles recently contributed by the writer to technical journals; but in the selection and co-ordination of this material, the scheme and purpose of the book have steadily been kept in mind.

Systems of distribution, whether in town or country, are not touched upon: the subjects dealt with cover only straight long-distance overhead transmission. It is true that, when treating of lightning protection, it is the machinery in the station buildings rather than the line itself that the various devices referred to are intended to protect; and, when considering the most economical system of transmission under given circumstances, a thorough

knowledge of the requirements and possibilities in the arrangement of generating and transforming stations is assumed; but these engineering aspects of a complete scheme of power development are not included in the scope of this book.

In the Appendix will be found reprints of some articles dealing with theoretic aspects of long-distance transmission which, although believed to be of interest to anyone engaged on the design of transmission lines, are not essential to the scheme of the book. In the Appendix will also be found complete specifications for a wood pole and steel tower line respectively: these should be helpful, not so much as models for other specifications —every engineer is at liberty to draw these up in his own way— but rather as containing suggestions and reminders that may be of service when specifying and ordering materials for an actual overhead transmission.

The writer desires to thank the editors of the following technical journals for permission to reprint articles or portions of articles which they have published from time to time: *Electrical World*, New York; *Electrical Times*, London; *Canadian Engineer*, Toronto; *Western Engineering*, San Francisco; *Journal of Electricity, Power, and Gas*, San Francisco.

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CHAPTER I

Introductory and General

An overhead electric power transmission line, consisting as it does of wires stretched between insulators on poles or structures the main purpose of which is to maintain the conductors at a proper distance above the ground level, may appear at first sight to be a very simple piece of engineering work. It is indeed true that the erection of an overhead line of moderate length, capable of giving good service on a comparatively low-pressure system, does not present any insurmountable difficulties to a man of ordinary engineering ability; but whether or not such a line will be the best possible line for the particular duty required of it, depends very much upon the knowledge, skill, and experience of the designer. By the best line should be understood a line which is not only substantially and lastingly constructed, but in connection with which economic considerations have not been overlooked.

It is an easy matter to design a bridge of ample strength for the load it has to carry, or a transmission line with conductors of so large a size, and supports so closely spaced and strong, that the electrical losses will be small and the risk of mechanical failure almost nil; but neither the bridge nor the transmission line will reflect credit on the designing engineer unless he has had before him constantly the commercial aspect of the work entrusted to him, and has so chosen or designed the various parts, and combined these in the completed whole, that all economic requirements are as nearly as possible fulfilled.

In the construction of electrical plant and machinery, such as generators, transformers, and switching apparatus, the economic

conditions are, as it were, automatically fulfilled, owing to the competition between manufacturers, each one of which is a specialist in his own particular line of business. This competition, it should be observed, is not merely in the matter of works cost or selling price, but in works cost *plus* efficiency and durability. It is not necessarily the cheapest nor the most costly manufactured articles that wins in the long run, but the one which is commercially best suited to the needs of the user.

In the lay-out of power plants; in the development of natural power resources and the transmission of electric energy from water falls or coal fields to the industrial centers, the engineer—who may or may not be influenced by possibly conflicting financial interests—has much scope for the reckless and unwise expenditure of other people's money. He must resist the temptation—if temptation it be—and devote himself to the careful study of all engineering problems from the economic standpoint.

The cost per mile of a finished transmission is not all-important. It may frequently be said to be of importance only in so far as it influences the *annual cost* of the line, which annual cost is understood to include interest on the capital sum expended on the line. If a heavy section of copper is used for the conductors, the loss of energy in overcoming resistance will be less than with a lighter section, but the initial cost will be greater: there is only one particular size of conductor which is economically the right size for any given line operating under definite conditions, and this is by no means easy to determine notwithstanding the apparent simplicity of what is usually referred to as Kelvin's Law.

Efficiency of service, which includes reasonably good voltage regulation and freedom from interruptions, must necessarily be merged into the all-important question of cost. By duplicating the transmission line and providing two separate pole lines, preferably on different and widely separated rights of way, insurance is provided against interruption of service over an extended period of time; but whether or not such duplicate lines shall be provided must be decided on purely economic grounds.

Again, lightning arresters may be provided in abundance at both ends of the line and at intermediate points, and assuming—what is not necessarily the case—that such profusion of protective devices will prevent interruptions which are otherwise liable to occur through lightning disturbances, it does not follow that they should be installed. Examples of this kind can be

cited to an almost unlimited extent, and, in the chapter dealing especially with economics, an attempt will be made to indicate a mode of procedure in designing a transmission line from this, the only standpoint of importance to the engineer; but the question is a large one which cannot adequately be dealt with by set rules or formulas. In such cases as the design of supporting structures, when the calculations for strength have been made, it is the designer of the transmission line and not the manufacturer of the steel tower who shall decide upon the factor of safety to be used, for this is the prerogative of the man who is going to be held responsible for the commercial success of the undertaking. If he is incompetent or timid, he will allow too high a factor of safety, or follow blindly in the footsteps of others who may have been equally incompetent or timid. If he is sure of himself, and has carefully checked his calculations and deductions, he may depart from precedent and construct a line which is cheaper—not only in appearance, but in fact—than any line previously constructed under similar conditions and within the same limitations. The usual spacing of wood poles for lines working at medium or low pressures is under 200 ft., yet the Madison River Power Company have, for nearly three years, had in operation a 50,000 volt three-phase line supported on ordinary 8-in. top 45-ft. and 50-ft. poles of Idaho cedar spaced 300 ft. apart, with many spans of 500 ft. and even more. Insulators of the suspension type are used. The designers of this line deserve commendation if the test of time proves their judgment to have been well founded.

Of course the climate and probable weather conditions have an important bearing on the safe span limit and mechanical design of the line generally. The effects of wind and ice will be referred to in Chapter VII.

A knowledge of the country through which a transmission line is to be carried is essential to the proper design of the line and supporting structures. Without a knowledge of the natural obstacles to be reckoned with, including the direction and probable force of wind storms, and whether or not these may occur at times when the wires are coated with ice, the nature of the supports and the economical length of span cannot properly be determined. On the Pacific coast, where there is rarely, if ever, an appreciable deposit of sleet on overhead conductors, it is possible that the spacing of supports may generally be greater than in

countries where the climatic conditions are less favorable. At the same time, it had been observed, in districts where the winters are severe and sleet formation on conductors of frequent occurrence, that the effects of storms in winter on wires heavily weighted with ice, and offering a largely increased surface to the wind, are less severe than in summer when much higher wind velocities are sometimes attained. These examples are here mentioned to emphasize the necessity for a thorough investigation of local conditions before starting upon the detailed design of a proposed transmission line.

There are obviously many preliminary matters to be considered and dealt with before the actual details of design can be proceeded with; but, although many of these are partly, if not wholly, engineering problems, they cannot adequately be dealt with in the limits of this book, or indeed within the limits of any book, since the differences in local conditions, in the scope and commercial aim or end of a transmission system, makes it next to impossible to formulate rules or devise methods of procedure which can be of general utility.

Assuming that it is proposed to transmit energy electrically from a point where the power can be cheaply generated to an industrial or populous center where there is a demand for it, a straight line drawn on the map between these two points will indicate the route which, with possibly slight deviations to avoid great differences in ground level, would require the smallest amount of conductor material and the fewest poles or supporting structures. There may be natural obstacles to the construction of so straight a line, as for instance, lakes that cannot be spanned, or mountains that cannot be climbed; but even the shortest route which natural conditions would render possible is by no means necessarily the best one to adopt. The right of way for the whole or part of the proposed line may have to be purchased, and the cost will often depend upon the route selected. By making a detour which will add to the length of the line, it may be possible to avoid crossing privately owned lands where a high annual payment may be demanded for the right to erect and maintain poles or towers. Again, by paralleling railroads or highways, the advantage of ease of access for construction and maintenance may outweigh the disadvantage of increased length. A slightly circuitous route may take the transmission line near to towns or districts where a demand for power may be expected in the near

future; and it may be wise to take such possibilities into account. The engineer in charge of the preliminary survey work (a section of transmission-line engineering which is not dealt with in this book) should bear all such points in mind and compare the possibilities of alternative routes. On a long and necessarily costly transmission line, it is rarely possible to spend too much time and thought on the preliminary work. Money so spent is usually well spent, and will result in ultimate economies.

Coming now to the problems of a more strictly engineering nature, one of the first things to be decided upon is the system of electric transmission, whether it shall be by continuous currents with its simple two-wire circuit and ideal power factor, or by single- two- or three-phase alternating currents with manifold advantages in respect to pressure transformations and adaptability for use with commutatorless motors, but handicapped by low power factors and other complications due to the inductance and electrostatic capacity of the circuit.

Although nearly all long-distance transmissions—especially on the continent of America—are by three-phase currents, the other systems will be referred to briefly in the following chapter; and since, with the latest improvements in continuous current machinery, the series system of power transmission by continuous currents may, under favorable conditions, hold its own in this country as it does in Europe, an entire chapter will be devoted to a discussion of the points for and against the use of continuous currents on long-distance power transmission lines.

The choice of system and determination of the most economical transmission voltage involve a knowledge of the cost and efficiency of generating and transforming machinery and controlling gear. It is obvious that a system of transmission that appears good owing to the low cost and high efficiency of the line itself, may yet be unsuitable and uneconomical because of the high cost or unsatisfactory nature of the machinery in the generating and receiving stations.

Apart from capital investment and power efficiency, a factor of the greatest importance, almost without exception, is efficiency and continuity of service. At the present time, the weakest link in a power system with long-distance transmission is probably the line itself. Electrical troubles may be due to faulty insulation, or they may have their origin in lightning or switching operations causing high frequency oscillations and abnormally

high voltages, leading to fracture of insulators or breakdown of machinery. Troubles are more likely to be due to mechanical defects, or mechanical injuries sometimes difficult to foresee and guard against. Trees may fall across the line, landslides may occur and overturn supports, or severe floods may wash away pole foundations; and against such possibilities the engineer must, by the exercise of judgment and foresight, endeavor to protect himself. The width of the right of way should depend upon the height of trees, and be so wide that the tallest trees cannot fall across the wires: poles and towers should, if possible, be kept away from the sides of steep hills where the nature of the ground suggests the possibility of falling stones or of landslides; and, in regard to floods, the inhabitants of the districts through which the line passes are usually able to furnish information of use in indicating where trouble from this cause may be expected. Other causes of mechanical failure are storms of exceptional violence, either with or without a heavy coating of ice on the conductors. When strong winds blow across ice-coated wires, the danger is not only that the wires themselves may break, but also that the resulting horizontal loading of the poles or towers may be great enough to break or overturn them.

Faulty mechanical design of the line as a whole, and improper supervision or inspection during construction, will account for many preventable interruptions to service. The transmission line considered as a mechanical structure will be dealt with at some length in Chapters VII and VIII, and the sample specifications for a wood pole and steel tower line respectively (see Appendix), together with detailed material schedules with approximate costs, given in Chapter III, will cover some practical details which should be helpful when designing a transmission line to operate under generally similar conditions.

Although the electrical and mechanical qualities of insulators and conductor materials are necessarily somewhat dependent upon each other, an attempt will be made to deal almost exclusively with electrical calculations and the electrical characteristics of transmission lines in the following chapter, and again in Chapters IV, V, and VI. The chapter treating particularly of the economic aspect of transmission-line design will follow immediately after Chapter II, because it is well to determine provisionally the system and voltage likely to be most suitable for a given scheme, before entering into the more detailed calcu-

lations of line losses and regulation, and considering the practical requirements in matters of insulation and lightning protection.

Before closing this chapter it may be well to refer to a few matters of general interest that are not dealt with in succeeding chapters.

Except for the sample specifications in the Appendix, previously referred to, no attempt has been made to describe complete transmission lines, with details of construction and operation; but the more purely practical details of construction have already been admirably presented by Mr. R. A. Lundquist in his book on Transmission Line Construction, to which the reader is referred. There is also an excellent article by Mr. L. J. Riter describing the transmission lines of the Utah Light and Railway Company, which was published in the *Electrical World* of Aug. 3, 1912.

In the matter of crossing highways or railroads with high-tension conductors, the engineer will usually have to abide by the rules and regulations of the local authority or railroad company, however unreasonable or unnecessary the particular requirements may be. The specification (Section No. 4) forming part of the report of the Committee on overhead line construction, read before the National Electric Light Association in New York City in 1911, appears to the writer reasonable and sound. The modern tendency is to avoid a multiplicity of devices intended to catch falling wires or ground them in the event of a breakage, and to rely mainly on short spans and exceptionally sound mechanical construction at places where the falling of charged wires would be a menace to life, and cause an interruption to traffic.

Reverting again to the all-important matter of uninterrupted service, it is obvious that reasonable provision should be made for the early resumption of service in the event of a stoppage. When two parallel lines are run all the way between generating and receiving stations, it is usual to provide section switches and by-pass connections about every 20 miles, at the points where patrolmen are stationed. Even if the line is not duplicated, it is usually wise, on important systems, to have patrolmen's houses every 15 or 20 miles, depending on the character of the country. Emergency houses, containing tools and sundry line materials such as wire and spare insulators, should be provided midway between the patrol houses.

For communication between generating station and patrolmen

and substations, a telephone circuit is almost essential. This can be run on the same poles or towers as the high-tension conductors; but, if possible, it should be run on separate poles. The extra cost of a separate pole line for telephone wires is, however, the reason that these wires are very frequently supported on the same poles as the transmission wires. This leads to trouble in the case of a ground on the high-tension wires and, indeed, almost invariably when there is a fault on the power system, except, of course, when the power service is entirely interrupted. Even when carried on a separate pole line, the telephone circuit is liable to be useless at times when it is most needed, and for this reason it is not unusual, on important lines, to provide telegraphic instruments in addition to the telephones, and men that are telegraph operators at the ends of the line and also at any intermediate points where switching stations may be provided.

As a protection against inductive troubles in neighboring parallel circuits, the wires of a transmission system should be *transposed* at regular intervals. A complete transposition of a three-phase line in a given distance is obtained by making each wire occupy the same position relatively to the ground and to the supporting structures over one-third of the distance. Where interference with other electric systems is to be feared, such transpositions of the power conductors have to be made. On a long line, a complete transposition every 10 miles should be sufficient; but, as a matter of fact, the present tendency is toward the avoidance of transpositions of the power conductors, particular attention being paid to the insulation and proper arrangement and transposition of the telephone wires. When these are carried on steel towers with spans of about 500 ft., there should be a half transposition at every tower, suitable insulating spacing pieces being provided, if necessary, to keep the two wires apart in the center of the span. On transmission lines of lower voltage, carried on wood poles spaced about 150 ft. apart, the transpositions of telephone wires are frequently made at every fourth or fifth pole; but it is generally more satisfactory to transpose at every pole, and this can be done without any appreciable increase in cost.

Closely related to the matter of continuous and efficient service is the question of duplication of lines, already referred to. This has to be decided mainly on economic grounds; but, at the same time, the purely engineering difficulties may become very

serious if an attempt is made to transmit very large amounts of power on a single set of conductors. It is not possible to lay down definite rules as to the practical limit for a single line; this will depend on the distance and voltage, and therefore on the current to be carried per conductor; but if the power to be transmitted exceeds 20,000 kw., it would generally be wise to duplicate the conductors, even if carried on the same set of supporting towers; and if the total power exceeds 40,000 kw., two separate tower lines, either on the same right-of-way or (preferably) following different routes, will, in most cases, lead to ultimate economy.

The introduction of automatic switches and similar devices designed to save labor and ensure the rapid changing over of the load from a faulty section to a sound section on a duplicated transmission line, is liable to lead to unlooked-for troubles; and even the generous provision of lightning arresters, especially on the extra-high-tension lines, is not necessarily good policy. Simplicity, and the avoidance of unnecessary joints, rubbing contacts (as in switches or cut-outs), fuses in the stations, and spark gaps or arresters along the line, should generally be aimed at; but there will always be exceptions to such rules.

Careful design in the matters of material, size and spacing of conductors, and in methods of support and insulation, together with scientific selection and lay-out of poles or towers, will lead to the construction of transmission lines which may ultimately prove to be the strongest instead of the weakest links in power transmission schemes; and this without the addition of more or less complicated and unreliable automatic and so-called protective devices, and at a cost which will make long-distance power transmission propositions more attractive from the stockholder's point of view than they have been in the past.

CHAPTER II

Principles and Theory—Elementary

The purpose of an electric transmission line is to transmit energy from one place to another; and it is the engineer's business to design and construct such a line to fulfil its purpose in the best and most economical manner.

The system to be adopted will affect the design of the generating plant and of the motors or other devices through which the electric energy at the receiving end of the line is converted for industrial purposes or public utility; but, in this chapter, references to alternative systems will be made only for the purpose of comparing them in the matter of line efficiency.

1. Losses in Transmission.—The principal cause of loss of power in a transmission line is the resistance of the conductors. For a given section of conductor, the power dissipated in the form of heat, in overcoming the ohmic resistance, is proportional to the square of the current. A definite amount of power can therefore be transmitted with less loss when the voltage is high than when it is low; but, on each particular transmission, there is a limit to the pressure beyond which there is nothing to be gained in the matter of economy. This limit is determined by the cost of generating and transforming apparatus (which will be greater for the higher voltages), by the greater cost of insulators and of the line generally—owing to the larger spacing required between wires—and also, when extra high pressures are reached, by the fact that the power dissipated is no longer confined to the I^2R losses in the conductors, but occurs also in the form of leakage current over insulators, and in the air surrounding the conductors. The means of calculating the dielectric losses will be explained in Chapter V, when treating of *corona* formation; and a method for determining the best voltage of transmission under any given conditions, will be outlined in Chapter III, because this is essentially an economic problem. For the present it is assumed that a total amount of power amounting to W watts has to be transmitted over conductors of known resistance; and losses through leakage or corona will be considered negligible.

2. Transmission by Continuous Currents.—If E is the voltage between the outward and return wires at generating end, the current is

$$I=\frac{W}{E}$$

and the losses in transmission are,

$$2r\times I^2$$

where r is the resistance of one conductor only.

The fact that continuous currents are not extensively used for the transmission of power to a distance is due mainly to the difficulty of providing sufficiently high pressures to render such transmission economical, and also to the necessity for using rotary machines with commutators to convert the transmitted energy into convenient form at the distant end of the line. The modern aspect of long-distance transmission by means of continuous currents will, however, be dealt with at some length in Chapter VI.

3. Transmission by Single-phase Alternating Currents.—The advantage that alternating currents have over direct currents is in the ease with which pressure transformations can be effected by means of static converters. On a constant-potential system, the distribution of power in scattered districts, at any voltage desired by the consumer, is a very simple matter.

In a single-phase two-wire transmission, the conditions would be similar to those of a direct-current transmission if not only the load, but the line also, could be considered as being without inductance or electrostatic capacity. The current and the line losses would be the same as if the transmission were by continuous instead of alternating currents.

In practice the inductance must always be reckoned with where alternating currents are used; this inductance is not only that introduced by the load (usually consisting in large part of induction motors), but is partly in the line itself, owing to the loop formed by the outward and return conductors. The charging current due to the capacity of the line is of less account on low-voltage transmissions, but becomes of considerable importance on long lines working at high pressures. The effects of inductance and capacity will be explained later.

Another difference between alternating and continuous cur-

rents is the fact that an alternating current has the effect of apparently increasing the resistance of the conductor: this is due to the uneven distribution of the current over the cross-section of the conductor. A small percentage of the alternating flux of induction is in the material of the conductor itself, and this generates counter e.m.fs. which are somewhat greater near the center of the wire than at the circumference, the result being that the current density becomes greater near the surface of the wire than in the center portions. This phenomenon is known as the *skin effect.* The additional resistance offered to the passage of alternating currents, and the correspondingly increased I^2R losses are, however, small and generally of negligible amount on low frequencies, unless the cross-section of the conductor is very large: it is with the high frequencies that this effect becomes of importance. Means of calculating increase of resistance due to skin effect will be given in Chapter IV.

4. Transmission by Two-phase Currents.—If four separate wires are run from generating to receiving station, as indicated

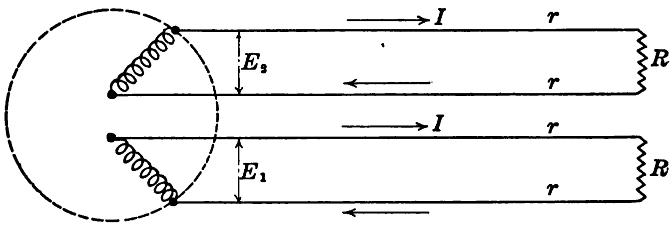

FIG. 1.—Two-phase transmission with four wires.

in Fig. 1, and if the load, R, is the same on both circuits, the total power transmitted, on the assumption of negligible inductance, is

$$\begin{aligned} W &= 2E \times I \\ &= 2 \times I^2 \times (R + 2r) \end{aligned}$$

where E stands for the terminal voltage E_1 or E_2 of either section; these pressures being assumed equal. The pressure lost in transmission is $2r \times I$, and the watts lost are $2(2r \times I^2)$; the system being simply a transmission of energy by means of two independent single-phase circuits. It will be seen, however, that, by combining two of the conductors to form a common return path for the current, the transmission of two-phase currents can be effected with only three wires, as indicated in Fig. 2. The

vector diagram for such a system of transmission is easy to construct *if it is permissible to assume the resistance r' of the common conductor to be negligible;* the current relations, with a quarter period time displacement between the currents in the

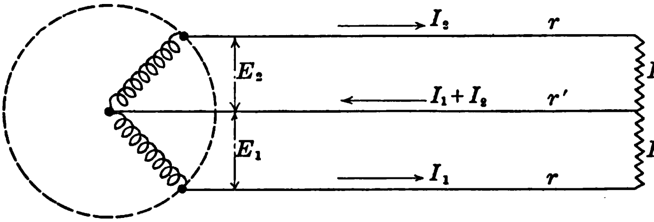

FIG. 2.—Two-phase transmission with three wires.

two phases, being as indicated in Fig. 3. The current I_3 in the common conductor is the vectorial sum of the currents I_1 and I_2. It has been drawn equal but opposite to the dotted resultant OA because it is generally convenient to assume the direction of all

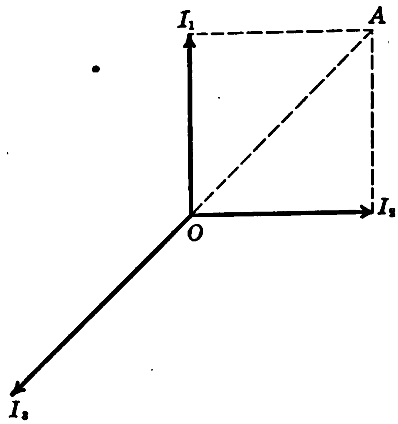

FIG. 3.—Vector diagram of currents in two-phase three-wire transmission.

currents to be positive when flowing away from the source of supply, in which case the condition

$$I_1+I_2+I_3=O$$

must be satisfied. The arrow on the central conductor in Fig. 2 indicates a flow of current *opposite* to the currents in the other two wires; but this is done merely to suggest the idea of currents going away from the source of supply by the two outer wires,

and *returning* by the common wire. When dealing with polyphase currents, such mental pictures of the actual physical occurrences are liable to be misleading, and they should be used sparingly.

On a long transmission, or in cases where the resistance of the common conductor cannot be neglected, the problem of two-phase transmission by means of only three wires becomes complicated. The resistance of the common wire has the effect of disturbing the phase relations, which are no longer the same at the receiving end as at the generating end of the line. It is difficult to explain in simple terms exactly how this occurs; but in Appendix C, at the end of this book, the peculiar effect of the common wire is discussed. As a matter of fact, there are few long-distance transmissions by two-phase currents, and on those that are in successful operation, four conductors are used. The advantage of three wires over four is, of course, in the saving of cost on the conductors. Instead of requiring four wires, each carrying I amperes, it is only necessary to provide two wires to carry I amperes and one wire of a cross-section sufficient to carry, not $2I$ amperes, but $\sqrt{I^2+I^2}$ or $\sqrt{2}I$ amperes; which leads to an appreciable saving in the total weight of conductors, if the current density is the same in all.

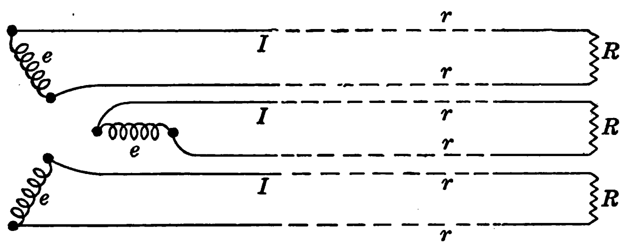

FIG. 4.—Three-phase transmission with six wires.

5. Transmission by Three-phase Currents.—If six separate conductors are run from generating to receiving station, as indicated in Fig. 4, the transmission is equivalent to three independent single-phase two-wire circuits; and if e is the potential difference at the terminals of each circuit, and I the current in each wire, the total power transmitted will be

$$W=3(e\times I)$$

or

$$W=3\times I^2\times(R+2r)$$

the assumption being, as in previous cases, that both inductance and capacity are of negligible amount.

The pressure lost in transmission will be $2r \times I$ and the total power lost in the three lines will be $3 \times I^2 \times 2r$.

Consider now the arrangement as in Fig. 5, where the three circuits have a common terminal at each end of the transmission

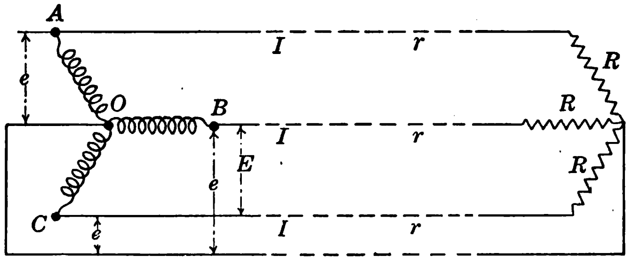

FIG. 5.—Three-phase transmission with three wires.

and three of the wires of the six-wire transmission are replaced by a common return conductor. The pressure at the generating end, between each of the three terminals of the alternator and the common return, or neutral point, is still e volts; and the total power transmitted is still $W = 3(e \times I)$; but, owing to the fact that the sum of the three outgoing currents is zero (since they differ in phase by 120 time degrees, as shown in Fig. 6, and any one current, such as OB, is exactly equal and

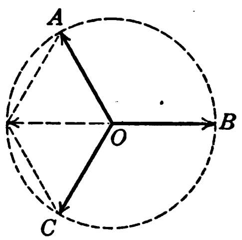

FIG. 6.—Vector diagram of currents in three-phase transmission.

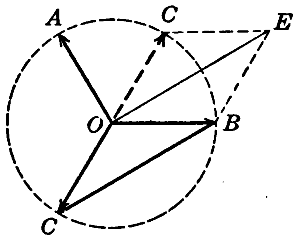

FIG. 7.—Vector diagram of e.m.fs. in three-phase star-connected system.

opposite to the resultant of the other two currents), there will be no current flowing in the common return conductor, which can therefore be omitted; and it follows that both pressure drop and I^2r losses in the lines are reduced to *one-half* of what they were with the arrangement of three separate circuits; the power

loss in the lines being now $3I^2r$. This clearly shows how the transmission by three-phase currents is more economical as regards line losses than single-phase transmission. But it must not be overlooked that, in order to obtain a reduction by half of the weight of copper in the lines, the pressure between the wires is greater on a three-phase system than on a single-phase system transmitting the same amount of power. Thus, the pressure, E (Fig. 5), between any two of the three transmission wires is equal to $e\times\sqrt{3}$, as shown by the diagram Fig. 7. Here the e.m.fs. in the three sections of the alternator windings are represented by the vectors OA, OB, and OC; and since the e.m.f., E, between any two terminals, such as B and C (Fig. 5), is the resultant of the e.m.fs. acting in the two windings OB and OC connected in series, one of these (as OC) must be subtracted from the other (OB). Thus the resultant is the vector OE (Fig. 7), obtained by *adding* to the vector OB an imaginary vector, OC', exactly equal, but opposite, to OC. This resultant is evidently equal and parallel to the line CB, joining the ends of the two vectors OB and OC, and it can be shown to be exactly $\sqrt{3}$ times greater than either of these vectors. Thus,

$$E=1.732\,e$$

The *power* of a three-phase circuit, which is three times $e\times I$, can evidently also be written

$$W=3\left(\frac{E}{\sqrt{3}}\times I\right)$$

or,

$$W=\sqrt{3}E\times I$$

where E is the pressure between any two of the three wires.*

6. Relative Cost of Conductors Required on the Various Systems.—Apart from all questions of voltage, or necessary insulation and spacing required between adjacent conductors and between the conductors and the supporting structures, the total I^2r losses will be the sum of the losses occurring in each conductor of the transmission system. Each wire can be considered as the outgoing conductor of a two-wire single-phase system in which the return wire has no resistance. Thus, in a balanced three-phase system as described in Fig. 5, wherein

*The Power Factor (cos. θ) does not appear in this formula because, owing to the assumed absence of inductance or capacity, it is equal to unity.

the common return wire is not required, since it carries no current, the total losses in the transmission wires are

$$3(I^2 \times r)$$

But this can be written,

$$(3I)^2 \times \frac{r}{3}$$

which shows that the total losses can be calculated by adding the currents in the respective conductors regardless of phase relations, and considering this total current as being transmitted over a single wire of the same weight or cross-section as would be obtained by connecting the individual conductors in parallel. This applies to any polyphase system with wires of equal resistance carrying equal amounts of current.

When comparing different systems of transmission, it is necessary to make some assumptions in regard to the voltage, so as to have a common basis of comparison. For instance, if it is desired to compare three-phase and single-phase transmission on the basis of *the same potential difference between wires*, apart from any question of voltage between wires and ground, the total power on the three-phase system will be

$$W = \sqrt{3} E \times I$$

and the (equal) power on the single-phase system would be written,

$$W = E \times (\sqrt{3} I)$$

Let r_3 = the resistance of each conductor on the three-phase system, and

r_1 = the resistance of each conductor on the single-phase system,

then, for equal total line losses,

$$3I^2 r_3 = 2(\sqrt{3} I)^2 r_1$$

and

$$r_3 = 2r_1$$

The weight of copper in the single-phase line is proportional to $2 \times \frac{I}{r_1}$, while in the three-phase line the weight is proportional to $3 \times \frac{I}{r_3}$.

Hence, the ratio,

$$\frac{\text{Weight of copper, single-phase}}{\text{Weight of copper, three-phase}} = \frac{2r_3}{3r_1}$$
$$= \frac{2(2r_1)}{3r_1}$$
$$= \frac{4}{3}$$

which indicates a saving of 25 per cent. of conductor material in favor of the three-phase system.

Consider now the condition of the various systems on the basis of the same efficiency (as in the above example), but on the further assumption that the potential difference between the earth or supporting structures and any one of the conductors is constant. This is equivalent to stating that the pressure stress

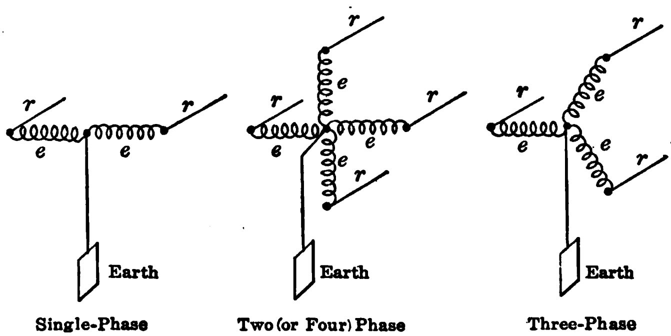

FIG. 8.—Different systems with same potential above ground.

at every point of support, where an insulator carries the conductor, is the same on all systems. This is shown diagrammatically in Fig. 8, where the voltage e, *per leg* is the same in all systems.

If I is the current per leg, and n the number of legs (or phases), the total power transmitted is

$$W = eI \times n$$

provided the power factor is unity. On the assumption of a balanced load, with the current lagging behind the voltage by the same number of time-degrees on each leg of all the systems, no complication will arise if the power factor of the load is taken into account. The total power transmitted, in every case, can, therefore, be written

$$W = eI \cos \theta \times n.$$

If r is the resistance of each line conductor, the total line loss for any system will be

$$w = I^2 r \times n$$

and, for the same line efficiency, the weight of copper per kilowatt transmitted will evidently be the same in all cases.

This leads to the conclusion that, *for any polyphase system*, the power lost in the line depends only upon the joint resistance of the conductors, the power transmitted, and the power factor, *provided the pressure between the conductors and the neutral point is constant.*

Still neglecting the inductance and capacity of the line itself, the *percentage power lost in transmission is*

$$\frac{w}{W} \times 100$$

If the loss w be expressed in terms of the total power W, it will be found that this ratio can be put in an interesting form. The symbol R_p will be used to denote the joint resistance of all the conductors in parallel; that is to say,

$$R_p = \frac{r}{n}$$

The power lost is

$$\begin{aligned} w &= nI^2 r \\ &= n^2 I^2 R_p \end{aligned}$$

but for n^2 can be substituted its equivalent value

$$n^2 = \frac{W^2}{e^2 I^2 (\cos\theta)^2}$$

hence,

$$w = \frac{W^2 R_p}{e^2 (\cos\theta)^2}$$

which shows how, for any given amount of power transmitted at a given pressure, the I^2R loss is directly proportional to the joint resistance of all the conductors, and inversely proportional to the square of the power factor of the load.

By substituting this value for w in the ratio for percentage efficiency, the latter quantity becomes,

$$\left.\begin{array}{r}\text{Percentage power lost in } \textit{any} \\ \text{balanced polyphase system}\end{array}\right\} = \frac{W R_p}{e^2 \cos^2\theta} \times 100$$

These formulas show very clearly the advantages of high power factors where economy of transmission is of importance.

7. Grounding the Neutral on High-Tension Overhead Transmissions.—The above comparisons of losses have been made on the assumption of a grounded neutral, but whether this common terminal of the polyphase circuit is grounded or not does not alter the fact that, *under normal conditions of working*, the neutral point is usually at about the same potential as the ground. Further, whether the generator or transformer windings on the high-tension side of a three-phase transmission system are delta connected or star connected is of very little importance if it is decided to run without a grounded point. The fact that the neutral point on a star-connected system is available for gounding purposes does not mean that it must necessarily be grounded.

The chief arguments in favor of grounding the neutral are (1) that the difference of potential between any conductor and the supporting structure or earth remains unaltered, and cannot become excessive in the event of the grounding of a high-tension conductor, and (2) that it is possible to detect instantly, and disconnect by automatic devices or otherwise, any portion of the system that may become accidentally grounded. The chief objection is that under such conditions, the grounding of any one conductor causes a short-circuit, and even if disconnected by the opening of a switch, leads to an interruption of supply. By inserting a resistance between the neutral and the ground connection, the current through the fault can be limited to just so large an amount as may be necessary to operate an automatic device, or give an indication that there is a fault on the line. Instead of opening the switches and disconnecting the line, the ground connection to neutral may be opened, thus leaving the conductor grounded until such time as it is convenient to carry out repairs; but this would be equivalent to running normally without grounded neutral. The chief advantage of transmitting with ungrounded neutral is that the grounding of one conductor only does not lead to immediate interruption of service. The chief disadvantage is probably the fact that the potential between earth and the other conductors is immediately raised; being $\sqrt{3}$ times greater, in the case of a three-phase transmission, than under normal conditions, when the voltages are balanced.

It is doubtful whether the question of grounding the neutral on a high-tension transmission can be so settled as to be applicable to all systems and voltages. A few years ago there was an

undoubted tendency on the part of engineers to ground all metal that would under normal conditions be at ground potential. At the present time the tendency appears to be in the direction of providing substantial insulation throughout the system, and omitting the grounding of the neutral. In an age when the individual appears to distrust the conclusions of his own intellect, there would appear to be much wisdom in the advice once given by Dr. Steinmetz, who suggested that when the engineer is in doubt as to the better course to pursue when two alternatives present themselves, he should *not* follow the one most favored by his fellow engineers, because in so doing he would in all probability merely adopt what happens to be the fad of the day. It is perhaps best practice to avoid grounding any point of a high-tension transmission unless the conditions are such that the grounding of the neutral point would appear to be the obvious remedy for troubles that may have been experienced or that are liable to occur.

8. Effect of Line Inductance on Transmission of Alternating Currents.—On account of the necessary spacing between the wires, the loops formed between outgoing and return conductors are of considerable area on a long-distance transmission; and the changing flux of induction in these loops will generate counter e.m.fs. in the conductors, which may be of considerable importance, especially in regard to their effect on the voltage regulation. Whether dealing with single-phase or polyphase transmissions, it will be found convenient to make calculations on single conductors only. Thus, instead of considering the resistance of the *complete circuit* (which is not convenient in the case of polyphase transmissions), the resistance of one conductor only, or the resistance *per mile* of single conductor is considered, and the ohmic voltage drop calculated for that portion of the complete circuit only. Similarly, in the matter of the counter e.m.f. due to the self-induction of the line, calculations are based, not on the total flux of induction in the loop or loops formed by outward-going and return wires, but on that portion of the total flux which is included between the center line of any one conductor and the *neutral* plane or line. Thus the induced volts per single conductor, or per mile of single conductor can be calculated, and the resulting total voltage drop can be computed for each conductor independently of the others. In the case of a single-phase two-wire transmission, the total loss of pressure is evidently just twice the amount so arrived at for a single conductor. In a

polyphase transmission, due attention has to be paid to the phase relations between the currents in the various conductors; but the same principle holds good, and calculations of any polyphase transmission can be made by considering each conductor separately, as will be explained later.

The induced volts will be directly proportional to the current, and will depend on the diameter of the wire and its distance from the return conductors. This will be again referred to in Chapter IV, but for the present the induced pressure can be calculated by means of the following approximate formula:

$$\left.\begin{matrix}\text{Volts induced per mile}\\ \text{of single conductor}\end{matrix}\right\} = 0.004656 \times f \times I \times \log \frac{D}{r} \quad \ldots\ldots\ldots (1)$$

where D and r stand respectively for the distance between outward and return (parallel) conductors and the radius or half diameter of the wire; these being expressed in the same units. The frequency f is expressed in cycles per second, and the current I, in amperes. In nearly all pocket books or hand books for the use of electrical engineers, tables are published giving inductive pressure drop for different diameters and spacings of wires; the assumption being always, as in the case of formula (1), that the current variation is in accordance with the simple harmonic law (sine wave).

9. Fundamental Vector Diagram for Line Calculations: Capacity Neglected.—In the diagram Fig. 9, the various quantities are represented as follows:

$O\,A$, or (I), is the current vector.

$O\,B$, or (E), is the vector corresponding to the pressure (wire to neutral) at the receiving end.

θ is the time angle by which the current lags behind the pressure at receiving end: $\cos\theta$ being the power factor of the load.

$B\,C$, or (E_R), which is drawn parallel to $O\,A$, is the quantity $I\,R$; being the voltage component required at the generating end to compensate for ohmic drop of pressure in the conductor.

$C\,D$, or (E_L), which is drawn at right angles to $O\,A$, is the quantity calculated by formula (I), being the voltage component required at generating end to compensate for loss of pressure due to the self induction of the conductor.

$B\ D$ is the sum of the vectors $B\ C$ and $C\ D$; being the total additional voltage required at the generating end to compensate for the impedence of the conductor.

$O\ D$, or (V), is the vector corresponding to the pressure (wire to neutral) at generator end of the line, required to maintain the pressure (E) at the receiving end when the current in the conductor is I amperes.

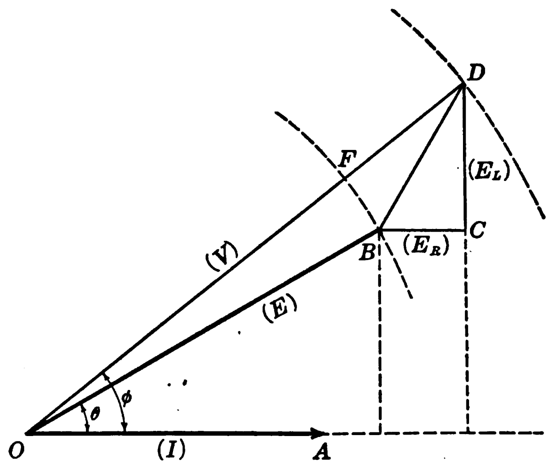

FIG. 9.—Vector diagram for line calculations—capacity neglected.

φ is the time angle by which the current lags behind the pressure at the generating end; cos φ being the power factor of the total load as measured at generating end.

$F\ D$ is the (arithmetical) difference between pressures at receiving and generating ends of the conductor. The *percentage* loss of pressure being

$$100 \times \frac{\text{length } FD}{\text{length } OF}$$
$$= 100 \times \frac{\text{length } FD}{\text{length } OB}$$

(The dotted circles being described from the center O.)

All graphical solutions of transmission line problems are based on this fundamental diagram. Some of them give results that are theoretically correct, while in others certain assumptions

are made to simplify the construction without introducing any appreciable error in the solution of practical problems.

Graphical and semi-graphical methods of making transmission line calculations are very convenient. Such methods have been proposed by Messrs. F. A. C. Perrine and F. G. Baum, by Prof. L. A. Herdt, by Mr. R. D. Mershon and others. A convenient diagram for determining the regulation of transmission lines, as used by the writer, will be explained in Chapter IV.

For those who prefer to use tables of trigonometrical functions, the required relations can easily be obtained from Fig. 9.

In the first place, the functions of the angle φ are:

$$\sin \varphi = \frac{E_L + E \sin \theta}{V} \tag{2}$$

$$\cos \varphi = \frac{E_R + E \cos \theta}{V} \tag{3}$$

$$\tan \varphi = \frac{E_L + E \sin \theta}{E_R + E \cos \theta} \tag{4}$$

From formula (3) it is seen that the required voltage at generating end is,

$$V = \frac{E_R + E \cos 6}{\cos \varphi} \tag{5}$$

and the volts required to overcome ohmic resistance are:

$$E_R = V \cos \varphi - E \cos \theta \tag{6}$$

As an example of the use of these formulas assume, in the first place, that the *material*, *size* and *spacing* of conductors is known, and also—in all cases—the power factor of the load ($\cos \theta$), and therefore the other trigonometrical functions of the angle θ, such as $\sin \theta$. Under these conditions, the quantities E_R and E_L can readily be calculated, and formula (4) can be used to obtain $\tan \varphi$; thence the angle φ and $\cos \varphi$ (the power factor at generating end). Then, by formula (5), the required voltage (V) at generating end is easily obtained.

Assume, in the second place, that the size of the conductors has to be determined. The spacing of conductors and the frequency being known, the induced volts E_L can be calculated approximately by estimating the value of r for use in formula (1). This estimate of the size of conductor will be based on the *required regulation*, or total voltage drop.

$$V = E + \text{allowable voltage drop}$$

now, since E_R is not definitely known, formula (2) will have to be used. This gives the value of sin φ with a sufficient degree of accuracy even if quite an appreciable error has been made in estimating the size of conductor for the purpose of calculating the induced volts (E_L). Having determined the angle φ, the function cos φ can be obtained from trigonometrical tables; and then, by using formula (6), the ohmic drop can be calculated. Thus the proper size of wire for use under given conditions may be calculated.

If the *power loss* in the line is the determining quantity, regardless of the voltage regulation, then, since this loss depends only on the voltage E_R (the current, I, being constant), the resistance and size of conductor is readily ascertained, and the unknown quantities would be calculated as in the first case considered.

10. Effect of Capacity on Regulation and Line Losses.—Although the effects of electrostatic capacity will be referred to again in Chapter IV, it will be well to consider briefly how the capacity on long lines may affect the voltage regulation and line losses.

Any arrangement of two conductors of electricity separated by an insulator, forms a condenser, of which the capacity will be large or small depending upon the nearness or otherwise of the conductors, and the nature of the dielectric between them. In the case of overhead conductors running parallel to each other and to the surface of the ground over a considerable distance, the electrostatic capacity between the individual conductors, and between these conductors and earth, becomes a matter of importance.

As in the case of inductance calculations, it is advisable, whenever possible, to consider the capacity of any one conductor as measured between the conductor and the neutral surface or neutral line. Thus the capacity current per conductor can be calculated independently of the current in the other conductors. Of course, in such cases, the potential difference causing the flow of current in and out of the condenser must be measured between the conductor and the neutral, and not between outgoing and return conductors. In the case of a transmission line, the capacity is *distributed* over the whole length of the line. It is incorrect to assume that the whole of this capacity is concentrated at either end; but, for the sake of simplicity, the total capacity will be supposed to be concentrated at the receiving

end of the line, and a correction will afterward be made in order to conform more nearly to actual conditions.

The following approximate formula may be used for calculating the capacity of overhead lines:

$$\left.\begin{array}{r}\text{Capacity in microfarads per}\\ \text{mile, between conductors}\\ \text{and neutral}\end{array}\right\} = C_m = \frac{0.0388}{\log \frac{D}{r}} \tag{7}$$

where D and r are the spacing between conductors and the radius of cross-section, exactly as in formula (1) for the calculation of the induced volts.

The charging current, in amperes, on the sine-wave assumption, can be calculated by the formula

$$I_c = 2\pi f C_m l V_n \times 10^{-6} \tag{8}$$

where l is the distance of transmission in miles, and V_n is the voltage as measured between the conductor and neutral. This charging current (I_c) is always a quarter period in advance of the voltage. This explains why, on a long line open at the distant end, or only very lightly loaded, there can be a *rise of pressure* at the receiving end of the line.

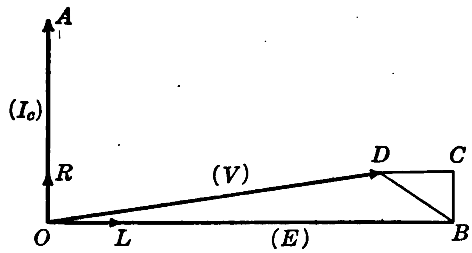

FIG. 10.—Vector diagram showing pressure rise due to capacity.

In the diagram Fig. 10, the pressure at the receiving end is represented by the vector $O\ B$, while $O\ A$, drawn at right angles to $O\ B$—in the forward direction—is the capacity current as calculated by formula (8). It is assumed that the load is entirely disconnected, and the current I_c is the total current on the line. The voltage component required at generating end to overcome ohmic resistance is $O\ R$, or $B\ C$, in phase with I_c, and the component required to balance the e.m.f. of self-induction, as calculated by formula (1), is $C\ D$, drawn 90 degrees *in advance*

of $O\,A$. The pressure required at generating end is $O\,D$, which may be *smaller* than $O\,B$. It is true that the capacity has been assumed to be concentrated at the receiving end of the line; but with distributed capacity, the same effect of a *rise* in pressure as the distance from generating end increases, will occur. It will be seen that this is due to the e.m.f. of self-induction of the charging current being in phase with the impressed voltage. If the lines were without inductance, there could be no pressure rise.

The effect of capacity on the line when the distant end is closed on the load, will depend upon the amount and nature of the load. If the load is heavy and largely inductive, the current put into the line at the generating end will be *less* than the load current, and the I^2R losses will therefore be smaller than if the line was without capacity.

At light loads, especially if the power factor is high, the line losses will be greater than if capacity were not present. On many overhead lines the effects of capacity are almost negligible; but on long high voltage lines, these effects become of great importance. Some idea of the magnitude of capacity effects on long lines may

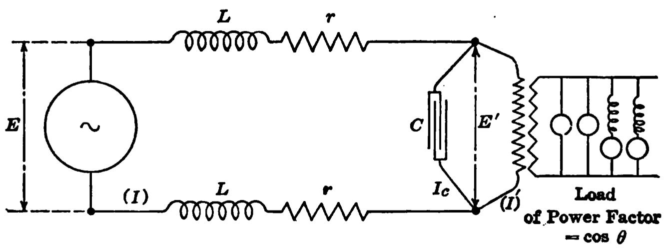

FIG. 11.—Transmission line with concentrated capacity.

be gathered from the statement that the Southern Power Company has 7000 kilovolt amperes (or apparent kilowatts) on its 100,000 volt lines when all the switches at the receiving end are open.

The effects of capacity under various conditions are best studied by constructing vector diagrams.

In Fig. 11, the current, I, is delivered to the line at the pressure E: each conductor has inductance L and resistance r, giving a pressure E' at the distant end, where the whole of the capacity, C, is supposed to be shunted across the wires. The load current is I'.

In the vector diagram, Fig. 12 (which may with advantage be compared with Fig. 9), *O B* and *O A* represent respectively the potential difference and current at the receiving end. The impressed voltage at the terminals of the imaginary condenser *C* (Fig. 11), will therefore be E' volts, and the vector for the condenser current must be drawn 90 degrees in advance of *O B*; this is the vector *O N*. The total current put into the line at the generating end will be *O M*, which is the (vectorial)

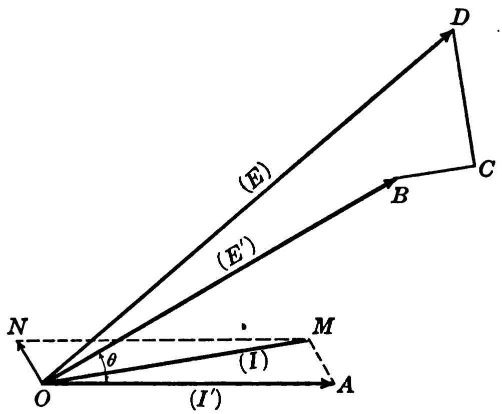

FIG. 12.—Vector diagram for transmission line of appreciable capacity.

sum of the currents I' and I_c. The pressure at generating end is made up of three components:

O B, the pressure available at receiving end;
B C, the pressure required to overcome resistance (drawn parallel to the total-current vector *O M*);
C D, the pressure required to counteract the e.m.f. of self-induction (drawn at right angles to *O M*).

By varying the angle θ and the length of the current vector *O A*, the effect of the capacity current with different power factors and loads can easily be studied.

This method of correcting the fundamental diagram to take account of capacity, is not theoretically accurate, because the capacity is never concentrated at one point of the line; but distributed over the whole distance of transmission. A correction, as used by the writer, which gives a close enough approximation for practical purposes, is explained in Fig. 13.

In this diagram, the vectors $O\,A$ and $O\,B$ and the angle θ have the same meaning as in the fundamental diagram, Fig. 9. They stand respectively for the current and pressure and time angle between these quantities at the receiving end of the line. The capacity current, calculated by formula (8), is represented by the vector $O\,N$, shown 90 time degrees in advance of the volts $O\,B$, which would be correct if the capacity were all concentrated at the receiving end. From A drop the perpendicular $A\,P$ on $O\,B$, and make $A\,M$ equal to $O\,N$, then join O to the point S midway between A and M. By using $O\,S$, lagging θ_1, time degrees behind the pressure vector, in place of $O\,A$, lagging

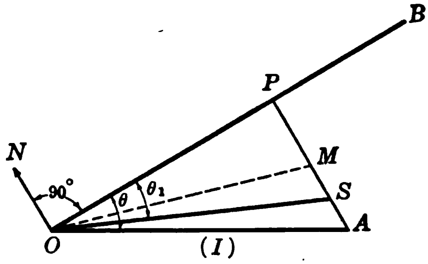

FIG. 13.—Vector diagram giving approximate correction for distributed capacity.

θ time degrees behind the same vector, the construction can be proceeded with as in Fig. 9, and the result will include a correction for capacity current which, although far from being theoretically satisfactory, will give, as a rule, a close enough approximation. This correction simply amounts to the assumption that, instead of the capacity, C, being distributed along the line, a capacity equal to $\frac{C}{2}$ is concentrated at the distant end.

If it is desired to avoid the graphical construction to obtain the corrected current vector $O\,S$ and the angle θ_1, trigonometrical tables can be used. Thus:

$$\tan\theta_1 = \frac{\text{length } P\,S}{\text{length } O\,P}$$

$$= \frac{P\,A}{O\,P} - \frac{S\,A}{O\,P}$$

$$= \tan\theta - \frac{\text{half capacity current}}{\text{load current} \times \cos\theta}$$

from which the other functions of the angle θ_1, together with the length of the vector $O\ S$ (giving the current value to use in calculations in place of the load current $O\ A$) can readily be obtained.

11. Use of Fundamental Diagram for Three-phase Calculations.—The vector diagram, Fig. 14, shows the relative phases of current and e.m.f. for a three-phase system with balanced load

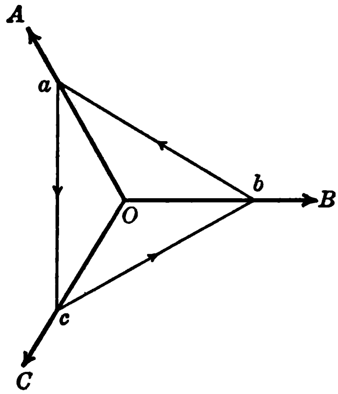

FIG. 14.—Vector diagram for three-phase system on non-inductive load.

when the power factor is unity. Here the three current vectors are $O\ A$, $O\ B$, and $O\ C$. The "star" voltages are,

$$O\ a = O\ b = O\ c = e$$

each being in phase with the corresponding line current; and the voltages measured between the three conductors of the transmission line are,

$$a\ b = b\ c = c\ a = E = \sqrt{3}\ e$$

The total power transmitted is,

$$\begin{aligned} W &= \sqrt{3}E \times I \\ &= 3\ e \times I \\ &= 3\ (O\ A \times O\ a) \end{aligned}$$

In Fig. 15 the diagram has been drawn for an inductive load. Here there is a certain displacement of the current phases relatively to the e.m.f. phases. It will be noticed that the vertices of the e.m.f. triangle no longer lie on the current lines as in the previous diagram. The three current vectors still subtend

the same angle of 120 degrees with each other; but they have been moved bodily round (in the direction of retardation) through an angle θ. The total power is evidently no longer equal to three times $O\ A \times O\ a$, but to $3 \times O\ A' \times O\ a$, where $O\ A'$ is the projection of $O\ A$ on $O\ a$; and cos θ is the power factor of the three-phase load.

It is a simple matter to complete this diagram by taking into account the effects of resistance and inductance in the line, because when the calculations of resistance drop and induced volts are made *per conductor* as previously explained, the construction can be carried out for each phase exactly as explained when describing the fundamental diagram (Article 9). It is only necessary to bear in mind that $O\ A$ and $O\ a$, in Fig. 15, cor-

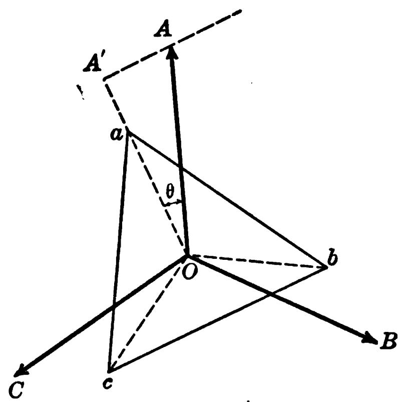

FIG. 15.—Vector diagram for three-phase system on partly inductive load.

respond with $O\ A$ and $O\ B$ in Fig. 9. When this construction has been carried out for each of the three phases, there will be a new set of star vectors which, when their ends are joined, will form a new e.m.f. triangle representing the necessary pressures at the generating end. This is shown in Fig. 16, where $a\ m$ and $m\ d$ are the vectors representing the required e.m.f. components to counteract the ohmic drop and reactive voltage respectively, due to the current $O\ A$. The same construction is supposed to be followed for the other two phases, and the resulting triangle $d\ e\ f$ indicates not only the magnitude of the potential differences between wires at generating end, but also their phase relations with

the other quantities. Thus the power factor at the generating end is not cos θ, but cos φ, all as explained in connection with Fig. 9. If it is required to take into account the effects of capacity, the correction *per phase* is made as explained in Article 10.

It is true that a symmetrical arrangement of conductors has been assumed; that is to say, the three conductors are supposed to occupy the vertices of an equilateral triangle, in which case the magnetic flux due to the current in one of the wires will neither increase nor decrease the amount of induction through the loop formed by the other two wires; or, in other words, the whole of the current in any one conductor may be considered as returning at a distance from this conductor equal to the side of the

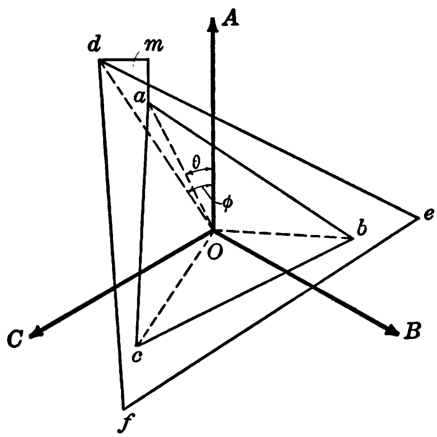

FIG. 16.—Complete vector diagram for three-phase transmission.

equilateral triangle. As a matter of fact, if the wires are arranged in any other practical manner, the effect of the induction due to any one wire on the loop formed by the other two wires is usually small; but a method of calculating the induced volts in any one conductor of a system of parallel conductors, whatever may be the arrangement or spacing of these conductors, is explained in Appendix A at the end of this book.

If the conductors of a three-phase transmission are regularly transposed, that is to say, if each of the three wires occupies a particular position relatively to the other wires for one-third of the total length of transmission, then the electrical calculations

can be based on an *equivalent* disposition of the wires at the vertices of an equilateral triangle of

$$\text{side } D = \sqrt[3]{abc} \tag{9}$$

where a, b and c stand respectively for the actual spacings between the three wires. This is a general statement of the particular problem considered in Appendix A, where the three conductors are assumed to lie in the same plane. A neat proof of formula (9) is given in Prof. H. B. Dwight's book on Transmission Line Formulas.[1]

[1] D. Van Nostrand Company, 1913.

CHAPTER III

Economic Principles and Calculations

That true engineering is essentially an economic science should be self-evident to every man who lays claim to the title of engineer; yet, there are many engineering undertakings, or portions of such undertakings, in which this fundamental principle has been disregarded. In the case of transmission lines, a certain system, or an exceptionally high pressure, may have been adopted because of its peculiar interest as an engineering problem; or duplicate lines, spare generating plant, and costly automatic gear may have been installed to ensure continuity of supply, apart from the economic value of such increased protection against possible interruption. This, however, is not engineering in the commercial sense. The determination of the economical size of conductors for the transmission of any particular amount of current, in accordance with the principle generally known as Kelvin's law, is but a very small part of the problem to be solved by the transmission-line engineer. An attempt will be made in this chapter to deal with the economics of the overhead power transmission line from a broad practical standpoint. Some approximate figures for use in getting out preliminary estimates will be given, but actual recorded costs of finished work carried out under various conditions can be obtained from other sources, and their inclusion in this book might tend to confuse rather than assist the reader.

It is proposed to deal here, in as small a space as possible, with economic principles, and to explain the methods by which the proper size of conductor for a given transmission can be calculated; and it is only when these principles have been grasped, and rightly understood, that the engineer can make the best use of cost data obtained from completed work.

When considering any scheme of power transmission from a generating plant of limited output, it is important to bear in mind that it does not pay to cover a distance greater than that within which there are reasonable prospects of supplying all the

power available at the generating station. The importance of this principle should be fairly obvious, yet there are instances which prove that it has been disregarded.

12. Choice of System.—On this continent it is usual to transmit electric power by means of three-phase alternating currents, the periodicity being 25 or 60 cycles per second. In Europe the Thury system of continuous current transmission at high voltages has met with success; it has much to recommend it, and there appear to be no reasons why it should not meet with equal success on this continent; but it is probable that three-phase transmission, at pressures even higher than those now in use, will hold its own for a considerable time to come.

13. Type of Transmission Line.—The structures for supporting the overhead conductors may be of wood, steel, or reinforced concrete. The wood supports may be of the ordinary single-pole type spaced 100 to 300 ft. apart, or they may be A or H frames built up of two poles suitably braced, and capable of supporting longer spans. The steel poles may be of the simple tubular type, or built up of three or four vertical tubes or angles. The more common construction for high-pressure transmission lines consists of light-braced towers with wide rectangular bases, except where the 'flexible' type of structure is adopted. These flexible towers are modeled generally on the A and H types of double wood pole supports. It is by no means an easy matter to decide upon the most suitable type of supporting structure to be used on any particular transmission scheme. In some cases a composite line including two or more types of support may be found advantageous. Among the factors influencing the choice of the supporting structures may be mentioned the character of the country, the means and facilities of transport, climatic conditions, the nature of the soil, and the scarcity or otherwise of suitable timber in the district through which the line will pass. In undulating or hilly country, advantage may frequently be taken of the heights, by erecting upon them comparatively low and cheap structures and spanning the depressions or valleys without any intermediate supports. The engineering features must, however, be very carefully studied in all such exceptional cases.

14. Length of Span.—Even when the type of structure has been decided upon, the height, strength, and cost of the structures will be dependent upon the distance between them. The

determination of the average length of span is indeed a very important economic question. The material of the conductor will, to some extent, influence the choice of span length, because aluminum conductors will usually have a greater summer sag than copper conductors, and this will necessitate higher supports to give the same clearance above ground at the lowest point of the span. In considering span length, the first cost of the individual support is not the only question which has to be taken into account; the cost of maintenance is almost equally important. The longer the span, the fewer will be the points of support; and if the line is well designed and constructed, there should be less trouble through faults at insulators. Again, where rent has to be paid for poles placed on private property, it is generally the rent per pole apart from the size of pole which has to be considered, and this is another factor in the determination of the best length of span. In level country, the economic span for steel tower construction is usually in the neighborhood of 550 ft. If substantial, braced wooden towers of considerable height are used in a district where such structures can readily be constructed, the economic span would probably be greater than 600 ft. It is hardly necessary to mention that, when comparing costs of various kinds of supports, the relative life and cost of upkeep of the poles or towers must be taken into account.

15. Effect of Span Variations on Cost of Steel Towers.—The height of towers in level country depends on (1) the minimum clearance between the lowest conductor and ground when the sag is greatest; (2) the voltage, since this has an effect on the spacing of the conductors and also to some extent on the clearance above ground level; and (3) the maximum sag. This last is determined by the length of span, the material and size of the conductors, the range of temperatures, and weather conditions generally. For the purpose of rough approximations suitable for preliminary estimates the writer has made use of the empirical formula

$$H = 35 + 0.3E_k + 0.6\left(\frac{l}{100}\right)^2 \qquad (1)$$

This gives the approximate overall height of towers in feet. The pressure E_k is expressed in kilovolts, and the length of span l, in feet. The formula is especially applicable to towers carrying a duplicate three-phase circuit. The constants have been worked out on the supposition that there are six No. 0000 aluminum

conductors and a grounded guard wire joining the tops of the towers. It is not intended to apply to spans greater than 600 ft. In the case of larger spans advantage is usually taken of inequalities in the ground levels. The cost of steel towers will depend not only upon the height, but also upon the stresses which the tower has to withstand. These again will be dependent upon the size of the wires, the length of span, and the weather conditions. It is probable that, with an increase in height, the cost increases less rapidly than the square of the overall length, but for approximate calculations it is convenient to assume the relation: cost varies as H^2. Also, if the weight of a tower measuring 60 ft. overall is taken at 1800 lb., and the price per pound of the finished (galvanized) tower at 5 cents,[1] the cost per tower in dollars is

$$\frac{H^2}{40} \qquad (2)$$

The cost of the flexible type of tower is appreciably less, being about seven-tenths of the cost of the rigid type with square base, or

$$\frac{H^2}{57} \qquad (3)$$

It is, however, usual to provide rigid strain towers in place of the flexible type at, say, every mile, and as the cost of such structures is about double the cost of the flexible tower, the cost of supports per mile of line may be calculated by assuming $n+1$ flexible structures per mile, when the actual spacing is n to the mile.

16. Duplicate Lines.—A point of great importance in connection with power transmission undertakings is the means adopted to guard against interruption of supply. If it is allowed that the least reliable part of a transmission system is the line itself, it is certainly advisable, when circumstances permit, to duplicate the lines, the two sets of conductors being connected in parallel under normal conditions. The best protection against interruption would be afforded by carrying the two sets of conductors on separate poles, perferably by different routes; but this would almost double the cost of the line, and it is usual to carry duplicate lines either on one set of poles, or on two sets of poles erected side by side on the same right of way. As an alternative to the duplication of the lines, the provision of reserve generating plant at the receiving end may be considered, and a comparison should be made between the relative advantages and costs of the various alternatives.

[1] This cost may be anywhere between 3 1/2 and 6 cents per pound.

A good example of steam-driven auxiliary plant in connection with hydro-electric power stations, is the recently completed oil-burning steam-generating station of the Southern California Edison Co., situated 25 miles from the city of Los Angeles and capable of connecting in parallel with the 60,000-volt and 30,000-volt systems ordinarily supplied by the Kern river and other hydro-electric generating stations of this company.

17. Costs of Typical Transmission Lines.—It would be possible to give a large number of figures relating to material and labor costs of completed transmission lines; but the conditions of transport of materials and quality of labor differ widely, and without complete knowledge of these conditions, such figures are liable to be misleading. For this reason two ideal preliminary estimates, one referring to a wood pole line, and the other to a steel line, are here reproduced, in the hope that they may be useful as a basis on which somewhat similar estimates may be shaped:

Preliminary Estimate No. 1

Wood pole transmission line, 20 miles long, carrying one three-phase line. Line pressure 22,000 volts. Span 130 ft. There is no grounded overhead guard wire; but two telephone wires are carried on the same set of poles. An allowance of 20 per cent. is made for extra insulators and fixtures to permit of doubling these on corner poles and in other selected positions.

Preliminary Estimate No. 2

'Flexible' type steel tower line, 60 miles long, with two sets of three-phase conductors. Line pressure = 80,000 volts. Average span, 480 ft. Spacing between wires, 8 1/2 ft. A Siemens-Martin steel cable, acting as grounded guard wire, joins the tops of all towers. Insulators of the suspension type. No telephone wires. Minimum clearance between lowest H.T. conductor and ground = 40 ft. Cost of right-of-way not included in estimate.

The cost of insulators increases rapidly with the rise of the working voltage. The curve of Fig. 17 gives approximate average prices of insulators complete with pins or suspension links for pressures up to 100,000 volts. The prices are per insulator or per series of insulator units. The suspension type

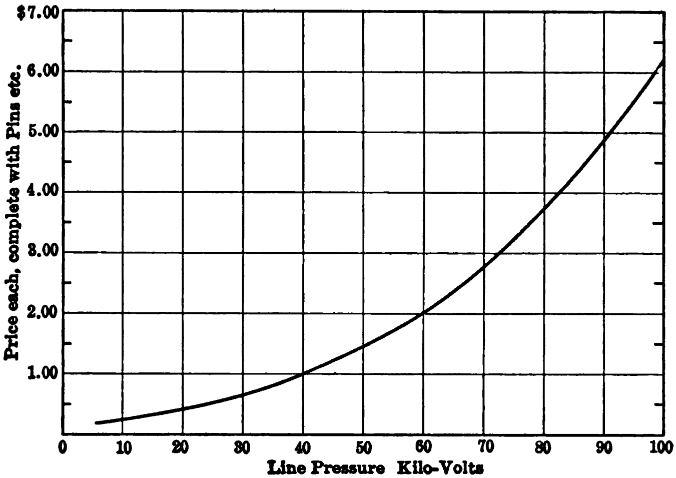

FIG. 17.—Approximate cost of line insulators.

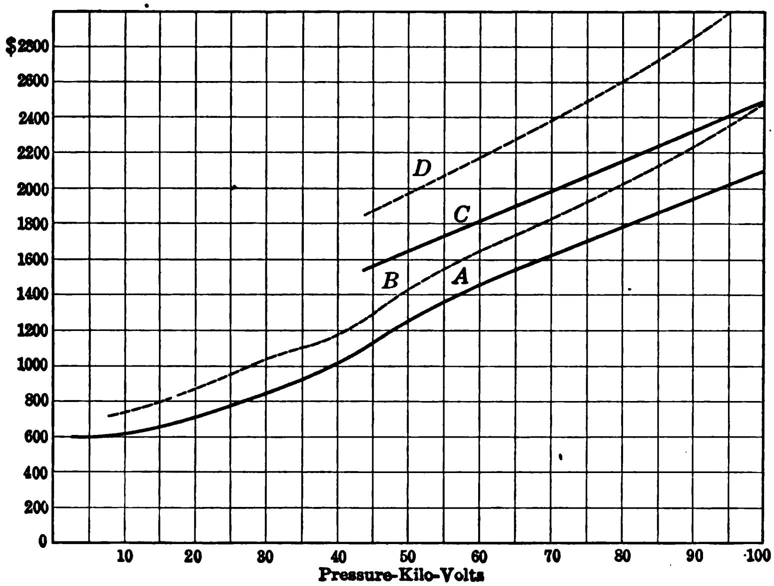

FIG. 18.—Approximate cost per mile of transmission line complete, but not including cost of conductors, right of way, or clearing ground in wooded country.

A—Single three-phase line without insulators or wires. *B*—Ditto, including insulators and stringing, but not cost of wires. *C*—Same as *A*, but for double three-phase line on single set of poles or towers. *D*—Ditto, including insulators and labor stringing wires, but not cost of wires.

of insulator consisting of a number of units in series is almost universally used for pressures exceeding 60,000 volts. One golden rule which applies to all overhead transmissions is that it is false economy to reduce first cost by putting in cheap and possibly unreliable insulators.

The curves of Fig. 18 are intended to supplement the figures of the typical estimates. They give approximate costs of transmission poles or towers, with and without insulators fixed in position. These costs are the averages of many actual figures, and give an approximate idea of the total expenditure per mile of line for various voltages; they do not include any clearing that may be necessary in wooded country, or payments for right-of-way. It is assumed that the conductors are of average size (No. 000 B. & S. gage), but the actual cost of the conductors, whatever the size, must be added to the costs indicated by the dotted curves *B* and *D* in order to arrive at the total cost of the finished line. These dotted curves, however, do include an amount to cover the labor cost of stringing the wires.

Preliminary Estimate No. 1

Materals (Excluding Conductors)

40 creosoted cedar poles, 35 ft. long, 8 in. diam. at top	$400.00
48 cross-arms, 3 1/2 by 4 1/2 in. by 4 ft. long.	14.50
96 galvanized-iron braces, 1 1/4 by 1/4 by 28 in. long	9.00
32 galvanized bolts, 5/8 by 12 1/2 in., with washers	
8 galvanized bolts, 5/8 by 16 in., with washers	
16 galvanized spacing rods, 3/4 by 16 in., with nuts and washers	7.50
48 galvanized lag screws, 1/2 by 3 1/2 in.	
96 galvanized carriage bolts, 3/8 by 4 1/2 in.	
1500 ft. galvanized 7-strand 5/16 in. guy wire	12.00
12 anchor rods with nuts and washers and necessary timber for anchor logs	7.00
24 galvanized guy clamps with bolts	3.00
8 galvanized sheet-iron bands to prevent cutting of poles by guy wire	0.50
12 standard thimbles for guy wire	0.40

20 galvanized-iron lightning conductors, with bolts	5.50		
20 ground plates or galvanized-iron pipes	8.00		
Staples and sundries, including allowance for breakages and contingencies	15.00		
80 telephone wire insulators (glass); 80 side brackets for same (wood); 5-in. wire nails	10.00		
144 H.T. porcelain insulators	36.00		
96 galvanized-iron insulator pins with porcelain bases	14.40		
48 special pole-top insulator pins, with bolts	19.20		
Total material cost per mile of line		$562.00	

Labor

Clearing 50 ft. on each side of pole line @ $30 per acre	363.00		
Distributing poles and other materials along the line	30.00		
Trimming poles, cutting gains, drilling holes, setting cross-arms	30.00		
Digging holes and erecting poles, including the necessary guying	80.00		
Fixing insulators and stringing wires, including telephone line	90.00		
Supervision and sundry small labor items	30.00		
Loss and depreciation of tools	10.00		
Management and preliminary work	25.00		
Total cost per mile for charges other than materials		$658.00	
Total cost per mile, excluding cost of conductor material			$1220.00

Conductors

16,000 ft. No. 1 copper conductors (hard-drawn); 700 ft. No. 4 copper for ties (soft); 10,800 ft. No. 10 copper for telephone circuit; 4550 lb. @ $15 per 100 lb	682.50
Total cost of finished line	$1902.50

Preliminary Estimate No. 2

Materials (Excluding Conductors)

10 flexible type, galvanized-steel, A-frame towers @ $85	$850.00		
1 galvanized-steel strain tower	170.00		
concrete foundations where necessary.	80.00		
5600 ft. 7/16 in. galvanized Siemens-Martin steel strand cable for guard wire and head guys on half-mile flexible towers	128.00		
4 anchor rods, complete with clamps and thimbles for guy wire	4.00		
90 sets of suspension-type insulators, including strain insulators and small allowance for breakages, complete with clamps	338.00		
Sundry small items or special material.	50.00		
Total material cost per mile of line			

Labor

		$1620.00	
Clearing 60 ft. on each side of line at average cost of $25 per acre	$363.00		
Distributing towers and other materials along the line	100.00		
Foundations for towers	75.00		
Assembly of parts and erection of towers	160.00		
Fixing insulators and stringing wires	170.00		
Supervision and sundry small labor items	50.00		
Allowance for loss and depreciation of tools	15.00		
Allowance for management and preliminary work	35.00		
Total charges other than materials, per mile		968.00	
Total cost per mile not including conductor material			$2588.00

Conductors

No. 00, hard-drawn, stranded-copper conductors; small amount of No. 2 soft copper for occasional ties; special clamps, shields, jointing materials, etc.; 13,350 lb. @ $15 per 100 lb	$2002.00
Total cost per mile of finished line, not including right-of way	$4590.00

The curves *A* and *B* of Fig. 18 (page 39) refer to wood poles or rigid steel towers (for the higher voltages) carrying three conductors; while curves *C* and *D* refer to a single set of poles or towers carrying six conductors. The cost of a line with flexible steel structures for voltages above 44,000 might be about 75 per cent. of the costs given by curves *A* and *C*. It is understood that the curves of Fig. 18 give only an approximate indication of the probable capital expenditure on the line. The actual cost will depend upon the character of the country, the nature of the ground, and other local conditions such as cost of labor and facilities for transportation. These, together with the weather conditions, force of wind and possible loading of wires with sleet or ice, will determine the most economical span and the average height of pole or tower. The cost, as previously mentioned, will also depend upon the material of the conductors, as a larger or smaller sag will influence spans and height of poles. The weight and diameter of conductors, by affecting the required strength of the supports, will be factors in determining the cost of the complete line, apart from any difference in the value of the conductors themselves. The actual cost of stringing very light or very heavy conductors will also differ from the average amount allowed for the purpose of plotting the curves. The number and style of lightning conductors, if any, and whether or not one or more grounded guard wires are strung above the conductors will obviously modify the average figures. Although steel or concrete poles, or steel towers, will generally be found more economical than a wood pole line for voltages above 44,000 on account of the heavier insulators, wider spacing between conductors, and generally greater height of support, it does not follow that wood poles or wood-pole structures may not prove economical, even for comparatively high voltages, in countries where suitable timber is plentiful and the ready means of transportation and erection of steel towers are wanting.

On the Pacific coast, near San Francisco, where overhead power lines are to be seen in abundance, there is an instance of a three-phase 100,000 volt transmission carried on single wood poles; but it does not follow that such a construction is economical.

Steel structures may be either galvanized or painted; the extra cost of galvanizing should be compared with the cost of

painting periodically, say every third or fourth year. The cost of concrete poles will usually be between 50 and 80 cents per 100 lb. weight. A pole 35 ft. high of square section 6 by 6 in. at top and 12 by 12 in. at bottom would weigh about 2000 lb.

The cost of foundations for towers varies greatly. In the case of fairly high steel towers with wide square bases in soil not requiring the use of concrete, the cost of excavating, setting legs, and back filling, not including erection of towers, will generally be between $10 and $20 per tower.

18. Cost of Conductors.—The capital expenditure on conductors will depend upon the material and the total weight. It is not proposed to discuss, in this place, the relative merits of copper and aluminum as conductor materials, but it may be well to point out that although, at the present market values of these metals, the use of aluminum may lead to some saving on first cost, there are many engineering points to be most carefully considered before definitely adopting either metal. The weight of the conductors necessary to transmit a certain amount of power over a definite distance will obviously depend upon the voltage, but apart from the engineering difficulties encountered at the higher voltages, there are economic considerations which determine the maximum voltage suitable for any given conditions. Among these may be mentioned a possible increase in the cost of generating plant for the higher pressures, the greater cost of step-up and step-down transformers and of the control apparatus, together with the line insulators, entering bushings, etc. The transmission line poles or towers will also, as previously mentioned, cost more for the higher pressures, because of the wider spacing between conductors. Then again, with the extra high pressures, the increased losses through leakage over insulators and possible corona losses may be quite appreciable.

Given a definite amount of power to be transmitted, and a definite line pressure, the current can be calculated; and the economic conductor cross-section—and therefore the weight and cost of the conductors—will be directly proportional to this current. It is only of recent years that this fact appears to have been generally recognized, and yet, so long ago as 1885, in his Cantor lectures delivered in London, Prof. George Forbes said: "The most economical section of conductor is independent of e.m.f.

and distance, and is proportional to the current." The determination of the current value to be used in the calculation of conductor sections is a real difficulty. It must not be supposed that even a knowledge of the load factor is sufficient by itself. The load factor, being the ratio of average load to maximum load, does not give the relation between the average I^2R loss and the I^2R loss of maximum output. The power lost in the conductors of a constant potential supply is proportional to the square of the power transmitted. On the basis of the average hydro-electric load curve, if the load factor is 50 per cent., the load on which the average transmission line losses should be based—being the square root of the mean of the square of the power—will probably be found to be more nearly 60 per cent. than 50 per cent. of the maximum load.

19. Economic Section of Conductor. Kelvin's Law.—Before considering to how great an extent the voltage can be raised, in order to keep down the current, without exceeding the limits determined by economic considerations, it will be well to examine, in some detail, the fundamental principle known as Kelvin's law, by which the proper size of conductor to carry a known current is determined. In this connection it is of no consequence whether the transmission is by direct or alternating currents, single phase or polyphase. If conductors have to be provided to carry a current of known amount, these may be of large cross-section and therefore of high initial cost, but of so low a resistance that the I^2R losses will be small; or they may be of small cross-section and high resistance, the capital expenditure on which will be small; but in which the I^2R losses will be large. The economical size of conductor for any given transmission will therefore depend on the cost of the conductor material and the cost of the power wasted in transmission losses; and the law of maximum economy may be stated as follows: *The annual cost of the energy wasted per mile of the transmission line, added to the annual allowance (per mile) for depreciation and interest on first cost, shall be a minimum.*

If it is assumed that the cost of poles or towers, insulators and other materials (apart from the conductors themselves) including the labor on erection and stringing of wires, is independent of the actual size of conductor, then the only variable item in the capital expenditure is directly proportional to the cross-section (or weight) of the conductor, and since the I^2R losses (for a given

current) are inversely proportional to the conductor cross-section, the law of maximum economy is greatly simplified, and in fact becomes Kelvin's law, which may be expressed as follows:

The most economical section of a conductor is that which makes the annual cost of the I^2R losses equal to the annual interest on the

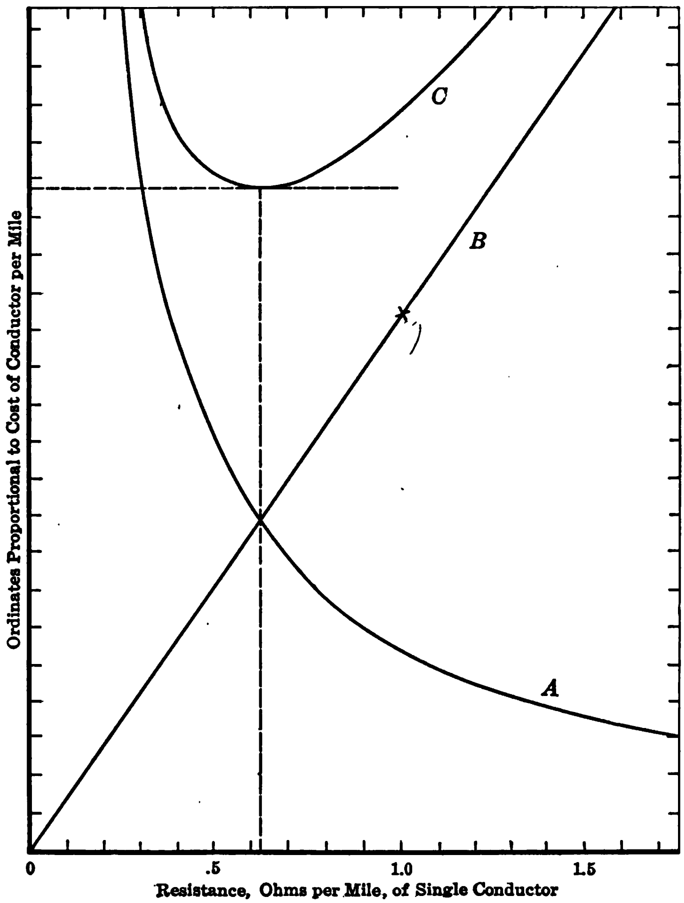

FIG. 19.—Graphical illustration of Kelvin's law.

capital cost of the conductor material, plus the necessary annual allowance for depreciation. The cross-section should, therefore, be determined solely by the current which the conductor has to carry, and not by the length of the line or an arbitrary limit of the percentage full-load pressure drop. If there are reasons

which make a large pressure drop undesirable, then, if necessary, economy must be sacrificed, and the line calculated on the basis of regulation only. It will, however, generally be found that the economic conductor will give reasonably good regulation.

The diagram, Fig. 19, will show clearly how the minimum total annual cost occurs when the cost per annum of the wasted power is equal to the capital cost expressed as an annual charge; and if desired a graphical solution of Kelvin's law can readily be obtained by this means. In Fig. 19, the horizontal distances measured to the right of the point *O* represent increasing conductor resistances; while the vertical distances represent money. The curve *A* shows how the annual charges depending on capital outlay decrease with increase of conductor resistance; while the straight line *B* indicates the growth of the cost of wasted power; this being directly proportional to the resistance. By adding the ordinates of curves *A* and *B*, the curve *C* is obtained, of which the minimum value corresponds with the resistance per mile of the conductor which will be the most economical to use, whatever may be the length of the line, or the pressure required at the receiving end. It will be observed that this minimum occurs where the two curves cross.

20. Practical Method of Applying Kelvin's Law.—The following formulas have been evolved with the view to facilitating the calculation of conductor sizes to give the most economical results on overhead transmissions. In every case the lesser factors which may, to a small extent, influence the results of the problem will be disregarded, but they may be taken into account when the final details of the transmission line are being considered. On the other hand, it will generally be found that the application of Kelvin's law in its simplicity, without regard to such influences as the possible variations in cost of supports, insulators, etc., depending upon the size of the conductors, will give results sufficiently accurate for practical purposes, and this for two important reasons:

1. A small variation in the diameter of the conductor either on the large or the small side is usually of very little consequence from the economic point of view.

2. As the standard size of conductor nearest in diameter to the theoretically correct size is generally selected, refinements or increased accuracy in the calculations will rarely affect the size of wire which is ultimately decided upon.

21. Economical Resistance Voltage Drop.—It is not generally realized that when the size of a conductor is determined by the application of Kelvin's law the ohmic drop of pressure per unit length of conductor is independent of the actual voltage or the current to be carried, and therefore bears no reference to the total amount of power to be transmitted. The economic data and assumptions alone determine the ohmic drop in volts per unit length of conductor, and this will be a constant quantity whatever the number of conductors or system of electric transmission adopted, the total amount of power to be transmitted, or the voltage ultimately decided upon. This fact very considerably simplifies the problem in its earlier stages.

The formula for the economic voltage drop may be arrived at as follows, bearing in mind that the annual charges to be considered are (1) an annual charge for interest and depreciation on the cost of the line wire; (2) the annual cost of the energy wasted in the conductor in the form of I^2R losses[1], and that the equality of these two items of cost determines the size of the most economical conductor.

Annual Charges Depending Upon Cost of Conductor.—Let p be the price to be paid for 100 lb. weight of conductor and a the percentage to be taken to cover the annual interest and depreciation, then, if r be the resistance in ohms per mile of the conductor,

$$\text{Annual charge} = \frac{a}{100} \times p \times \frac{1}{r} \times K \tag{4}$$

where K is a constant depending upon the material of the conductor.

Annual Cost of Energy Lost.—Let p_1 be the cost per h.p.-year of the wasted energy; then,

$$\begin{aligned} \text{annual cost per mile of conductor} &= p_1 \times \text{h. p. lost per mile} \\ &= p_1 \times \frac{I^2 r}{746} \\ &= p_1 \times \frac{e_r{}^2}{746 \times r} \end{aligned} \tag{5}$$

where e_r stands for ohmic drop in volts per mile of conductor.

[1] Other losses due to leakage over insulators and through the air should be taken into account when considering the choice of e.m.f., especially if this should exceed 60,000 volts.

In order to fulfil the condition of equality between the values (4) and (5) one must put

$$\frac{a \times p \times K}{100 \times r} = \frac{p_1 \times e_r^2}{746 \times r}$$

which gives

$$e_r^2 = \frac{746 \times K \times a \times p}{100 \times p_1}$$

If the material of the line is copper, the constant K may be taken as 8.76, while for aluminum it works out at 4.32. Inserting these values in the last formula, the economic resistance volts per mile of copper conductor become:

$$e_r = 8.1\sqrt{\frac{a \times p}{p_1}} \qquad (6)$$

and for aluminum:

$$e_r = 5.66\sqrt{\frac{a \times p}{p_1}} \qquad (7)$$

If preferred, these formulas can be put in the form:

$$\text{Circular mils per ampere (copper)} = 6750\sqrt{\frac{p_1}{a \times p}} \qquad (8)$$

and,

$$\text{Circular mils per ampere (aluminum)} = 15800\sqrt{\frac{p_1}{a \times p}} \qquad (9)$$

22. Economical Voltage, and Calculation of Conductor Sizes.—Having ascertained what will be the most economical ohmic drop of pressure per mile of conductor without reference to the total amount of power to be transmitted, the size of the conductor cannot be determined unless the value of the current is known, and this will depend upon the pressure at which the energy will be transmitted.

If the cost of the conductors forming the transmission line, and the I^2R losses therein, were the only considerations, a high voltage would in all cases be desirable on account of the corresponding reduction of current for a given amount of energy to be transmitted. But, apart from the extra cost of the line due to the better insulation and wider spacing of wires required by the higher pressures, the cost of generation and transformation of high-pressure energy must be taken into account, and as the extra cost per kilowatt of equipment for generating at high pressures will depend largely upon the total output required, it

follows that the most economical pressure will bear some relation to the total power to be transmitted. This is apart from the distance of transmission, which is the most important factor governing the choice of voltage. If the distance is great it is obvious that the reduction of material cost and power losses in the line due to the employment of higher pressures will be relatively of far greater importance than the increased cost of plant in generating and transforming stations. On the other hand, the employment of very high pressures even on a comparatively long line might not be justified if the total amount of power to be transmitted were very small.

In the early days of electric transmission Mr. C. F. Scott suggested as a rough-and-ready rule that the total distance of transmission in miles divided by three would give a figure approximating to the required pressure, expressed in kilovolts. This rule does not take into account the amount of power to be transmitted; but an empiricial formula which the writer has used for preliminary estimates, and which agrees generally with modern practice is:

$$\text{Pressure in kilovolts} = 5.5\sqrt{L} \tag{10}$$

where L = distance of transmission (in miles) $+ \dfrac{\text{horse-power transmitted}}{200}$

Given the amount of power to be transmitted and the length of line, one can with the aid of formula (10) decide upon a standard voltage and proceed with the calculations for current and size of conductor; but it is necessary always to bear in mind that a transmission line cannot be considered by itself; it must be treated as part of a complete scheme of transmission and distribution, and the best voltage to use on any given system can generally be arrived at only by a system of trial and error, taking into account the costs of the various parts of the complete system as influenced by alterations in the transmission voltage. No accurate formula can be evolved which would be applicable to all the varied conditions encountered in actual work; but a practical method of attaining the required end will be explained later.

23. Example Illustrating Quick Method of Determining Economic Size of Conductors.—For the purpose of working out a practical example the following assumptions have been made:

Total horse-power to be transmitted, $P = 16{,}000$.

System, three-phase.

Power-facter $= 0.8$.

Distance of transmission $= 120$ miles.

Copper conductors to be used, the cost p being \$15 per 100 lb.

Percentage to be taken to cover depreciation and annual interest on cost of copper, $a = 12.5$.

Estimated cost of wasted power per horse-power-year, $p_1 = \$16$.

The economic voltage drop per mile of single conductor will be, by formula (6),

$$e_r = 8.1\sqrt{\frac{12.5 \times 15}{16}}$$
$$= 27.7 \text{ volts}$$

The transmission voltage as given by formula (10) is:

$$\text{Kilovolts} = 5.5\sqrt{120 + \frac{16{,}000}{200}}$$
$$= 77.7$$

or, say, 80,000 volts at the receiving end.

The current per conductor will be:

$$I = \frac{e.h.p. \times 746}{\sqrt{3} \times E \times \cos\theta}$$
$$= \frac{16{,}000 \times 746}{\sqrt{3} \times 80{,}000 \times 0.8}$$
$$= 107.5 \text{ amp.}$$

Resistance of conductor per mile $= \frac{e_r}{I} = \frac{27.7}{107.5} = 0.257$ ohm, and since No. 4-0 B. & S. wire has a resistance of 0.259 ohm per mile, that is the standard size which should be adopted unless a more careful study of the complete scheme should lead to a different decision in regard to the pressure of transmission.

Since, for a given amount of power to be transmitted, the current will vary inversely as the pressure, it follows that the resistance per mile of conductor to give the economic voltage drop per mile (27.7 volts in this particular example) will be directly proportional to the pressure at which the power is transmitted. Thus if 100,000 volts were found to be a more economical pressure than 80,000, the ohms per mile of conductor

would be $\frac{0.257 \times 100}{80} = 0.321$, the nearest standard size being No. 3-0 (ohms per mile $= 0.3265$).

Power lost in Line.—If w stands for the total I^2R watts lost in the three conductors, based on the calculated value of the resistance, then

$$\begin{aligned} w &= 3 \times \text{length of line} \times I \times e_r \\ &= 3 \times 120 \times 107.5 \times 27.7 \\ &= 1{,}070{,}000 \text{ watts} \end{aligned}$$

on the assumption that a transmission pressure of 80,000 volts is adopted; and since the total horse-power transmitted is 16,000, the *percentage* power loss is:

$$\frac{1070 \times 100}{16{,}000 \times 0.746} = 9 \text{ per cent.}$$

Voltage Regulation.—The drop in pressure per conductor, due to ohmic resistance only, will be:

$$e_r \times \text{length of line} = 27.7 \times 120 = 3330 \text{ volts}$$

or $3330 \times \sqrt{3} = 5750$ volts between wires, since the system is three-phase and the volts e_r refer to a single conductor only. The *percentage* ohmic drop is, therefore:

$$\frac{5750 \times 100}{80{,}000} = 7.2 \text{ per cent.}$$

This figure alone does not, however, give much indication as to what will be the actual regulation of the line, as the effects of inductance and electrostatic capacity must be taken into account and the resultant difference of pressure between the transmitting and receiving ends of the line calculated by any one of the usual methods. The resultant pressure drop may be found to be excessive; it may be such as cannot readily be dealt with in a practical scheme, and in such a case the economy of the line may have to be sacrificed by putting in larger conductors.

It is obvious that other conditions may render it inexpedient or impossible to adopt the most economical size of conductor as calculated by the application of Kelvin's law, but in such cases experience and common sense will usually indicate the right course to follow. If the economic size of wire is small there may be trouble due to excessive heating, want of me-

chanical strength, or loss of power through cornona discharges if high pressures are used. If, on the other hand, the conductor diameter is very large, there may be difficulties in handling and in taking the strain on the individual insulators. The remedy in this case is obviously to subdivide the single circuit into two or more parallel circuits, and, in fact, there are many advantages in doing so rather than running very heavy single conductors. One particular aspect of the question of subdivision of transmission lines is dealt with in Appendix B.

Again, even from the economic point of view, the case might arise of a temporary installation intended to give a quick return on capital invested, and an exceptionally small size of wire giving a large I^2R loss might produce the best results. This, however, leads to the consideration of the most important factor in the whole problem, namely, the correctness of the estimates of costs, depreciation allowances and power transmitted, upon which the value of the calculated results will mainly depend. It is here that the experience, foresight and sound judgment of the engineer must necessarily play an important part, and it is not possible in this chapter to do more than indicate a few considerations which must not be overlooked.

24. Estimation of Amount and Cost of Energy Wasted in Conductors.—The correct value of the power (P) from which the value of the current (I) is determined is frequently very difficult to estimate. This is a point which is best considered when determining the cost of the wasted energy. It is, however, clear that the annual amount of energy wasted will depend not only on the average value of I^2, but also on the *time* during which the average amount of power may be considered as being transmitted by the wires. If, therefore, it is desired to estimate accurately the amount of energy wasted annually in the lines, a probable load curve for the year should be drawn and the average I^2 calculated therefrom. This will give a value for I which, if considered as flowing in the wires continuously throughout the year, will lead to a certain watt-hour or yearly horse-power loss, the cost of which it is desired to know.

Now, the annual cost of production of an additional electrical horse-power, considered apart from the total cost of production, is always difficult, if not impossible, to estimate accurately, but where coal is the source of energy there is at least the extra cost of coal consumed to be taken into account when

estimating the production cost of the lost energy. The case is different in a water-power generating station, where the cost of running the station at full output is very little in excess of the cost of running at one-quarter or one-tenth of maximum output, and it is even more difficult to decide upon a figure which shall represent the cost of wasted energy (p_1 in the calculations) with sufficient accuracy to make the calculations of the economic conductor of real practical value.

There are two points in connection with water-power propositions which must never be lost sight of:

If the amount of water-power available is limited, while the demand for power is unlimited, the cost (p_1), of the wasted power may be taken at the price which the user would be actually prepared to pay for this power were it available for useful purposes.

If the water-power is unlimited as compared with the demand for power, the cost of wasted power is practically *nil*, except for the fact that a generating plant has to be installed of a somewhat larger capacity than would otherwise be necessary; and the works cost of the wasted power must, of course, include a reasonable percentage to cover interest and depreciation on this extra plant.

25. Estimation of Percentage to Cover Annual Interest and Depreciation on Conductors.—So far as interest is concerned, if cash is to be paid for the conductors, the figure to be taken for interest on capital should be on a par with the expected percentage profit on the complete undertaking; but if the conductors are mortgaged, it is the annual amount of the mortgage which should be taken.

In regard to depreciation, the probable life of the conductor must be estimated, and this, to a certain extent, may depend upon the life of the transmission line considered as a whole.

26. Economic Voltage.—It should be clearly understood that the foregoing articles deal only with the determination of the correct size of conductors based on certain assumptions as regards voltage and power to be transmitted. The cost of generating and transforming plant and buildings, as influenced by the voltage, must be carefully considered, together with the type and cost of pole line, so far as these are influenced by the size of the conductors. The character of the country, too, will have some bearing on the design of the transmission line, and the final choice of

voltage may depend to some extent upon whether a wood pole line with comparatively short spans and (preferably) small spacing between wires is likely to be more economical than a line with steel towers which will permit of longer spans with wider spacing between wires. In other words, the total cost of the whole undertaking and the total annual losses of energy from all sources, as influenced by any change of voltage, must be considered before the line pressure as given by formula (10) can be definitely adopted as being the most economical for the undertaking considered as a whole.

Clearly, the choice of the transmission voltage is a very important matter; and as it is possible to determine the proper voltage on purely economic grounds, the use of exceptionally high pressures merely because of their interest from an engineering point of view, should be discouraged. On the other hand, it would appear that most transmission-line troubles occur on lines working at pressures between 30,000 and 80,000 volts; and an important consideration to bear in mind is that more trouble may be experienced with heavy currents than with high voltages, owing to the more serious effects of interruptions or transient disturbances when the current is large, so that greater security may sometimes be obtained by increasing the voltage with a view to reducing maintenance and operating costs. A remarkable instance of high pressure being used for a short-distance transmission occurs in Germany at Lauchammer. This is the first 110,000-volt installation in Europe. The line is only 35 miles long, but the power to be transmitted is considerable, being 20,000 kw. The engineers claim that, owing to large fluctuations expected through rolling-mill load, low voltages would be uneconomical. Under ordinary circumstances, tne trend of modern practice would indicate something above 60,000 volts as the best pressure in this case; but when there is little difference between the cost of a 60,000-volt and an 80,000-volt scheme, it is wise to adopt the higher pressure.

When figuring on the best voltage for any particular scheme, the capital cost of all works, buildings, or apparatus which is liable to be influenced by the transmission-line pressure, together with all operating and maintenance charges which may be similarly influenced, must be taken into account. It will usually be found convenient to reduce all such costs or differences of cost to the basis of annual charges.

27. Costs Others than Transmission Line, Liable to be Influenced by Voltage Variations.—The cost of a generating station complete with all plant and machinery, but not including transmission line, may be anything from $20 to $200 per horse-power installed. It will depend on total output, that is, on the size of the station, on location, and transport and labor facilities. The cost of a hydro-electric station will depend on the head of water, the amount of rock excavation, the size of dam, etc.

The figures given in the accompanying table are approximate costs per kilowatt (not including the transmission line) for a medium head hydro-electric development suitable for a total output

	Transmission-line voltage		
	30,000	60,000	100,000
Hydraulic works outside power-station buildings	$15.00	$15.00	$15.00
Power-station building, including excavations..	5.00	5.06	5.10
Receiving-station building..................	1.00	1.03	1.05
Switch-gear (both ends).....................	1.20	1.35	1.70
Electrolytic lightning arresters................	0.34	0.66	1.20
Transformers (both ends)....................	2.50	2.90	3.50
Generators and exciters......................	8.00	8.00	8.00
Cables in buildings, entering bushings, etc	0.40	0.40	0.50
Crane, sundries, and accessories, including preliminary work..........................	2.00	2.10	2.20
Turbines and hydraulic equipment............	10.00	10.00	10.00
Total cost per kilowatt..................	$45.44	$46.50	$48.25

in the neighborhood of 10,000 kw. to be transmitted over two outgoing three-phase feeders. The usefulness of these figures lies mainly in the indication they give of the probable differences in cost with the variation of transmission-line pressure.

28. Annual Charges Depending on Voltage.—These charges may be summarized as follows:

1. A percentage on all capital expenditure, whether for generating station, transmission line, or receiving stations, which is not constant irrespective of voltage.

2. The yearly cost of the power lost in the transmission line.

3. The yearly cost of power lost in generators and transformers (the efficiency of the electrical plant will not necessarily be the same for all voltages).

4. The yearly cost of maintenance and operation. This may depend upon length of spans in transmission line, and on the

necessary plant, switch-gear, etc., to be attended to, and kept in working order.

The percentages referred to under item (1) must include interest on capital invested, and depreciation. The accompanying table gives the percentage to allow for depreciation for various

DEPRECIATION TABLE

(On basis of 5 per cent. compound interest earned by money put aside annually.)

Life (years.)	Depreciation (per cent.)	Life (years.)	Depreciation (per cent.)
2	48.70	28	1.710
4	23.20	30	1.505
6	14.70	32	1.325
8	10.50	34	1.175
10	7.95	36	1.045
12	6.28	38	0.928
14	5.10	40	0.828
16	4.23	42	0.740
18	3.55	44	0.662
20	3.03	46	0.593
22	2.60	48	0.532
24	2.25	50	0.477
26	1.96		

terms or years. Depreciation—which may include what is sometimes referred to as obsolescence—is the amount to be set aside annually in order to reproduce, at the end of a term of years, the capital originally invested. This term of years is the 'life' of the works or materials on which the percentage depreciation is to be calculated. It is assumed that, at the end of this term of years, the value of such works or materials is *nil.* It is also assumed that the amount put aside annually earns interest at the rate of 5 per cent. compound.

29. Example: Method of Determining Most Economical Voltage.—Consider the case of a typical medium head hydroelectric power station:

Distance of transmission = 50 miles.

Duplicate three-phase line with copper conductors.

Cost of conductors = $15 per 100 lb.

Power demanded = 15,000 h.p. or 11,200 kw. (It is assumed that this power will be required continuously day and night for

industrial purposes, and that it is the probable limit of the water-power available.)

Power factor = 0.8.

Selling price of power = $21 per horse-power-year.

Interest on capital invested; allow 6 per cent.

The economic drop of voltage per mile of single conductor as given by formula (6) is:

$$e_r = 8.1 \sqrt{\frac{a \times p}{p_1}}$$

Where p is the price in dollars of 100 lb. weight of conductor (in this example $p = 15$), a is the percentage to cover annual depreciation and interest on cost of conductors, and p_1 is the cost per horse-power-year of the wasted power. The proper value for a may be arrived at by estimating the term of years corresponding to the life of the conductors, at the end of which they are supposed to be of no value. Taking 20 years as the life of the conductors, the depreciation to be allowed according to the table herewith is 3.03, which makes $a = 6 + 3.03$, or, say, 9 per cent.

With regard to p_1, if the demand for power were equal to the available supply from the time of the power-plant being put into operation, the works cost of waste power would be the same as the selling price; but, on the assumption that the supply exceeds the demand during the first five years of operation, and that the cost of waste power during this period is only $7 per horse-power-year,[1] the average cost of wasted power during the 20 years life of the conductors is:

$$p_1 = \frac{(5 \times 7) + (15 \times 21)}{20} = \$17.50$$

The economic voltage drop is therefore:

$$e_r = 8.1 \times \sqrt{\frac{9 \times 15}{17.5}} = 22.5 \text{ volts per mile.}$$

[1] The actual works costs of the wasted power is always difficult to determine exactly. It must, however, be remembered that even with unlimited power, and no appreciable increase in maintenance and operating charges with increase of losses, the greater capital cost of the plant installed to provide this waste power has to be taken into account and expressed in the form of an annual charge per horse-power wasted, whether this waste occurs in the generating and transforming plant or the line itself.

A first approximation to the required line voltage may be obtained by formula (10):

$$V_k = 5.5\sqrt{\text{distance} + \frac{\text{horse-power}}{200}}$$

$$= 5.5\sqrt{50 + \frac{15000}{200}}$$

$$= 61.5, \text{ or say } 60{,}000 \text{ volts.}$$

In order to calculate the cost of the line losses, it will be necessary to adopt a figure for the horse-power transmitted, which, when squared, will give the average square of the power during the estimated life of the conductors.

Assuming an average figure for the output during 12 months, a table showing probable demand for power can be constructed as follows:

Period	H. p. demanded	H.p. squared × years
First year of working	5 by 1000	25 by 10^6 by 1 = 25 by 10^6
Second year of working	6 by 1000	36 by 10^6 by 1 = 36 by 10^6
Third year of working	7 by 1000	49 by 10^6 by 1 = 49 by 10^6
Fourth year of working	9 by 1000	81 by 10^6 by 1 = 81 by 10^6
Fifth year of working	12 by 1000	144 by 10^6 by 1 = 144 by 10^6
Remaining 15 years of estimated life of conductors	15 by 1000	225 by 10^6 by 15 = 3375 by 10^6.

Total of last column, 3710×10^6.

Average, 185.5×10^6.

Average horse-power for purpose of calculating cost of waste power $= \sqrt{185.5 \times 10^6} = 13{,}600$ approximately.

When the section of the conductors is such as to satisfy Kelvin's law of economy, the yearly cost of the I^2R losses is equal to the amount representing annual depreciation and interest on first cost of conductors; and the total annual charges on active line material, for a three-phase line, will therefore be:

$$2 \times \frac{I^2 R \times p_1}{746} \times 3 \times l$$

where R is the resistance per mile of conductor. But

$$I = \frac{P \times 746}{\sqrt{3} \times E \times \cos\theta}$$

where P stands for the horse-power transmitted.

Also: IR = voltage loss per mile = e_r. So that the formula for the total yearly charges on conductors can be written

$$\frac{2\times\sqrt{3}\times e_r\times P\times p_1\times l}{E\times\cos\theta} \tag{11}$$

which in this example becomes

$$\frac{2\times\sqrt{3}\times 22.5\times 13{,}600\times 17.5\times 50}{60{,}000\times 0.8}=\$19{,}300$$

An amount which is independent of the fact that the transmission, in this particular instance, is by two three-phase lines ordinarily connected in parallel.

30. Closer Estimate of Economical Voltage.—In order to take into account first cost, life, annual maintenance, and operating charges of every portion of the complete undertaking which may be affected by a change in the transmission voltage, the costs, worked out on an annual basis, may be arranged in tabular form as here shown, where the total charges for the 60,000-volt scheme are compared with the estimated charges for an 80,000-volt transmission. In this particular example, the figures are favorable to the higher voltage; but the difference is small. It would be useless to repeat the process for a voltage lower than 60,000, because the cost would certainly be higher.

It will be understood that the accompanying estimate of total annual charges of the two selected voltages does not include any items other than those that are liable to vary with changes in the line voltage. An estimate covering the complete undertaking would, in addition to the items named, have to take account of riparian rights for dam, reservoir, etc., preliminary legal and other expenses; cost of providing proper access for materials to site of works; dam and hydraulic works outside station building; turbines; electric generators, and exciters; auxiliary plant; sundries and contingencies.

In the case of a short distance transmission with a line pressure not exceeding 11,000 volts, and the possibility of winding the generators for the full pressure, the relative costs and efficiencies of generators wound for different voltages should be taken into account.

COMPARISON OF COSTS AT DIFFERENT VOLTAGES

Portion of complete undertaking affected by change of voltage	Estimated life (yr.)	Depreciation (from tables)	Depreciation plus 6 per cent. interest	Total cost voltage 60,000	Total cost voltage 80,000	Annual charges voltage 60,000	Annual charges voltage 80,000
Line conductors (copper) of most economic section (annual cost varies as $\frac{1}{\text{voltage}}$)	20	....	....	....	....	$19,300	$14,475
Steel tower transmission line, without conductors, but otherwise complete (from curves, Fig. 18)	18	3.55	9.55	$108,000	$130,000	10,310	12,410
Generating-station buildings	40	0.828	6.828	56,600	57,000	3,870	3,895
Substation buildings	30	1.505	7.505	11,500	11,900	863	893
Transformers	18	3.55	9.55	32,500	35,200	3,100	3,360
Switch-gear, including lightning arresters, cables in buildings, and entering bushings	14	5.10	11.10	27,000	34,000	3,000	3,775
Assume unaltered: Yearly cost of power lost in generators and transformers. Yearly cost of operation and maintenance. Right-of-way and clearing. Difference in favor 80,000 volts = $1635.						$40,443	$38,808

CHAPTER IV

Electrical Principles and Calculations

31. Materials.—Under ordinary circumstances, the choice of material for the conductors of an overhead H. T. transmission line lies between copper and aluminum. It is possible that under certain conditions, especially for the transmission of continuous currents, galvanized iron or steel might prove satisfactory and economical; and compound wires or cables such as copper-clad steel, and aluminum cables with galvanized steel core, are used where great mechanical strength is of more importance than high conductivity. Much has been written on the relative advantages of copper and aluminum for transmission-line conductors, and some writers, who have not been interested in the sale or manufacture of conductor materials, have no doubt treated the subject impartially, and stated the case for either metal with clearness and ability; but there is usually a tendency to give too much weight to the question of first cost. It is very difficult to make a comparison between various metals which shall be of general utility, because not only the electrical, but also the mechanical properties have to be taken into account, and the requirements in the latter respect will depend largely on local conditions. Then again, with market fluctuations, and tariffs controlling the prices of raw materials in different countries, together with varying costs of freight from manufacturers' works, a comparison of costs, except when based on current quotations, is of little value. For these reasons no direct comparison between conductors of different materials will be made here, but leading particulars will be given, together with such notes as the writer's experience may suggest, which it is hoped will be helpful to the transmission-line engineer in deciding upon the right material to use under any circumstances. Tables of resistances, sizes and weights, and other physical properties of the materials will be found in the various engineering handbooks and manufacturers' catalogues; and only such particulars will be given here as may be useful for preliminary calculations.

32. Copper.—It is probably safe to assert that, *apart from the question of cost,* the high conductivity combined with the great strength and elasticity of hard-drawn copper, give this material the advantage over all others for use on the average high-tension electric transmission line.

The ultimate tensile strength of hard-drawn copper is greater per square inch of section in the smaller wires, being approximately as follows:

Gage No. B.&S.	Diameter (inches)	Breaking stress; pounds per sq. in.
000	0.410	52,000
0	0.325	55,000
2	0.258	58,000
4	0.204	60,000
6	0.162	62,000
8	0.128	64,000
10	0.102	65,000
12	0.081	66,000
14	0.064	67,000
16	0.051	67,500
18	0.040	68,000

A stranded cable, in which the pitch is usually between 12 and 16 diameters of the cable, will generally break under a load slightly smaller than the combined breaking loads of the individual wires. The tensile strength of a stranded cable should, however, not be less than 90 per cent. of the combined strengths of the single wires.

The elastic limit of hard-drawn copper wires is about 60 per cent. of the breaking stress; but it may be as high as 70 per cent. and even 75 per cent. of the ultimate stress.

33. Aluminum.—The conductivity of hard-drawn aluminum wire is between 60 per cent. and 61 1/2 per cent. by Matthiessen's standard; pure copper being 100 per cent. The weight of an aluminum conductor is almost exactly half that of the copper conductor of equal resistance, and it is about 77 per cent. as strong as the equivalent copper cable (safe working stress). Comparing aluminum of 61 per cent. conductivity with copper of 97 per cent. conductivity, the diameter of the equivalent aluminum cable would be 1.26 times the diameter of the copper cable.

The ultimate tensile stress of hard-drawn aluminum wire usually lies between 23,000 and 30,000 lb. per square inch,

depending upon the size of wire and hardness; if carried beyond a certain point, high tensile strength is a disadvantage, because the conductivity is lowered and the wire becomes "short." Some recent tests made on the strands composing an aluminum cable of 61 per cent. conductivity gave the following results:

Diameter of wire (inch)	0.1092	0.116	0.138
Number of tests	8	......	33
Breaking stress, highest	34,500	......	28,900
lowest	28,200	......	24,200
average	32,100	29,300	26,100

The elastic limit of hard-drawn aluminum wire is a little over 50 per cent. of the breaking stress.

Special care is required in the handling of aluminum conductors, as they are easily scratched and cut. This metal is readily attacked by alkaline substances, and coils of cable should not be left lying on wet marshy ground liable to contain alkalies, or in old stables where ammonia may be present.

Aluminum is not easily soldered, because of the thin film of oxide which quickly forms on the surface exposed to the atmosphere. The tin must be *mechanically* worked through the oxide coating with the aid of an old file or, preferably, a scratch brush with bristles of 0.01 in. diameter steel not more than 1 in. long. A little experience is needed for neatly soldering aluminum into cable sockets, etc., partly for the above reason and also because the metal is a good conductor of heat, and the parts to be tinned will cool rapidly unless special precautions are taken.

34. Steel.—The ordinary commercial galvanized steel strand cable, as used for guy wires, has a breaking strength averaging 70,000 lb. per square inch, and a conductivity of about 11 1/2 per cent. by Matthiessen's scale. When used to convey alternating currents, the high permeability of iron increases the so-called skin effect, with the result that the resistance to the flow of current may be greatly increased, depending upon the size of the cable and the frequency.

The weight of a steel cable would be about 7 1/2 times that of the copper cable of equal resistance, but with the higher grade (and stronger) steels, this multiplier might be as high as 10. High-grade steel conductors can be used to advantage for very long spans, or where the climatic conditions are such as to subject the cables to abnormally great stresses.

Whatever may be the material of the conductor, a stranded

cable made up of a large number of small wires will be stronger than a cable of the same sectional area made up of fewer large wires.

The accompanying Table No. 1 gives the most important physical constants for various conductor materials. It will be noticed that aluminum has a larger temperature coefficient than copper. This has an important bearing on the economic length of span; the difference in sag between summer and winter temperatures is often considerable with aluminum conductors, but this difference is, of course, more noticeable on the shorter spans such as occur with a wood pole construction: on long spans, the difference in sag due to temperature changes is very small, whatever metal is used.

TABLE I

PHYSICAL CONSTANTS OF CONDUCTOR MATERIALS

Properties of conductor materials	Copper, hard-drawn solid or stranded	Aluminum, hard-drawn stranded	Copper-clad steel	Steel, galvanized stranded guy wire
Breaking stress: pounds per square inch of cross-section	50,000 to 62,000	22,000 to 30,000	60,000 to 100,000	50,000 to 90,000
Breaking stress (average)	56,000	26,000	80,000	70,000
Maximum working stress (average)	28,000	13,000	40,000	25,000
Elastic limit (lb. per square inch)	30,000 to 35,000	14,000 to 17,000	50,000	30,000 (approx.)
[1]Modulus or Coefficient of elasticity (Young's modulus)	15×10^6	9×10^6	22×10^6	25×10^6
Cœfficient of linear expansion of wire per degree Fahrenheit $= a$	9.6×10^{-6}	1.28×10^{-5}	6.7×10^{-6}	6.5×10^{-6}
Weight per cubic inch	0.323 lb.	0.097 lb.	....	0.287 lb.
[2]Weight per mile per circular mil.	0.016 lb.	0.0048 lb.	0.015 lb.	0.0142 lb.
[3]Cœfficient k	0.485	0.146	0.45	0.43
[4]Resistance; ohms per mile per circular mil.	54,700	89,600	137,000	470,000
Relative resistance	1.00	1.64	2.50	8.60
Relative Conductivity	1.00	0.61	0.40	0.116

An argument often advanced in favor of aluminum conductors is that the weight of these, for any given transmission scheme, is only about half that of copper. This is certainly an advantage

[1] Being ratio $\frac{\text{stress in lb. per square inch}}{\text{extension per unit length}}$.

[2] To obtain weight per mile of any size of wire, multiply these figures by the cross-sectional area expressed in circular mils.

[3] Used in sag calculations.

[4] To obtain resistance per mile of any size of wire, divide these figures by the number of circular mils in the cross-section.

in the handling of the wire, but otherwise it is at least counterbalanced by the fact that the wind effect is greater on the increased diameter and that the towers must often be higher than if copper is used, partly on account of the higher coefficient of expansion of aluminum, but mainly because of the lower permissible stress. The advantage of lighter weight is largely discounted by the fact that the equivalent aluminum conductor can only be drawn up to a tension equal to about three-quarters of the permissible maximum tension of the copper cable. On the other hand, the larger diameter of the aluminum cable may be an advantage on very high-pressure transmissions, because it raises the critical voltage at which corona losses become appreciable.

At the present time, the metal market in the United States is so regulated that there is no economic advantage in using aluminum in place of copper for transmission lines; but on the continent of Europe and in Canada the former metal has found favor and is much used.

Table II gives the approximate resistances and weights of the usual sizes of cable, whether of copper or aluminum, but makers' lists should be consulted for exact particulars, as the method of building up the stranded conductors necessarily modifies to a small extent the average figures here given. The figures in the

TABLE II

RESISTANCE AND WEIGHT OF STRANDED CONDUCTORS

Size: cir. mils and B. & S. gage	Diameter (inches) approx.	Circular mils (nominal)	Area (approx.)	Copper		Aluminum	
				Ohms per mile	Weight per mile (pounds)	Ohms per mile	Weight per mile (pounds)
500,000	0.81	500,000	0.393	0.1095	8100	0.182	2430
450,000	0.77	450,000	0.354	0.1210	7300	0.202	2187
400,000	0.73	400,000	0.314	0.1363	6500	0.227	1944
350,000	0.68	350,000	0.275	0.1566	5650	0.260	1701
300,000	0.63	300,000	0.236	0.1818	4880	0.303	1458
250,000	0.58	250,000	0.1965	0.2192	4060	0.364	1215
4/0	0.53	211,600	0.1661	0.260	3448	0.430	1028
3/0	0.47	167,800	0.1317	0.326	2730	0.542	816
2/0	0.42	133,100	0.1045	0.410	2165	0.684	647
0	0.37	105,600	0.0830	0.518	1705	0.862	513
1	0.33	83,700	0.0657	0.655	1346	1.085	407
2	0.29	66,400	0.0521	0.826	1067	1.370	323
3	0.26	52,600	0.0413	1.040	850	1.728	256
4	0.23	41,700	0.0327	1.313	675	2.185	203

table are intended for quick slide rule calculations, and the resistances are approximately correct for a temperature of 60° F. The sizes of the smaller conductors are given in the B. & S. gage because this is generally used on this continent. With this system, when the area of any particular gage number is known, it is only necessary to double this in order to get the area of the third size larger; or if instead of multiplying by two, the multiplier 1.261 is used, this will give the area of the next size larger in the B. & S. series. It is convenient to remember that No. 10 B & S. copper wire measures almost exactly 1/10 in. in diameter, and has a resistance of 1 ohm per 1000 ft.

When the resistance, R, per mile of a stranded conductor is known, the *weight* per mile is approximately:

$$\text{For Copper; pounds per mile} = \frac{885}{R}$$

$$\text{For Aluminum; pounds per mile} = \frac{440}{R}$$

35. Copper-clad Steel.—By welding a coating of copper on a steel wire, a compound wire known as hard-drawn copper-clad steel wire is produced. This has been well tested, and experience has shown it to be an excellent material for many purposes. The wire can be made up in the form of cables if desired, which, when used as conductors for overhead transmissions, will have greater strength than cables made entirely of copper, and lower resistance than cables made entirely of steel. The two metals are intimately and permanently welded together by means of a special copper-iron alloy, and the relative quantities so adjusted that the finished wire has a conductivity of 35 per cent. to 40 per cent. of a copper wire of the same diameter. The ultimate tensile strength of commercial copper-clad wire of various sizes is approximately as below:

Gage No. B. & S.	Diameter, inches	Breaking weight, pounds	Number of times stronger than copper of same diameter
000	0.410	7600	1.15
0	0.325	5400	1.20
2	0.258	3700	1.23
4	0.204	2700	1.38
6	0.162	1750	
8	0.128	1200	1.47
10	0.102	780	

36. Stranded Cables with Steel Wire Core.—The central wire of a stranded conductor may be of galvanized steel, or a small diameter steel cable may be used for the core. This increases the strength, especially in the case of aluminum cables, and a compound conductor of this sort is useful for long spans on an aluminum wire transmission line.

It is usual to neglect the current-carrying capacity of the steel core, and calculate the conductivity on the assumption that all the current is carried by the strands of the higher conductivity metal. Composite cables can be made of steel and copper wires, but the strength of hard-drawn copper is so great that the gain due to the addition of the steel core is comparatively small.

The impedence of a steel core conductor will be higher than that of a conductor made entirely of non-magnetic material, but experiment has shown that the increase of impedence is practically negligible when there are two layers of copper wires spiralled in reverse directions on a central steel core: it would seem as if the current divided itself in two equal parts circulating in opposite directions, thus neutralizing any tendency to magnetize the steel core.

Hemp Core Cables.—When a stranded conductor is made up of one material only, the central wire is subjected to a greater strain than the wires that are spiralled around it. This difficulty can be overcome by using hemp for the central core. A hemp core cable will have slightly increased diameter for the same conductivity and greater smoothness of surface than a metal core cable, and these features will raise the critical voltage at which corona will form.

37. Skin Effect.—Imagine a straight length of cable of fairly large cross-section, through which a steady continuous current is flowing, the return circuit being a considerable distance away. The magnetic induction due to this current will not be only in the non-conducting medium surrounding the wire, but a certain amount—due to the current in the central portions of the cable—will be in the substance of the conductor itself. In other words, the magnetic flux surrounding one of the central strands of the cable will be greater than that which surrounds a strand of equal length situated near the surface. It follows that, if the circuit be now broken, the current will die away more quickly near the surface of the conductor than at the

center; and, for the same reason, on again closing the circuit, the current will spread from the surface inward.

If, now, the conductor be supposed to convey an alternating current, it is evident that, with a sufficiently high frequency (or even with a low frequency if the conductor be of large cross-section), the current will not have time to penetrate to the interior, but will reside chiefly near the surface. This crowding of the current toward the outside portions of the conductor has the effect of apparently increasing the resistance; and it follows that if I is the total current in a cable of ohmic resistance R, the power lost in watts would no longer be I^2R, as in the case of a steady current, but I^2R', where R'—which stands for the apparent resistance of the conductor—is k times greater than R, its true resistance. The multiplier k may be read off the diagram Fig. 20, or if preferred, it can be calculated by means of the formula:

$$k = \frac{1+\sqrt{1+F^2}}{2} \qquad (1)$$

where F is a factor proportional to the vertical distances on the diagram, that is to say, to the quantity *area of cross-section* $\times$*frequency*. The value of F for copper is:

$$F = 0.0105\ d^2f$$

and for aluminum,

$$F = 0.0063\ d^2f$$

where d is the diameter of the conductor in inches, and f is the frequency in periods per second. This formula and the curves of Fig. 20 are based on the assumption that the return current is at an infinite distance; but this assumption introduces no appreciable error when dealing with overhead transmission lines.

It will be observed that, so long as the product d^2f remains unaltered, the multiplier k is constant provided the material remains the same. Thus if, when doubling the frequency, the sectional area of the (circular) conductor is halved, the ratio $\dfrac{\text{resistance to alternating currents}}{\text{resistance to continuous currents}}$ remains unaltered.

In regard to the *material* of the conductor, the value of F in the formula is directly proportional to the specific conduc-

tivity of the metal so long as the frequency remains constant. Thus if *F* (or the value of the ordinates in the diagram, Fig. 20) is known for a conductor of given diameter, made of copper, its

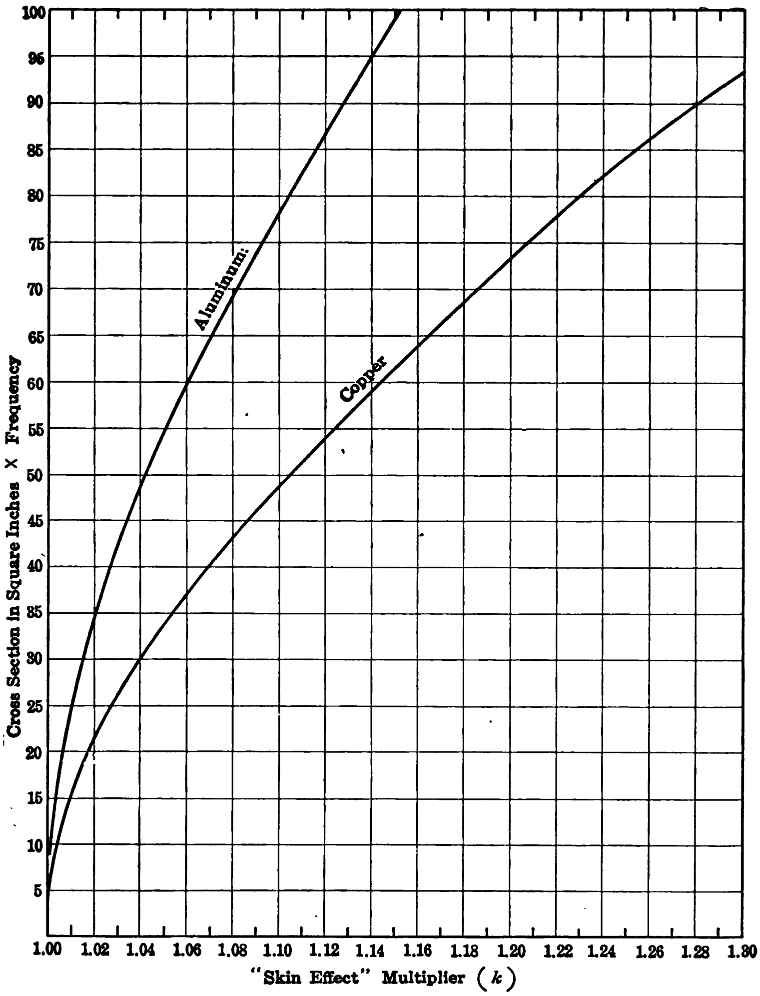

FIG. 20.—Diagram giving "skin effect" coefficient.

value for any other "non-magnetic" material is given by the ratio:

$$\frac{\text{conductivity of metal of conductor}}{\text{conductivity of copper}}$$

If the conductor is of iron (or other "magnetic" material), the value of k may be much greater than this ratio would indicate.

It is a not uncommon belief that when aluminum conductors are used in place of copper, the larger diameter necessary to give the same conductivity will lead to a greater loss through "skin effect"; but the above multiplying ratio makes it clear that the percentage increase of losses with alternating currents of the same frequency will be independent of the material of the conductor (iron excepted), because the greater sectional area necessary to maintain the same ohmic resistance of the lines when a wire of lower conductivity is used, is evidently exactly balanced by the higher specific resistance of the metal.

The increased pressure drop and I^2R loss on overhead lines at normal frequencies and with conductors of average size are usually very little greater with alternating than with continuous currents; but when the material is iron or steel the difference may be very noticeable, and in such cases as the rail return of an alternating current traction system, it should be taken into account.

38. Inductance of Transmission Lines.—In Chapter II, the formula for calculating the counter e.m.f. due to inductive reactance was given in Article (8) as

$$\left.\begin{array}{r}\text{Volts induced per mile}\\ \text{of single conductor}\end{array}\right\} = 0.004656 \times f \times I \times \log \frac{D}{r}$$

This is convenient and simple, but not strictly accurate, as it does not take into account the magnetic flux within the material of the conductor.

Maxwell's formula is:

$$\left.\begin{array}{r}\text{Millihenrys per mile}\\ \text{of single conductor}\end{array}\right\} = 0.08046 + 0.741 \log \frac{D}{r}.$$

This can be put into more convenient form thus:

$$\text{Millihenrys} = 0.741 \left[\log \frac{D}{r} + \frac{0.08046}{0.741}\right]$$

but the antilogarithm of the second term in the brackets is 1.284, therefore the inductance can be written,

$$\text{Millihenrys} = 0.741 \times \log\left(1.284 \frac{D}{r}\right)$$

$$= 0.741 \times \log\left(2.568 \frac{D}{d}\right)$$

and since the back e.m.f. due to inductive reactance (on the sine wave assumption) is

$$\text{Henrys} \times 2\pi \times f \times I$$

the correct formula for the reactive volts can be written:

$$\left.\begin{array}{r}\text{Volts induced per mile}\\ \text{of single conductor}\end{array}\right\} = \frac{0.741 \times 2\pi}{1000} \times f \times I \times \log\left(2.568\frac{D}{d}\right)$$

$$= 0.004656 \times f \times I \times \log\left(2.568\frac{D}{d}\right) \qquad (2)$$

where d is the diameter of the conductor.

Excellent *tables* giving inductive reactance in ohms per mile for different spacings and sizes of wires are given in the Standard Handbook for Electrical Engineers; these figures, when multiplied by the value of the current flowing in the conductor, give the induced volts as calculated by formula (2).

39. Impedence Drop. Regulation Diagrams.—The reader is asked to refer back to the fundamental regulation diagram (Fig. 9) on page 23.

There $C\ B$ is the resistance drop (E_R) equal to $I \times R$, where R stands for the true resistance multiplied, if necessary, by the "skin effect" factor. The reactance voltage (E_L) is represented by $D\ C$, and the *impedence drop* is $D\ B$, of which the numerical value is

$$\sqrt{(E_R)^2 + (E_L)^2}$$

This quantity does not, however, represent the difference in pressure between the generating and receiving ends of the line; this may be calculated as explained in Chapter II, and it will depend not only on the resistance and size and spacing of the conductors, but also on the power factor of the load, since this will determine the position of the point B on the dotted circle. It is the position of the point B on the circle that modifies the ratio of the length $F\ D$ to the length $B\ D$, even if the proportions of the impedence triangle $B\ C\ D$ remain unaltered. Problems can be solved graphically by drawing the diagram, Fig. 9, to the proper scale; but the objection to this method is that the radius $O\ B$ is generally large in proportion to the quantities represented by the impedence triangle, and the process is either tedious or the results are unsatisfactory. The field for ingenuity in the construction of practical charts based on the fundamental diagram (Fig. 9) is very great. One of the theoretically correct methods

of obtaining graphical solutions is with the aid of the Mershon diagram.

In Fig. 21 curves concentric with the dotted circles of Fig. 9

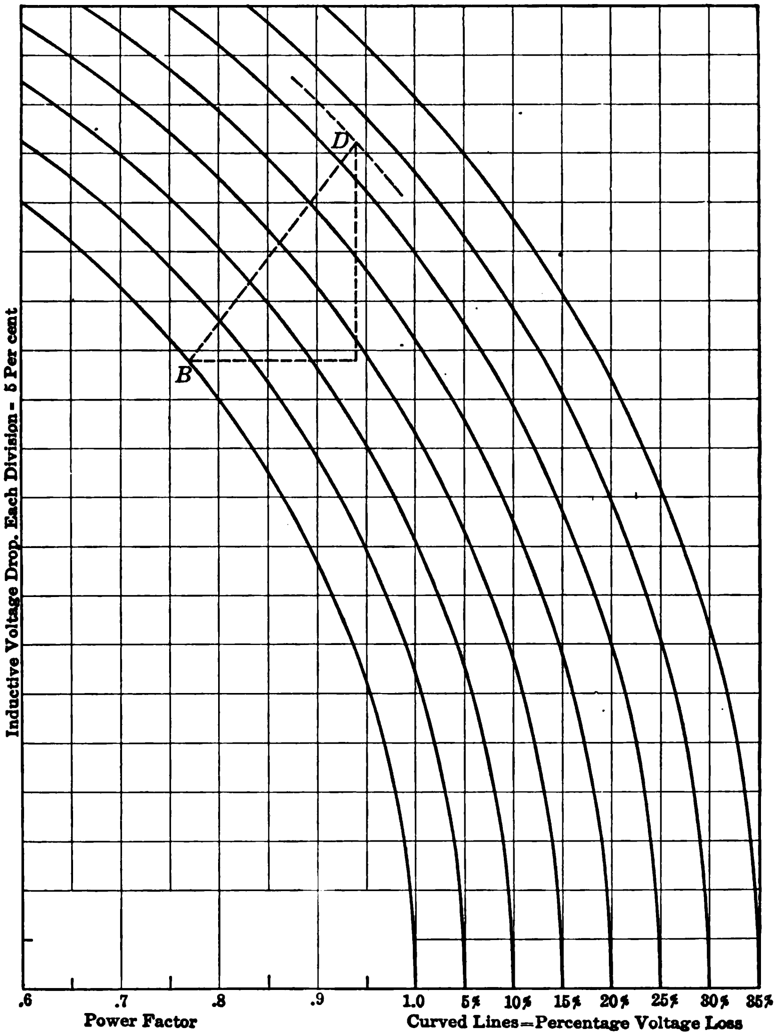

FIG. 21.—Mershon diagram for determining voltage regulation.

are drawn on a piece of squared paper from a center which lies on the prolongation of the base line, but at a considerable distance outside the diagram. The radius of the inner circle is 10

(or 100) divisions in length, and the projection on the horizontal axis of any point B is therefore the cosine of the angle $B\ O\ A$ of Fig. 9 and it indicates directly the power factor at the receiving end. By expressing the calculated resistance and reactive voltage drops as *percentages* of the receiving end pressure, the impedence triangle can readily be drawn to the proper scale, and by making the spaces between the circles equal to the side of the squares on the divided paper, the regulation, or difference between generating and receiving end pressures ($F\ D$ in Fig. 9), can be read off the diagram as a percentage of the receiving end pressure.

As an example of the use of the diagram, suppose the power factor of the load is 0.77, and that the calculated components of the pressure drop are,

Resistance volts = 17 per cent. of receiving end pressure.
Reactance volts = 22 per cent. of receiving end pressure.

From the division on the horizontal axis corresponding to power factor 0.77 follow the vertical ordinate until it meets the

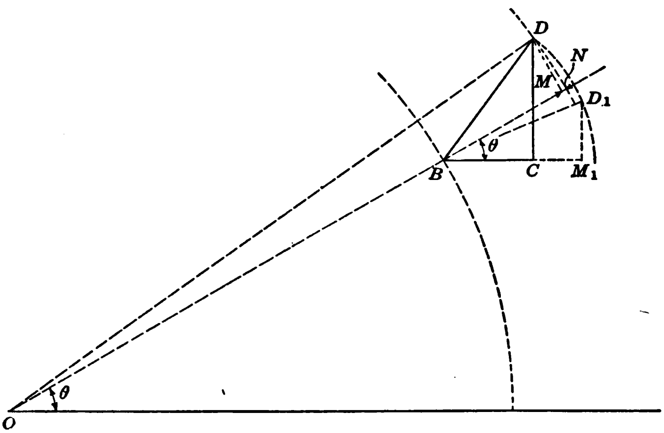

FIG. 22.—Vector diagram illustrating approximate method of determining regulation.

inner circle at B; then measure horizontally 17 divisions, and vertically 22 divisions, and the point D which lies on the dotted circle 27.5 divisions larger in radius than the inner circle (which is described with a radius equal to 100 divisions) indicates that the difference in pressure between generating and receiving ends of the line is 27.5 per cent. of the receiving end pressure.

Consider now Fig. 22, which is merely a repetition of the

fundamental diagram, Fig. 9, with the addition of a few lines. Drop the perpendicular $D\ M$ on the radius $O\ B$ extended beyond the point B. It will be seen that when the angle $D\ O\ B$ is small, that is to say, when there is little difference between the power factors at the receiving and generating ends of the line, the distance $M\ N$ will be very small, and for nearly all practical purposes the voltage regulation may be expressed by the ratio $\frac{B\ M}{O\ B}$ instead of $\frac{B\ N}{O\ B}$, this last being theoretically correct and as given by the Mershon diagram. By adopting the alternative construction, and replacing the arc $D\ N$ by a straight line perpendicular to either $O\ D$ or $O\ B$, the necessity for drawing circles from a center outside the limits of a practical diagram is avoided.

The method used by Professor L. A. Herdt for the calculation of transmission lines (originally described in the *Electrical World* of Jan. 2, 1909) employs this approximation; and it is also employed in the method about to be described, which the writer has found very quick and convenient for practical calculations.

It will be observed that if the impedence triangle $B\ C\ D$ (Fig. 22) be moved round on the point B through an angle θ, so that the hypotenuse $B\ D$ now occupies the position $B\ D_1$, the perpendicular dropped from D_1 on the extension to the horizontal line $B\ C$, meets this line at the point M_1, the distance $B\ M_1$ being obviously equal to $B\ M$. Thus, by revolving the impedence triangle through an angle θ such that $\cos\theta =$ the power factor of the load, the projection of the hypotenuse on any line parallel to the current vector will be a measure of the volts lost in transmission.

To apply this method in practice, nothing more is required than a piece of squared paper and a piece of tracing paper. The squared paper is divided into any convenient number of equal parts to represent, horizontally, the percentage ohmic drop of voltage, and, vertically, the percentage inductive voltage drop, as indicated in Fig. 23. On the vertical axis on the left-hand side of the diagram, a power factor scale is provided. This is merely an arbitrary length divided into ten equal parts with suitable subdivisions so chosen as to make use of the horizontal ruling of the squared paper. This scale is used for turning the hypotenuse of the impedence triangle through the proper angle, as will be explained shortly.

The method of using the diagram is best explained by working

out an example. The data used for illustrating the Mershon chart will be suitable:

Power factor of load $= \cos\theta = 0.77$
Ohmic volts $= 17$ per cent.
Inductive volts $= 22$ per cent.

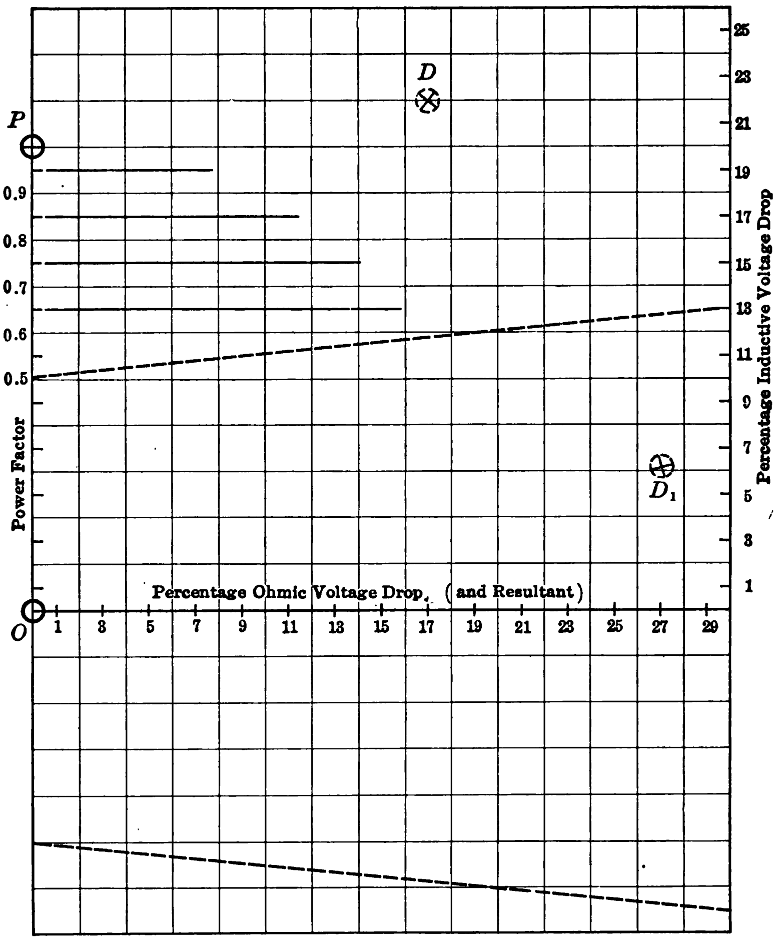

FIG. 23.—Author's diagram for determining voltage regulation.

Place a piece of tracing paper over the diagram (Fig. 23) and mark upon it the point D, 17 divisions to the right of the vertical axis, and 22 divisions above the horizontal axis. Then, with a pin or

pencil held at the point O, move the tracing paper round through an angle of 39 degrees 40 minutes ($\cos 39° 40' = 0.77$), bringing the point D to D_1, and read on the horizontal axis the distance 27.1, which is the difference between the pressures at the two ends of the line, expressed as a percentage of the receiving-end pressure. The result, as read off the Mershon diagram was 27.5, which might at first sight be thought to be more nearly correct; but, as a matter of fact, the writer's method will usually give more accurate results notwithstanding that the solution is not theoretically correct. This is because the impedence triangle is very much larger for the same size of chart than in the Mershon diagram; and the subdivisions are more easily read. When the point D_1 falls between the two inclined dotted lines drawn on the diagram (Fig. 23), this is an indication that the error introduced by substituting the chord for the arc is less than half of 1 per cent.

The use of the power factor scale will now be explained. It is not necessary, as suggested in working out the example, to calculate the angle θ from the value of the power factor and then set out this angle on the diagram. If in addition to marking the point D on the tracing paper, the position of the point P is also marked, it is merely necessary to move the tracing paper round (on the center O) until the point P falls on the horizontal line representing the required power factor, as this will ensure that the point D has been moved through the proper angle. The reason of this will be obvious to anyone possessing even the most elementary knowledge of trigonometry.

40. Capacity of Transmission Lines.—The formula given in Chapter II (Article 10) for the capacity in microfarads per mile of conductor, as measured between wire and neutral was:

$$C_m = \frac{0.0388}{\log \frac{D}{r}} \tag{3}$$

This formula is not theoretically correct and would not be applicable if the distance D were very small in relation to the diameter of the wire (or the radius r); but for overhead transmissions it is a serviceable formula, and, in the writer's opinion, it may be used in all practical calculations. The question of capacity on overhead lines is, however, one of very great importance, especially in view of the increasing pressures and distances of transmission; and it is felt that some space should be devoted

to it, even if it be only to sum up our present knowledge on this subject, and refer the reader to sources from which he can obtain more complete information.

The exact formula,[1] which gives the linear capacity in microfarads per mile between two cylindrical parallel wires is

$$C_m = \frac{0.0194}{\log\,[a+\sqrt{a^2-1}]} \tag{4}$$

where $a = \frac{D}{2r}$; but it is more generally useful to consider the capacity as being measured between one wire and the neutral potential surface. This will be *twice* the value of the capacity as measured between the two wires; but, when calculating the charging current, it is the voltage *between wire and neutral surface* that must be taken, if this latter value of the capacity is used.

The formula (4) may be put into another form which is very convenient if tables of hyperbolic functions are available.

In formula (4) common logarithms are referred to in the denominator; but by making the proper correction to the numerator and substituting naperian logs. the denominator becomes $\log_e(a+\sqrt{a^2-1})$ which is the quantity of which the hyperbolic cosine is a. Thus, the inverse hyperbolic cosine of a or $\cosh^{-1}a$ is the equivalent of $\log_e\,(a+\sqrt{a^2-1})$; and with the corrected numerator, the formula (4) becomes,

$$C_m = \frac{0.0447}{\cosh^{-1}a} \tag{5}$$

If the capacity per mile of single conductor, measured between wire and neutral, is required, the numerators of these formulas must be doubled, and the correct formula can be written either

$$C_m = \frac{0.0388}{\log\,(a+\sqrt{a^2-1})} \tag{6}$$

or,

$$C_m = \frac{0.0895}{\cosh^{-1}a} \tag{7}$$

Some excellent practical diagrams based on these formulas are to be found in an article by Dr. A. E. Kennelly which appeared in the *Electrical World* of Oct. 27, 1910.

The approximate formula given in Chapter II may be written

$$C_m = \frac{0.0388}{\log 2a}$$

[1] H. Pender and H. S. Osborne in *Electrical World*, Sept. 22, 1910, p. 667.

and, by comparing this with the correct formula (6), it will be seen that the first gives results slightly smaller than the true values; but when a is large, that is to say, when the distance between wires is many times the diameter, the error is negligible. The error only becomes appreciable if a is less than 10, and even if a is as small as 4 (a quite impossible state of things on an overhead transmission with bare wires), the error would be only 0.8 per cent.

41. Capacity of Three-phase Lines.—The formulas in the last article give the capacity between two parallel wires as measured from wire to neutral, and in the case of a single-phase transmission, the capacity between the two wires would, as it were, consist of two such capacities in series, and would therefore measure *half* the value given by these formulas, all as previously mentioned. It should, however, be noted that it makes no difference

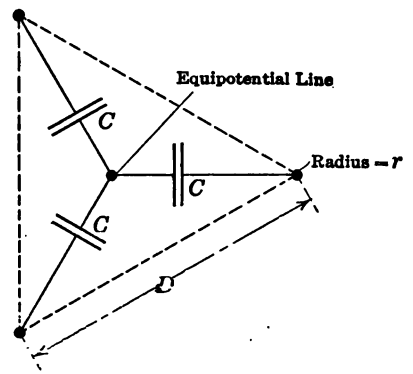

Fig. 24.—Distribution of capacity. Three-phase transmission.

which value of the capacity is taken for the purpose of calculating the charging current, provided proper attention is paid to the potential difference available for charging the condenser. In the case of the single-phase transmission, the pressure available for charging the two imaginary condensers in series, is exactly twice the pressure between one wire and neutral, so that it is not possible to make any mistake in the calculation of charging currents.

Consider, now, a three-phase transmission with the conductors occupying the vertices of an equilateral triangle, as indicated in Fig. 24. If the diameter r of the conductors and the distance D between them are the same as in the case of a single-phase

transmission, then the capacity *as measured between the wire and neutral* is the same for the three-phase as for the single-phase transmission; but the charging current is different because the potential difference across each imaginary condenser is no longer $\frac{E}{2}$ but $\frac{E}{\sqrt{3}}$, where E stands for the voltage between wires. By treating the three-phase system—or indeed any polyphase system—as a combination of several single-phase systems each having a condenser connected between conductor and ground, the calculation of capacity currents becomes a comparatively simple matter, unless great refinements and scientific accuracy are aimed at.

It has been shown by Mr. Frank F. Fowle[1] and other careful investigators in this field, that the presence of the conducting ground or other neighboring circuits affects only very slightly the capacity between the conductors of an overhead transmission. By systematic transposition of wires on a long transmission, so that each conductor occupies the same position relatively to ground and parallel circuits over the same portion of the total distance, even these slight unbalancings of the charging currents can be corrected if desired.

42. Value of Charging Current.—The charging current in amperes on a line l miles long will be, according to formula (8) in Chapter II,

$$I_c = 2\pi f C_m l V_n \times 10^{-6}$$

where C_m is the capacity in microfarads per mile, as given by the formulas (6) and (7), or by the approximate formula (3). The formula for calculating the charging current is based on the supposition that the e.m.f. variation is harmonic (simple sine wave); but, owing to irregularities and peaks in the pressure wave, the charging current in practice is generally greater than as calculated by the above formula. It is usually safe to add 25 per cent. to the calculated value, and it does not need much irregularity in the wave form to increase the charging current 30, and even 40, per cent. It is for this reason that an error of less than 1 per cent. in the calculation of electrostatic capacities is of no practical importance. Then again, the capacity is not concentrated at one point, but is distributed all along

[1] "The Calculation of Capacity Coefficients for Parallel Suspended Wires," *Electrical World*, Aug. 12, 19, and 26, 1911.

the line. A close approximation is obtained by assuming the whole of the capacity concentrated at a point midway along the line; or two separate capacities, each equal to half the total, may be assumed to be connected one at each end of the line. These approximations involve no errors of practical importance; but it is not impossible to solve transmission line problems without making more or less arbitrary assumptions of concentrated capacities. A notable solution of these problems with distributed capacities was given by Dr. Harold Pender in the *Electrical World* of July 8, 1909. Dr. Pender has successfully evolved comparatively simple working formulas where previous workers in this field have expressed the results of their calculations in complicated equations.

43. Pressure Regulation on Transmission Lines.—Notwithstanding that a rise of pressure at the distant end of a transmission line may be obtained on open circuit or on light loads, owing to the combined presence of inductance and capacity, it does not follow that the presence of distributed capacity is an advantage from the point of view of regulation. The capacity of a transmission line will to some extent counteract the effects of induction; but the power factor of the load will be an important factor in determining the extent to which this tendency will have practical results. Moreover, if the load is heavy and the current large, the induced volts may be out of all proportion to the capacity, seeing that they are dependent upon the amount of the main current, whereas the capacity depends only on the size and spacing of the wires. Much may be done by the splitting up of one heavy circuit into two or more parallel circuits, the conductors of which will be of smaller diameter and will carry smaller currents. This will not only considerably reduce the reactive volts because of the smaller current in the several circuits, but on account of the reduction in the *diameter* of the wires, it will, with the same spacing between wires, increase the inductance and reduce the capacity as will be evident from a study of the foregoing formulas.

When abnormal conditions cannot be remedied by the subdivision of the circuit, the ratio of leading to lagging currents can be controlled by introducing inductance in series with the line, or—when the opposite effect is required—by connecting condensive reactance at suitable points. It is well known that synchronous alternating-current motors will act as rotary con-

densers if the fields are over excited, because the component of the armature current required to counteract the excess of field excitation will be 90 time degrees in *advance* of thee.m.f. By installing synchronous machines at the receiving end, the power factor can be controlled; but the use of such auxiliary machinery is best avoided if possible.

The power factor of the load is not always easy to estimate; it may consist of induction motors of various sizes, together with lighting circuits, all having different power factors. If several circuits of different power factors are connected in parallel, the joint power factor can be calculated by the formula:

$$\text{Cos}\ \theta = \frac{1}{\sqrt{1+\left(\frac{I_1 \sin\theta_1 + I_2 \sin\theta_2 + \ . \ . \ .}{I_1 \cos\theta_1 + I_2 \cos\theta_2 + \ . \ . \ .}\right)^2}}$$

where I_1, I_2 . . . are the currents taken by the various circuits of power factors cos θ_1, cos θ_2, . . . etc.

If the pressure drop on a straight transmission is considerable at full-load, the pressure can be kept within reasonable limits at the receiving end by raising the voltage at the generating end as the load increases. This method may not be applicable if there

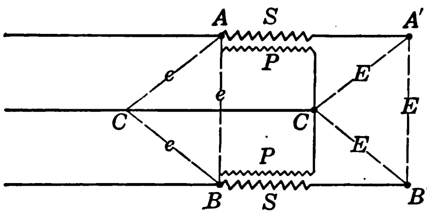

FIG. 25.—"Boosters" on three-phase system.

are several substations or branch circuits on the main line. In such cases, regulation may be necessary at the receiving points. Variable ratio transformers may be used, either of the type with movable iron core, or of the more usual type with tappings from primary or secondary windings taken out to a regulating switch. Unless the current to be dealt with is very large, such regulating devices are more conveniently placed in the connections from the secondary side of the step-down transformers.

On a delta-connected three-phase system it is not necessary to provide more than two single-phase regulators as these may be connected up as indicated in Fig. 25. ere H*P* and *S* represent

respectively the primary and secondary windings of the variable-ratio transformers. That these are capable of raising the voltage equally on all three phases to the extent of the volts induced in the secondary coil S will be clear from an inspection of Fig. 26. In this diagram, $A\ B$, $B\ C$, and $C\ A$ are the three vectors representing the pressures e before boosting up: $A\ A'$ and $B\ B'$ represent the added volts between the terminals A and A' or B and B' (Fig. 25). These added volts are evidently in phase with the pressures indicated by the vectors $C\ A$ and $B\ C$ respectibely,

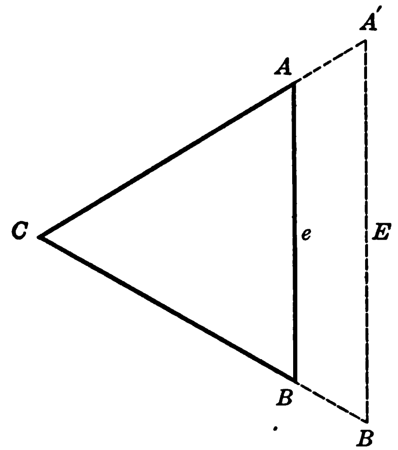

FIG. 26.—Vector diagram—two "boosters" on three-phase system.

because the potential difference at the secondary terminals of a well designed transformer is always in phase with the primary impressed e.m.f. It is only necessary to complete the triangle $C\ A'\ B'$ to see that the two transformers connected up in the manner described will do all that is required in the way of raising the pressure on the three-phase circuit.

44. Effect of Boosting Voltage at Intervals along a Transmission Line.—If a long transmission line, designed for working at a given pressure of (say) 100,000 volts, can be worked as a 100,000-volt line at all times through its entire length, it will be more efficient than if only a portion of it is working as a 100,000-volt transmission while portions farther from the generating end are working at (say) 80,000 volts. By installing boosters along the line to maintain the pressure at or near the maximum working value, whatever the load may be, economies may sometimes be

effected. It is true that the energy put into the line at intermediate points cannot be cheaper—and, indeed, is usually more costly—than the energy supplied to the line at the generating end; but the booster system allows of the pressure being kept up all along the line, thus effecting economy; provided always that the losses in the boosters themselves, their maintenance, and the necessary allowances for interest and depreciation, do not counterbalance the saving.

As an example, consider a single-phase line conveying a current of 100 amp. at an initial pressure of 20,000 volts. Suppose the drop in pressure in the whole length of line to be as great as 10,000 volts; this will leave only 10,000 volts at the receiving end.

The power put into the line at generating end $=2000$ kw.
The loss in the line $=1000$ kw.
The power available at receiving end $=1000$ kw.
Hence, *efficiency of line* $=50$ per cent.

Now imagine a booster to be introduced at a point half-way along the line. This booster may be considered as a highly insulated alternator of 500 kw. capacity, capable of generating

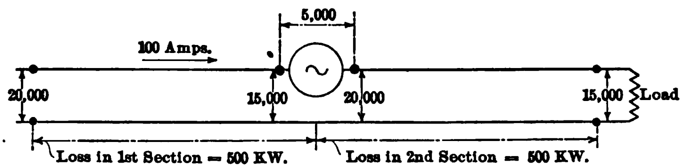

Fig. 27.—Method of maintaining pressure on long line.

100 amp. at 5000 volts, the arrangement being as shown in Fig. 27. The drop in the first section of the line is, as before, 5000 volts; and the drop in the second section is evidently similar—namely, 5000 volts—which means that the total amount of power dissipated in the line is the same as it was before the booster was introduced. But by providing this booster at the middle point of the line, it has been possible to raise the pressure at this point up to the initial value of 20,000 volts, with the result that 15,000 volts (1500 kw.) are available at the receiving end. The additional power available for useful purposes has, of course, cost something to produce; but the point to be noted is this: by keeping up the pressure, it has been possible to transmit a greater

amount of energy to the receiving end of the line *without increasing the losses in the conductors.* If, for the sake of simplicity, the losses in the booster are neglected, the line efficiency is arrived at thus:

Power supplied to the line $=2000+500=2500$ kw.
Power lost in the line $=1000$ kw.
Power available at receiving end $=1500$ kw.

$$\text{Line efficiency} = \frac{1500}{2500} = 60 \text{ per cent.}$$

Boosters may be arranged to take their power from the generating end of the line; that is to say, they may take the form of variable-ratio transformers, with hand or automatic regulation,

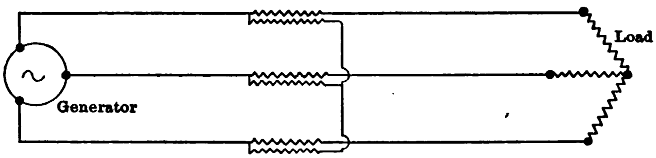

FIG. 28.—Transformers connected as "boosters" on transmission line.

connected up as indicated in Fig. 28. Transformers so connected will provide the additional volts at the cost of a corresponding loss of current.

45. Fault Localizing.—The location of broken insulators, crossed or fallen wires, or any fault leading to unsatisfactory or interrupted service, must be dealt with by the operating staff; and any but the briefest reference to these matters would be beyond the scope of this book.

The usual methods of testing are explained in many text-books, and in the electrical pocket-books and hand-books; but the well-known Varley and Murray loop tests are not always satisfactory on high-voltage transmission lines. Then, again, the conditions in regard to grounded or ungrounded neutrals, transformer connections, positions of section switches, telephonic facilities, etc., are so variable that it would be a difficult matter to lay down rules to be followed in emergencies, except in connection with a particular system; but such rules should be laid down by the chief operating engineer, and rigidly adhered to. By giving careful attention to the position of patrolmen's houses, switching and telephone stations, as referred to in Chapter I, much may be

done toward preventing long interruption of service in the event of accidents.

46. Surges: Standing and Travelling Waves.—The remainder of this chapter will be devoted to a brief consideration of the principal causes leading to the somewhat complicated phenomena still unfamiliar to the majority of practical engineers, which Dr. C. P. Steinmetz so ably treats of in his book on "Transient Phenomena," and which would lead to astonishing revolutions in the transmission of power by alternating currents if the length of line, or the frequency, or both, could be sufficiently increased to make use of some valuable properties peculiar to travelling waves.

The phenomena referred to depend mainly on the relation between the inductance and capacity of the circuit, and it is thought that a brief examination of the relations existing between these quantities on practical overhead transmission lines may be helpful in clearing the ground for further investigations; but, for a thorough study of the subject, the reader is referred to such authorities as Dr. Steinmetz, Dr. Kennelly, and others.

47. The Quantity $L \times C$.—The diagram Fig. 29 shows the arrangement of conductors of an overhead transmission which may

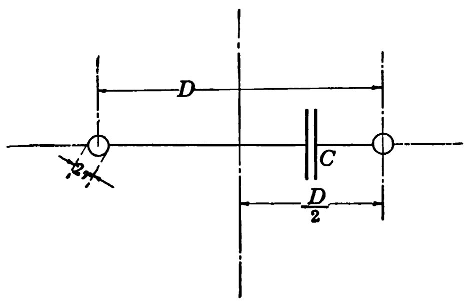

FIG. 29.

be either by single-phase or three-phase currents; but, if the latter, it must be understood that the vertical dotted line is no longer the neutral plane of a two-wire system, but that $\frac{D}{\sqrt{3}}$ is now the distance of any one of the three conductors from the neutral line, and C is the capacity measured between one conductor and neutral.

The approximate formula for the voltage drop *per mile of single*

conductor, due to the inductance of the line when the current is 1 amp., is

$$E_L = 0.004656 \times f \times \log \frac{D}{r} \tag{8}$$

where f is the frequency in cycles per second,

The coefficient of self-induction, in henrys, is

$$\mathbf{L}_h = \frac{E_L}{2\pi f} = 0.000742 \log \frac{D}{r} \tag{9}$$

The approximate formula for capacity per mile of conductor is,

$$C_m = \frac{0.0388}{\log \frac{D}{r}} \tag{3}$$

where C_m is the capacity in microfarads between one wire and neutral. Obviously, the product of (9) and (3) is constant irrespective of the diameter and spacing of the wires, and

$$C_m \times \mathbf{L}_h = \frac{1}{34700} \tag{10}$$

This relation becomes more interesting when the capacity and inductance are expressed in the same units.

One henry $= 10^9$ electro-magnetic units, and

One microfarad $= 10^{-15}$ electromagnetic units; thus,

$$\mathbf{L}C = \text{quantity } (9) \times 10^9 \times \text{quantity } (3) \times 10^{-1}$$

and the reciprocal, or

$$\frac{1}{\mathbf{L}C} = 3.47 \times 10^{10}$$

The square root of this quantity is

$$\frac{1}{\sqrt{\mathbf{L}C}} = 186{,}000$$

which is the velocity of light in miles per second.

Thus the reciprocal of the quantity $\sqrt{\mathbf{L}C}$, on an overhead transmission line, is constant and approximately equal to the velocity of light. (The velocity of propagation of electric waves is practically the same as that of light.)

That the quantity $\frac{1}{\sqrt{\mathbf{L}C}}$ has the *dimension* of a constant velocity is seen by reference to the fundamental dimensions of **L** and C. In a medium such as air, of which both the permeability

and inductive capacity are unity, the product $\mathbf{L} \times C$ is of the dimension

$$\text{length} \times \frac{(\text{time})^2}{\text{length}}$$

and the dimension of the quantity $\frac{1}{\sqrt{\mathbf{L}C}}$ is therefore $\frac{1}{\text{time}}$, which is a constant velocity.

The formulas (9) and (3) are approximate only: the former does not take into account the flux of induction within the conductor itself; and, for a closer approximation for transmission line calculations, it will be better to use

$$\frac{1}{\sqrt{\mathbf{L}C}} = 183{,}000 \qquad (11)$$

as expressing the constant relation between capacity and inductance.

48. Relation between Charging Current and Inductive Pressure Drop.—The constant relation between inductance and capacity on a practical transmission line suggests the possibility of simplifications in certain transmission-line calculations. Mr. R. S. Brown[1] has, indeed, evolved a method of determining the electrical characteristics of a long line, based on the fundamental relation as given in formula (11). He points out that this relation will not be correct in the case of lead-sheathed cables or where the capacities between overhead conductors of a transmission line are appreciably influenced by the nearness of the earth or other conductors.

Since there is a definite and constant relation between the inductance and capacity of overhead conductors; there must obviously be a definite relation between charging current and induced volts, whatever may be the diameter or disposition of the wires. It will be interesting to determine this relation.

By inserting for $\mathbf{L}_h$ in formula (10) its equivalent value $\frac{E_L}{2\pi f}$ the relation $C_m = \frac{2\pi f}{34700\, E_L}$ is obtained; which, when corrected for the closer approximation to the actual speed of propagation of electric waves, as previously referred to, becomes

$$C_m = \frac{f}{5300\, E_L} \qquad (12)$$

[1] *Proc. Amer. Inst. E. E.*, Nov., 1911.

This is the relation between capacity to neutral, in microfarads, and, the induced volts *per ampere* for a single conductor of any overhead transmission.

On the assumption that the voltage alternates according to the simple harmonic law (sine wave), the charging current *per mile of single conductor*, due to electrostatic capacity, is

$$I_c = 2\,\pi f C_m E \times 10^{-6}$$

where E is the transmission voltage measured between conductor and neutral. By inserting for C_m its equivalent value by formula (12),

$$I_c = \frac{E \times f^2}{E_L \times 8.43 \times 10^8} \tag{13}$$

Thus, knowing the volts induced per mile of single conductor, per ampere of current, the charging current per mile of single conductor can readily be calculated. This formula will, on account of the sine wave assumption, give a result generally too small: the charging current, as measured on a practical transmission line, would probably be about 25 per cent. greater.

49. Relation $\frac{L}{C}$: Surges.—Given an electric circuit in which there is alternating or oscillating electric energy which is not dissipated in the form of heat through the ohmic resistance of the conducting circuit, it is obvious that, at the instant when the current wave is at zero value, the whole of the energy must be in the electrostatic field; and, at the instant when the pressure wave is at zero value, the whole of the energy will be in the electromagnetic field.

It can readily be shown that the electromagnetic energy stored in a given circuit is $\frac{LI^2}{2}$ watt-seconds; where L is the inductance in henrys, and I the maximum value of the current wave; and the electrostatic energy stored in the dielectric (the air surrounding the conductors) is $\frac{CE^2}{2}$ watt-seconds, where C is the capacity in farads, and E the maximum value of the e.m.f. wave.

Now, since these quantities must obviously be equal so long as the interchange of energy from one form to the other continues,

$$LI^2 = CE^2$$

or

$$\frac{E}{I} = \sqrt{\frac{L}{C}}$$

The quantity $\sqrt{\frac{L}{C}}$ is therefore seen to be of the nature of a resistance. This can be checked by referring to the fundamental dimensions of an inductance (or coefficient of self-induction) and of an electrostatic capacity.

The ratio $\frac{L}{C}$ is length $\div \frac{(\text{time})^2}{\text{length}}$ or $\frac{(\text{length})^2}{(\text{time})^2}$, the square root of which is $\frac{\text{length}}{\text{time}}$, being a *velocity*, and having the properties of a *resistance* or *reactance* which may be expressed in ohms. This quantity, $\sqrt{\frac{L}{C}}$ is what Dr. Steinmetz has called the *natural impedance* of the circuit.

In the case of an overhead transmission line, the ratio $\frac{L}{C}$ can be obtained from formulas (9) and (3). Thus

$$\frac{\text{Henrys}}{\text{Farads}} = 0.000742 \log \frac{D}{r} \div \frac{0.0388 \times 10^6}{\log \frac{D}{r}}$$

$$= 0.0191 \left(\log \frac{D}{r}\right)^2 \times 10^6$$

and

$$\sqrt{\frac{L}{C}} = 138 \log \frac{D}{r} \text{ ohms.} \qquad (14)$$

In practical overhead work, the limiting values for the ratio $\frac{D}{r}$ will probably be 800 and 50; which, when inserted in formula (14), show that the "natural impedance" of an overhead transmission line must lie between 400 and 230, or, to be well on the safe side, between (say) 500 and 200 ohms.

A knowledge of this quantity renders it possible to determine the maximum value of any surge pressures that can possibly occur on the line due to the sudden interruption of the current. Thus, if the "natural impedance" is 300 ohms, and the instantaneous value of the current at the crest of the wave is 200, the surge pressure, however suddenly the current is interrupted, cannot possibly exceed $200 \times 300 = 60{,}000$ volts; because this is the maximum value of the pressure wave necessary to store in the electric field the whole of the energy stored in the magnetic field at the moment when the current was interrupted. It is safe to say that, on a practical transmission line, the surge pressure is never likely to exceed 200 times the current in amperes; but, with heavy

currents, this may well be sufficient to break down insulation and cause considerable damage to power plant. It must not be overlooked that it is often more difficult to handle heavy currents at comparatively low pressures than small currents at the very highest pressures yet attempted. When the current is large, the opening of switch or fuse on full-load, or an accident causing a break in the circuit, with or without the formation of an arc across the gap, may lead to insulation troubles on many widely separated parts of the system; but, on a high-pressure system, even if the current were as large, the insulation is frequently so good that it will withstand without injury the stress imposed on it by the highest possible value of the surge pressure.

Of course these considerations do not take into account the effects of lightning, either by direct stroke or by induction, because in such cases a pressure from an outside source is impressed upon the circuit, and the potential of these atmospheric charges may be tens of times greater than any surge voltage due to a redistribution of the energy stored in the circuit itself.

Whether or not oscillations will be set up in the circuit, due to the interchange of stored energy between the electromagnetic and the electrostatic fields, will depend upon the resistance of the circuit in which the energy can be dissipated. This circuit need not be entirely metallic: it may include an air path through which the current can expend energy in the form of an arc in addition to what is spent in heating the conductors. If a long transmission line of negligible resistance be closed through a resistance equal to the "natural impedance" of the circuit, the discharge will take place without oscillations. Thus if

$$R > \sqrt{\frac{L}{C}}$$

there will be no oscillations set up on closing the circuit: the suddenly impressed voltage will not give rise to a surge of energy, but to a travelling wave of decreasing amplitude carrying energy along the line without oscillations. But if

$$R < \sqrt{\frac{L}{C}}$$

energy will oscillate between the magnetic and electrostatic fields with decreasing amplitude of pressure and current waves,

until the energy is dissipated in the resistance of the circuit.[1] In an imaginary circuit having appreciable capacity and inductance, but no resistance, what is known as a standing wave would be produced; that is to say, there would be a stationary oscillation of energy surging between the magnetic and dielectric fields, but no propagation of energy along the line; and this action would continue indefinitely with undiminished amplitude of current and pressure waves, provided the further assumption is made that no loss could occur over insulators or through "dielectric hysteresis."

A mechanical analogy to this state of things is the action of a pendulum hung from a frictionless pivot, and swinging in a frictionless medium, periodically converting its store of kinetic energy into potential energy, and *vice versa.*

In a practical circuit there can be both travelling waves and stationary oscillations of energy, the frequency of which is entirely independent of the working frequency of the system.

50. Natural Frequency of Circuit: Wave Length.—If C is the capacity in farads of a condenser, and E the maximum value of the potential difference in volts, the total charge in coulombs at the end of each half period will be $C \times E$. But,

$$\text{Quantity} = \text{Current} \times \text{Time}$$

hence the maximum charge in coulombs is:

$$I_a \times \frac{1}{4f}$$

where I_a is the *average* value of the current between zero and its maximum, and $\frac{1}{4f}$ is the time, in seconds, taken by the current to change from its maximum to its zero value. It follows that

$$\frac{I_a}{4f} = CE$$

If I is the *maximum* value of the current wave, this will, on the sine wave assumption, be $\frac{\pi}{2}$ times the *mean* value,

thus, $$\frac{I}{2\pi f} = CE$$

[1] The condition leading to an oscillatory discharge of a condenser is $R < 2\sqrt{\frac{L}{C}}$ but this assumes *massed* capacity, inductance, and resistance. The case of a transmission line with *distributed* capacity and inductance is somewhat different.

or,

$$\frac{1}{2\pi fC}=\frac{E}{I}$$

but, as previously shown, when energy is oscillating between the magnetic and electric fields, the ratio $\frac{E}{I}$ is equal to the "natural impedence," or to $\sqrt{\frac{\mathrm{L}}{C}}$, therefore,

$$\frac{1}{2\pi fC}=\sqrt{\frac{\mathrm{L}}{C}}$$

and

$$f=\frac{\sqrt{C}}{\sqrt{\mathrm{L}}\times 2\pi C}=\frac{1}{2\pi\sqrt{\mathrm{L}C}}$$

This is the rate, in periods per second, at which energy will oscillate in a circuit of negligible resistance. It is sometimes called the "natural frequency" of the circuit.

In transmission lines, if L and C are in absolute units *per mile* length of line,

$$f=\frac{1}{2\pi l\sqrt{\mathrm{L}C}}$$

where l is the length in miles. But the quantity $\frac{1}{\sqrt{\mathrm{L}C}}$ is, in this case, approximately equal to the velocity of light, as previously shown; and the natural frequency of an overhead transmission line is, therefore, approximately,

$$f=\frac{183000}{2\pi l}=\frac{29200}{\text{length in miles}}$$

The length of the complete circuit must be taken at twice the distance of transmission.

The length of the travelling waves will be,

$$\frac{\text{velocity of propagation of electric waves}}{\text{periodicity}}=2\pi l \text{ miles}$$

Thus it is seen to be an easy matter to calculate the frequency and wave lengths of oscillations set up on overhead transmission lines through suddenly applied charges of electric energy, or such impulses as are liable to be set up through sudden changes of e.m.f. or current. The problem is, however, greatly complicated when the effect of transformers and other apparatus connected to the line has to be taken into account.

The length of a circuit of overhead conductors along which an electric impulse will travel and return to the starting point in the time of one-half period is obviously,

$$\frac{183000}{2f}$$

and the distance of a straight transmission will be,

$$\frac{183000}{4f}$$

This is the *quarter wave length transmission,* the characteristics of which are quite different from those of the ordinary line.

Without artificial loading of the line to alter the ratio of **L** to C, which is not an easy matter, the distance of transmission equal to one-quarter of a wave length is very great at ordinary frequencies. Thus, with a periodicity of 60, the distance would be 760 miles; and, on a transmission of half this distance, or 380 miles (which might be commercially practicable), the frequency would have to be raised to 120.

The peculiar properties of such a line may be said to be largely due to the fact that the "half wave action" provides the charging current for the line, so that the generators have only to supply the load and the losses. The inductive pressure drop and the capacity current are, in fact, wiped out by the peculiar overlapping of the travelling waves of energy. The power factor of such a line would therefore be very nearly 100 per cent. on all loads, and the regulation, even if the load were inductive, would be surprisingly good.

CHAPTER V

Insulation and Lightning Protection

51. Insulator Materials.—The material most commonly used for insulators on high-tension overhead lines is porcelain; but glass, which is cheaper than porcelain, may sometimes be used to advantage on the lower-voltage lines. Insulators made of special molded materials such as "Electrose" have the advantage over both glass and porcelain in that they are lighter in weight and less liable to fracture from mechanical shocks or high-pressure discharges. This material is used in preference to porcelain by the Canadian Niagara Power Co. on its high-voltage transmission lines. Glass is a material of high resistance and dielectric strength and makes excellent insulators for pressures up to about 20,000 volts. The fact that it is pervious to light tends to discourage spiders and cocoon-spinning insects, which ordinarily find their way to the interior of insulators; but, on the other hand, more moisture will condense on the inside surfaces of glass than on porcelain insulators, thus attracting dust and dirt. Glass cannot withstand great and sudden changes of temperature so well as porcelain.

52. Design of Insulators.—The design of an insulator, to comply with any given specification, is a matter which concerns the manufacturer, who has been compelled of late years, owing to the rapid increase of working pressures, to devote his attention to the principles underlying the correct and economical design of insulators for high-pressure lines, and who is therefore something of a specialist on this particular subject. The transmission-line engineer should understand the principles underlying the correct design of overhead insulators; but it is suggested that, however great his knowledge of the subject, he will be well advised to leave details of design to the manufacturer, and even to use standard types when possible.

The design of insulators for the lower voltages is a comparatively simple matter, and the difficulties increase in proportion to the increase of pressure, although the introduction of the suspen-

sion type, which permits of many units being connected in series, considerably simplifies the problem so far as the higher pressures are concerned.

It is important to bear in mind that every insulator is necessarily a more or less complicated condenser, and it can generally be considered as a number of separate condensers in series, the dielectric being alternately air and porcelain. The current passing from line to ground is partly a leakage current over the surfaces (the leakage *through* the porcelain being generally negligible), and partly a capacity current. This capacity current spreads itself over the high resistance surfaces of the insulator material in a way which will depend upon the surface conductivity and on the spacing and disposition of the various parts. It is well to keep the electrostatic capacity as low as possible, but it is of equal if not greater importance so to distribute it by a scientific arrangement of the component parts that abnormal stresses will not occur locally, as these may puncture or damage the insulator at one particular part, while a more carefully designed insulator of lighter weight may withstand a greater total breakdown pressure, because proper attention has been given to this important matter of capacity distribution. The effect of rain on the exposed surfaces of an insulator is to increase the capacity, and this will generally lower the flash-over point; but the increased surface conductivity has the effect of equalizing the potential distribution; and in the case of a large number of condensers in series, such as occurs especially with the suspension type of insulator, it has actually been observed that this equalizing of the potential distribution may cause the flash-over pressure of the wet insulator to be no lower than the flash-over pressure of the same insulator when dry.

If I is the charging current passing through a condenser, and E the potential difference causing this flow of current, the relation between the capacity, C, and these two quantities is $E = \frac{I}{C}$ in the case of an alternating current of definite frequency.

When a number of condensers are connected in series, the current, I, is the same through all the condensers, and the potential difference across any one condenser is therefore inversely proportional to the capacity. This is one of the main points to bear in mind in the design of insulators for high voltages.

With the pin type of insulator, a number of sheds or petticoats

hanging close to the pin, with small air spaces between them, will not be effective, because, although the leakage path may be long, the capacity is high. The air spaces between the partially conducting surfaces of the porcelain may, in such cases, be little thicker than the porcelain sheds, but the dielectric flux-constant of air is only about one-fifth that of porcelain. The remedy consists in spreading the petticoats away from the pin, the outer shed in some designs being almost horizontal. This outer shed has, in some cases, been replaced by a metal shield. The proposal to depart to this extent from previous practice was made by a firm of German manufacturers,[1] and it is the result of a scientific study of insulator problems. Apart from the advantage of lightness, which permits of a thin metal shed being made of larger diameter than would be permissible if the material were porcelain, the charging current will spread itself more uniformly over the surface of the outer shed, and so prevent the concentration of potential at the point where the conductor is tied to the insulator. An insulator of this type may flash over at a somewhat lower pressure than if the upper hood were of porcelain, but under wet conditions, the flash-over pressure may be higher.

With the *suspension type* of insulator, the conductor is hung below the point of support (which is usually grounded) at the end of a string of insulator units connected to one another by metal links. The potential difference which will cause a flash-over or breakdown on such a series of insulators will not be in direct proportion to the number of insulators in the string. This is due to the unequal distribution of the potential differences, which is again a question of relative capacities. The design of the individual units may appear to be good, and yet a string of such insulators, if these are not specially designed to fulfil certain requirements, may give surprisingly unsatisfactory results. A factor of importance is the ratio $\frac{\text{mutal capacity}}{\text{capacity to ground}}$ which determines the potential distribution; and this ratio will depend not only on the shape and size of the porcelain units, but also on the metal caps or means of attachment, and the spacing between units. With increasing separation between the units, the flux density at the points of attachment diminishes up to a certain distance at which it becomes constant, notwithstanding any further increase of the separation. Mr. F. W. Peek, Jr., in his able paper read

[1] The Hermsdorf Porcelain Company.

before the American Institute of Electrical Engineers[1] uses the term "string efficiency" to denote the ratio

$$\frac{\text{arc-over voltage of } n \text{ insulators}}{n \times \text{arc-over voltage of single insulator}}$$

and this is a very convenient term for use in comparing strings of different insulators, each consisting of a series of units of one particular size and shape.

In actual tests made by Mr. Peek, the dry arc-over required 1 3/4 times the pressure of the wet arc-over on a single unit; but, on a string of nine such units, the wet and dry flash-overs occurred at the same voltage. The "string efficiencies" of these insulators on the dry test were as follows:

No. of units in series	Flash-over voltage	Efficiency, per cent.
1	74,000	100
2	137,000	93
3	186,000	84
4	238,000	80.5
5	281,000	76
6	318,000	72

Another series of tests, quoted by Mr. W. T. Taylor,[2] gives efficiencies as below:

No. of units in series	Flash-over dry test	String efficiency dry	Flash-over wet test	String efficiency wet
1	90,000	100	56,000	100
2	160,000	89	90,000	80
3	220,000	81.5	130,000	77.5
4	274,000	76	175,000	78
5	310,000	69	220,000	78.5
6	340,000	63	265,000	79

When the mutual capacity is small relatively to the ground capacity, there soon comes a point beyond which it is useless to put more insulators in the string. Short strings, of well-designed and properly spaced units, will often be more effective and less costly than a longer series of insulators of which the single units may have excellent insulating properties, but which have not been

[1] *Proc. A. I. E. E.*, Vol. XXXI, No. 5, May, 1912.
[2] *Journal Inst. E. E.*, Vol. XLVI, No. 207. May, 1912.

designed for the particular requirements on scientific lines. It is sometimes possible to increase the arc-over voltage of a string of insulators by *reducing* the distance between consecutive units; a fact that it is difficult to understand unless the importance of the proper capacity distribution has been realized. A well-arranged string of properly designed units not necessarily similar in shape and size (in fact preferably not identically of same size), is less liable to damage by lightning and kindred phenomena than a series of insulators of low "string efficiency."

It is well to bear in mind that an insulator may, and does, behave differently when subjected to high-frequency charges as induced by lightning disturbances, than when the difference of potential is applied at the normal frequency of the transmission line. The distribution of potential among a number of condensers in series has already been referred to, and this should be independent of frequency, but a high-tension insulator consists of many capacities in series with resistances, such as the leakage paths through the body of the material and over the surfaces, and the distribution of the total potential drop across a condenser and resistance in series, even if the latter may be considered non-inductive, is very different at high than at low frequencies. This is probably one of the chief reasons why insulators that will flash over rather than puncture on tests conducted at the ordinary line frequency, will sometimes fail through puncture during atmospheric electrical disturbances.

53. Pin Type Insulators.—This type of insulator is used for pressures up to 60,000 volts, but the suspension type, made up of two or more units, will generally be more satisfactory and economical for pressures above 50,000 volts. The pin type of insulator becomes too heavy and costly when designed for the higher voltages, and, owing to the great length of the supporting pin, the bending moment near the point of attachment to the cross-arm tends to become excessive.

Wooden supporting pins are not recommended for high-tension work; they are rarely used on lines working at pressures above 33,000 volts. Metal pins are generally preferable.

Fig. 30 shows the sparking distances as usually measured. The dotted line on the left side of the drawing shows the length of path on a dry flash-over. When the exposed surfaces of the insulator are wet with rain, these surfaces are looked upon as conductors, and the flash-over distance is the sum of the separate

distances *A*, *B*, and *C*; the lines *A* and *B* being usually drawn at an angle of 45 degrees to the vertical. In this particular case, the line *C* is also drawn at 45 degrees because the metal pin is surrounded by a porcelain base. The length of the pin should be

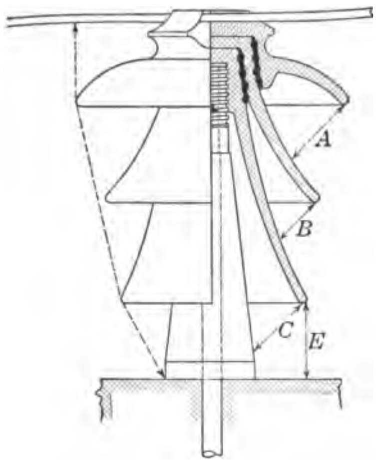

FIG. 30.—Sparking distances on pin type insulator.

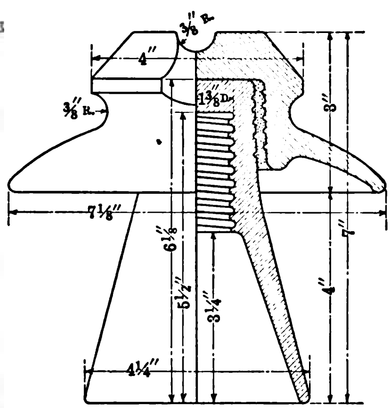

FIG. 31.—Pin type insulator.

such that the distance *E* to cross-arm is slightly greater than the sparking distance *C* from edge of inner petticoat to pin.

The flash-over distances as measured on actual insulators of the pin type are approximately as given below.[1]

Voltage, R.M.S. value	Flash-over distance, inches
40,000	3
60,000	4 1/2
80,000	6 3/8
100,000	8 1/2
120,000	11
140,000	14
160,000	18 1/4

Figs. 31, 32 and 33 are illustrations of typical pin type insulators as made by R. Thomas & Sons of East Liverpool, Ohio. The

[1] Average figures taken from curves given by Mr. J. Lustgarten in the *Journal of the Inst. E. E.*, Vol. XLIX, No. 214, July, 1912.

leading particulars relating to these insulators, as furnished by the makers, are as follows:

	Fig. 31	Fig. 32	Fig. 33
Dry-test voltage	80,000	90,000	150,000
Rain-test voltage	50,000	60,000	100,000
Leakage distance, in	12 1/2	18 3/4	42
Arcing distance, in	4 1/2	4 3/8	9 1/2
Net weight each. lb	4 1/2	8 1/4	24 1/2

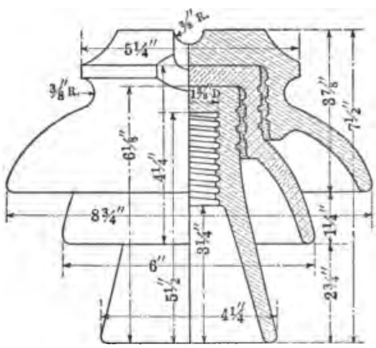

FIG. 32.

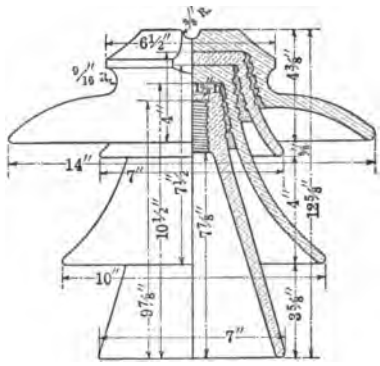

FIG. 33.

FIGS. 32 and 33.—Pin type insulators.

54. Suspension Type Insulators.—Apart from the great advantages from the point of view of insulation, which are obtained by suspending the conductor from a string of insulators connected in series, this arrangement, as now generally adopted for pressures above 60,000 volts, has the further advantage that the conductor is less liable to be affected by lightning disturbances, since, at every point of support, the wire is hung *below* the attachment to the supporting structure which, in almost every instance is a well-grounded steel structure. Another advantage is the comparative flexibility of the attachment, which very considerably diminishes the possibility of crystallization of the conductor material, such as is liable to occur when the wire is rigidly attached to the pin type of insulator; this effect being more noticeable with aluminum than with copper.

In designing cross-arms for the attachment of the suspension type of insulator, it is not necessary to pay much attention to torsional stresses, which, however, must be carefully considered

when pin-type insulators are used on heavy long-span lines; but, on the other hand, taller towers are necessary with the suspension type of insulator. Another possible disadvantage is the wider spacing between wires generally adopted when this type of insulator is used; and it is certain that rather more skill and experience are necessary in the stringing of the conductors than when pin-type insulators are used. It is usual to sling the wire in the first instance from snatchblocks attached to the cross-arms of every tower over a distance of about a mile, or between

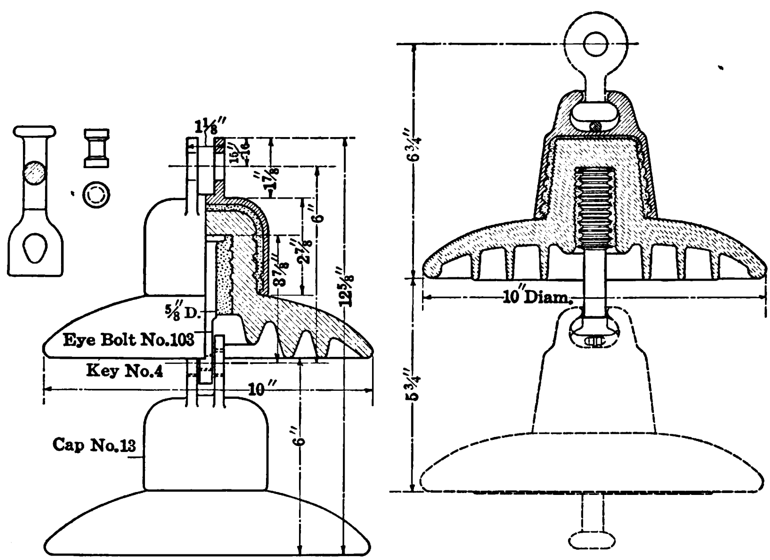

FIG. 34. FIG. 35.

FIGS. 34 and 35.—Suspension type insulators.

anchoring or corner towers, and then draw up tight; the conductor being subsequently transferred at every point of attachment from the snatch block to the permanent suspension clamp at the end of the lowest insulator in the series.

In the interlink type of suspension insulator, the porcelain between the metal links is in compression, and in the event of the shattering of the porcelain parts, the conductor remains suspended; but although, from the mechanical point of view, this type is to be recommended, it is more liable to puncture owing to excessive electric stress in the thickness of porcelain between

the metal links, than the more usual type of insulator with cap and bolt cemented to the porcelain parts as shown in the accompanying illustrations.

Fig. 34 shows an insulator manufactured by Messrs. R. Thomas & Sons, and Fig. 35 is an insulator of generally similar type made by the Ohio Brass Co., of Mansfield, Ohio. A string of ten insulators as illustrated in Fig. 35 is being used by the Eastern Michigan Power Co. for a working pressure of 140,000 volts. The weight of each unit is 9 3/4 lb. net. The flash-over dry is 92,000 volts, and wet, 53,000 volts for the single unit. The illustration, Fig. 36, shows a series of two-part insulators as supplied by the Locke Insulator Manufacturing Co. for the 108,000-volt line of the Sierra & San Francisco Power Co., the first transmission line on the Pacific coast to use the suspension type of insulator.

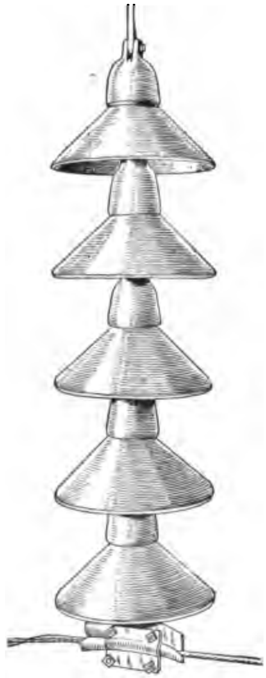

Fig. 36.—Suspension type insulator for 108,000 volts.

55. Weight of Insulators.—It is convenient to know the approximate weights of porcelain insulators for estimating cost of handling and freight, and for determining the loads to be carried by supporting structures. The following average figures will be found sufficiently accurate for calculations in connection with preliminary estimates.

Working voltage	Weight of insulators, pounds
20,000	3
40,000	11
60,000	23
80,000	40
100,000	60
120,000	80
140,000	100

56. Entering Bushings.—When overhead high-tension conductors have to be brought into buildings, the design of insulating

bushings should receive careful consideration. The fact that every bushing acts as a condenser or series of condensers between wire and ground, must steadily be recognized just as in the case of the pole line insulators, except that the effect is even more marked in connection with bushings entirely surrounding the conductor than with insulators that support the wire at one end only. When the climate and weather conditions are favorable, it is well to avoid bushings entirely. In such cases, the wires cannot be brought down through the roof of the building, but they must enter at the side; a suitable protecting hood or roof being placed above the wires on the outside of the building.

The smallest dimension of the opening in brick, stone, or concrete wall, according to usual practice, should be as follows:

For working line pressure of:	Feet
20,000 volts	$1\frac{1}{2}$
40,000 volts	2
60,000 volts	3
80,000 volts	4

On each side of the wall opening, the conductor is carried by line insulators, of the pin or suspension type, as the voltage may require, these being so arranged as to maintain the conductor in the center of the opening, with a slight downward incline toward the outside of the building to prevent rain-drops being carried to the inside.

When bushings are used, these must necessarily be thicker at or near the center, where the ground potential is brought up close to the conductor, than at either end. Fig. 37 shows a porcelain bushing built up of four parts, as supplied by the R. Thomas & Sons Co., suitable for a working pressure of 70,000 volts.

In the case of a roof bushing, permitting of the conductor passing vertically downward to the inside of the building, a hollow elongated barrel-shaped insulator (that is, of greater diameter at center, where it is supported on the outside, than at the two ends) filled with an insulating substance of higher specific inductive capacity and greater dielectric strength than air, makes a satisfactory arrangement provided there is no danger of the oil or

other insulating filling leaking out of the containing shell. A good insulating oil will frequently give good results, but it is liable to leak out at the joints, and, moreover, the use of oil calls for a larger and heavier bushing than if the filling has a specific capacity more nearly equal to that of the porcelain shell (porcelain being the material most generally used). It will be understood that if the substance filling the space between metal conductor and hollow bushing had the same specific capacity as the material of the bushing, there would be no change in the potential gradient at the inner surface of the bushing. Thus, the specific capacity of

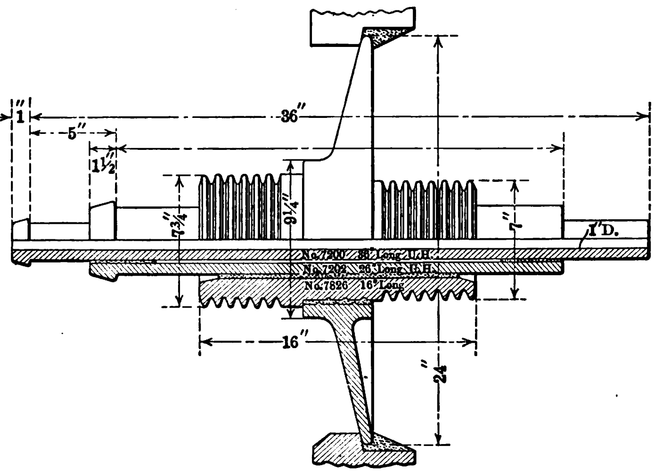

FIG. 37.—Porcelain entering bushing for 70,000 volts.

air being 1, porcelain is 4.4, while sulphur is 3.9 (not very different from that of porcelain), and a petroleum oil will be about 2. The thickness of the porcelain shell and the distance between inner surface of shell and conductor surface should be proportioned in accordance with the insulating material intended for use as a filling.

The arrangement shown in Fig. 38 is bad. Here the thickness of the porcelain bushing is large compared with the air gap between wire and inner surface of bushing, the result is that a brush discharge between wire and bushing is very liable to occur, the potential gradient near the surface of the wire being greater than if the porcelain bushing were entirely removed.

Condenser Type of Bushing.—By separating thin concentric layers of insulating material by tubes of tin-foil or other metal,

and so proportioning the *lengths* of the tin-foil tubes that the *areas* remain the same, notwithstanding the variations in diameter, it is possible to design a bushing which virtually consists of a number of condensers of *equal* capacity all connected in series. In this manner the potential gradient can be made uniform throughout the thickness of the insulating bushing. Commercial entering bushings and transformer terminals have for some time past been built on this principle. The design of such terminals is not quite so simple a matter as this brief reference to the main principle involved might suggest, but the commercial insulators of this class have, on the whole, given reasonably good results.

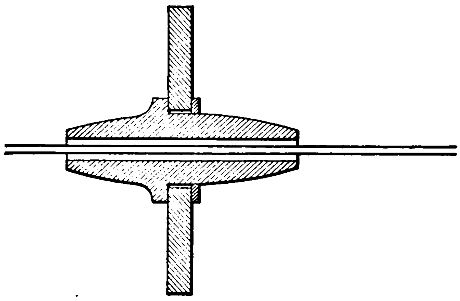

FIG. 38.—Type of bushing liable to cause brush discharge on surface of wire.

Before considering such matters as factors of safety, spacing between wires, and probable limits of pressure on power transmission lines, it will be well to review briefly what is known about the brush discharges and corona formation which are liable to become important factors when transmitting at the higher voltages.

57. Formation of Corona, and Accompanying Losses of Power.—When the pressure on an overhead transmission system exceeds a certain critical value depending upon the spacing and diameter of the wires, there will appear on the surface of the conductors a halo-like glow to which the name "corona" has been given. Apart from this luminous effect, the appearance of the corona is accompanied by a certain loss of power proportional to the frequency and the square of the amount by which the pressure between conductors exceeds a certain value known as the disruptive critical voltage. If the distance between outgoing and return conductors is comparatively small (less than fifteen times the diameter of the wire) there will be a spark-over when the dis-

ruptive critical voltage is reached; but with the greater separation such as occurs on practical high-tension transmission lines, the effect of the high potential at the conductor surface is to break down the resistance of the air in the immediate neighborhood of the conductor surface. In this way a luminous cylindrical coating of air, acting as a conductor of electricity, is formed, the diameter of which will depend on the amount by which the actual value of the applied potential difference between wires exceeds the disruptive critical value of the potential difference. The result is equivalent to an increase of the diameter of the conductors, thus raising the value of the voltage necessary to break down new concentric layers of surrounding air, until it is approximately equal to the voltage impressed on the wires. During the last few years much light has been thrown on the formation and effects of the corona. Among the earlier workers in this field were C. F. Scott, Harris J. Ryan, J. J. Thomson, H. B. Smith, Signor Jona, Lamar Lindon, E. A. Watson, and, among the latest investigators, J. B. Whitehead[1] and F. W. Peek, Jr.[2]

The more important features and effects of the corona of interest to the practical engineer may be summarized as follows:

1. The loss due to leakage of current from the conductor into the surrounding air is practically negligible for pressure values below the disruptive critical voltage; no account need be taken of such leakage on alternating-current circuits operated at pressures below 44,000 volts. On 80,000 volts, however, the loss may be appreciable, and a visible corona may even be formed if the wires are small in diameter.

2. The current passing from the wires into the air on an alternating system is an energy current in phase with the pressure.

3. On alternating-current systems, the critical break-down voltage will depend upon the maximum value of the e.m.f. wave, and therefore on the "form factor."

4. The break-down voltage—or, more properly, the disruptive critical voltage—is determined by the potential gradient at the conductor surface; it is therefore dependent upon the diameter and spacing of the wires; being higher with the larger diameters and spacings; it is also dependent upon the density of the air, and therefore on the temperature and barometric pressure.

5. The loss of power due to corona formation is proportional

[1] *Proc. A. I. E. E.*, Vol. XXIX, p. 1059 (1910).

[2] *Proc. A. I. E. E.*, Vol. XXXI, p. 1085 (June, 1912).

to the frequency (within the usual commercial range), and to the square of the excess of line voltage over critical voltage.

6. The disruptive critical voltage is highest when the conductor surface is smooth and quite clean. It is lowered by roughness or dirt on the conductors, also by smoke and fog in the atmosphere, sleet on wires and falling sleet, rain, and snow storms, but especially the last. All these causes tend, therefore, to *increase* the corona *losses*.

7. The visual corona occurs only at a pressure above the disruptive critical voltage and is an indication that there is loss of power in the air.

When considering the effects of the corona formation on overhead wires, it is convenient, as in the case of the majority of electrical problems connected with transmission lines, to consider each wire separately in relation to the neutral plane or line. Since the formation of the corona depends upon the electric stress at the surface of the conductor, it is the potential gradient in the immediate neighborhood of the wire which, as previously mentioned, is the determining factor in corona formation. This gradient may be expressed by the ratio *volts per centimeter* at the boundary between the surface of the wire and the surrounding air; its actual value being 29.8 kv. What is known as the disruptive critical voltage for any particular wire under specified atmospheric conditions, is that voltage which produces the disruptive gradient at the conductor surface. It will, as previously mentioned, depend upon the diameter of the wire and the distance of the wire or wires forming the return conductor; also upon the density of the air and, to some extent, upon the surface condition of the wire. Mr. F. W. Peek, Jr., who is an authority upon this subject, has given us for the disruptive critical voltage between wire and neutral, the formula:

$$e_o = m_o \delta g_o r \log_\epsilon \frac{s}{r} \qquad (1)$$

in which $r =$ radius of conductor in centimeters,

$s =$ distance between centers of the outgoing and return (parallel) conductors, in centimeters,

$m_o =$ a factor depending upon the surface condition of the conductor,

$= 1$ for polished wires,

$= 0.98$ to 0.93 for roughened or weathered wires,

$=0.87$ to 0.83 for stranded cables,

$\delta=$ a factor depending on the air density,

$=\frac{3.92b}{273+t}$

in which b is the barometric pressure in centimeters of mercury, and t is the temperature in degrees Centigrade.

g_o = the disruptive gradient of 29.8 kv. per centimeter corresponding to the *maximum* value of the alternating-current wave. If the cyclic variation is assumed to follow the sine law, then the value to take for g_o is 21.1 kv. per centimeter, and the value of e_o then given by the formula will be the root of the mean square, or effective value of the critical pressure (kilovolts to neutral).

The luminosity, or visible halo of light surrounding the conductor, does not occur until a higher pressure has been reached, the increase over the critical disruptive voltage being dependent upon the diameter of the conductor. Mr. Peek's formula for the visual critical voltage (kilovolts to neutral) is:

$$e_v=m_v\delta g_o r\left(1+\frac{0.301}{\sqrt{r}}\right)\log_e\frac{s}{r} \tag{2}$$

where the surface factor m_v has the same value as m_o for wires, and may be taken at 0.82 for a decided visible corona on seven-strand cables. The notation is otherwise as above.

The formula for loss of power in fair weather, *per kilometer of single wire*, as given by Mr. Peek, is:

$$P=\frac{344}{\delta}\times f\times\sqrt{\frac{r}{s}}\times(e-e_0)^2\times10^{-5} \tag{3}$$

where f is the frequency in cycles per second, and e is the actual (effective) pressure between wire and neutral, expressed in kilovolts. The approximate loss under storm conditions is obtained by taking e_0 as 80 per cent. of its (effective) value as calculated by formula (1).

For the purpose of the transmission line engineer, it is more convenient to use inch units and common logs in the formulas; and, if the assumption is made that $\delta=1$ (which will be true when $t=25$ and $b=76$ cm.) and also that, for the usual sizes of seven-strand cables, the value of m_0 may be taken at 0.85, the following approximate formulas, for fair-weather power-loss calculations, may be used:

$$e_0 = 105R \log_{10} \frac{D}{R} \tag{4}$$

$$P_m = 0.00553f\sqrt{\frac{R}{D}} \times (e - e_0)^2 \tag{5}$$

Here R = radius of conductor in inches.
D = distance between conductors in inches.
P_m = loss in kilowatts per mile of single conductor.

Practical Example.—Consider a 100-mile, three-phase, 110,000-volt, 60-cycle, transmission on No. 1 seven-strand conductors arranged in the form of an equilateral triangle with 6-ft. spacing. What will be the approximate corona loss?

The diameter of the conductor is 0.33 in.; hence R = 0.165.

By formula (4), the critical disruptive voltage is:

$$e_0 = 105 \times .165 \times \log \frac{72}{0.165} = 45.7 \text{ kv.}$$

By formula (5) the loss of power per mile of single wire is:

$$P_m = 0.00553 \times 60 \times \sqrt{\frac{0.165}{72}} \times \left(\frac{110}{\sqrt{3}} - 45.7\right)^2 = 5 \text{ kw nearly.}$$

The total loss to be expected under fair-weather conditions is therefore:

$$5 \times 3 \times 100 = 1500 \text{ kw.}$$

58. Corona Considered as 'Safety Valve' for Relief of High-frequency Surges or Over-voltage Due to Any Cause.—The loss, as calculated in the above example, is not small; but since it is proportional to the *square* of the excess of pressure over the disruptive critical voltage, a small increase of pressure will lead to an enormously increased dissipation of energy in the air. Thus, if the pressure of 110 kv. in the above example be supposed to increase only 10 per cent. the total dissipation of power, instead of being 1500 kw., would be 2750 kw. It has indeed been stated that on the 110,000-volt system of the Grand Rapids-Muskegon Power Co., the line loss due to corona discharge actually increases 100 per cent. for a 10 per cent. rise in pressure.

This property of the corona suggests the possibility of working high-voltage transmission lines at a normal pressure in the neighborhood of the critical disruptive voltage where the loss would be inappreciable. An extra high voltage discharge, due either to atmospheric lightning, or to internal causes, would then be largely dissipated in the corona itself. This may, to some extent, account for the fact that fewer lightning troubles are experienced

on the very high voltage transmissions than on the lower voltage lines. The insulation of the conductors being such as to withstand, without breakdown, pressures considerably in excess of the disruptive critical voltage of the corona, a large amount of oscillating energy can be dissipated in the air before the voltage rises to such a value as to pierce or shatter insulators or damage apparatus connected to the line. On the other hand, too much reliance should not be placed on the corona as a means of dissipating large amounts of suddenly impressed energy; because lightning and similar disturbances, being to a great extent local, must discharge their power locally, and the corona losses over a *short* section of the transmission line cannot under any circumstances be very great.

59. Factors of Safety, and Tests.—When selecting insulators and deciding upon the spacing and arrangement of conductors suitable for a given voltage, the factor of safety to cover abnormal pressure-rises is a matter of great importance, since it is obviously bad engineering to provide insulation in excess of what experience has shown to be a reasonable safeguard against interruption of service. Generally speaking, the insulators should, when dry, withstand a pressure test of $2\frac{1}{2}$ to 3 times the working pressure to ground, applied for 10 to 15 minutes, and a wet test of not less than $1\frac{1}{2}$ times the working pressure. This would generally be considered too small a margin of safety; but the ratio between test pressure and working pressure will depend upon whether the line voltage is high or low. The following safety factors, representing ratio between wet-test pressure and working pressure are generally in accordance with usual practice; but the engineer should use his judgment in a matter of this sort. It is clear that, on the coast, where gales and salt sea mists are prevalent, the factor of safety should be rather higher than in a district where the climatic conditions are more favorable.

Working voltage	Safety factor (wet test)
20,000	2.5
40,000	2.2
60,000	2.0
80,000	2.0
100,000	1.8
120,000	1.8

As the wet or "rain" test will give different results, depending on the method of conducting the tests, there should be a clear understanding between the purchaser and manufacturer on this point. A very common specification is that the spray shall be directed at an angle of 45 degrees, under a pressure of 40 lb. per square inch at the nozzles; the flow being regulated to give a precipitation of 1 in. in 5 minutes. The method of attaching test wire and ground connection to the insulator should also be clearly defined. The test pressure is usually measured by means of a spark gap; and the alternating e.m.f. used should conform as nearly as possible to the sine wave.

The striking distances in air between No. 3 *sharp* needles, as given by the Locke Insulator Mfg. Co., are as follows:

Kilovolts	In.	Kilovolts	In.
20	1.00	200	20.50
40	2.45	225	23.05
60	4.65	250	25.60
80	7.10	275	28.30
100	9.60	300	31.00
125	12.25	350	36.10
150	15.00	400	41.20
175	17.80		

It will be observed that for pressures above 100,000 volts the gap between needle points is approximately 1 in. per 10,000 volts. The pressures referred to in the table are the effective or root-mean-square values of the test pressure, on the sine wave assumption.

The mechanical strength, or rather the fragility of the porcelain, is sometimes tested by discharging a shot-gun at the insulator, mounted in position on the pole; but unless all insulators on a transmission line are so tested with a view to eliminating those that may be mechanically defective, it is doubtful whether such a test is of much value.

60. Distance between Wires.—The spacing of the wires is determined by considerations partly electrical and partly mechanical. With the longer spans, the spacing should be greater than with the short span, apart from voltage considerations. The diagram Fig. 39 will be found to give spacings between conductors generally in accordance with present-day practice.

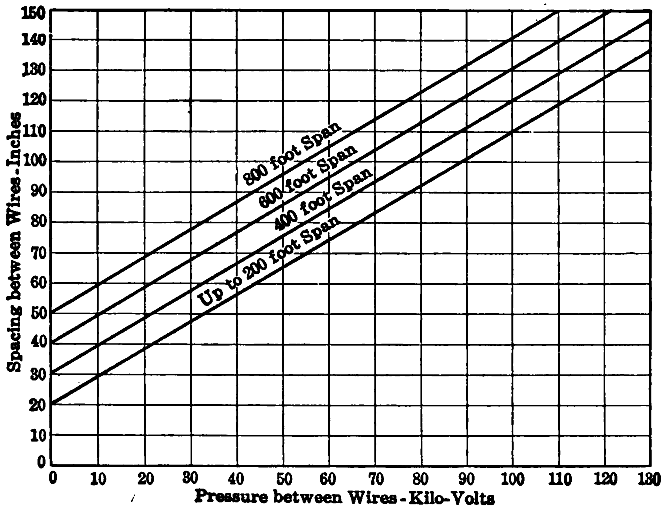

FIG. 39.—Usual distances between overhead conductors.

61. Distance between Conductors and Pole or Tower.—It is usual to allow a clearance between conductor and tower or pole approximating to the following distances:

Line voltage	Clearance
Under 10,000	9 inches
10,000 to 14,000	12 inches
14,000 to 27,000	15 inches
27,000 to 35,000	18 inches
35,000 to 47,000	21 inches
47,000 to 70,000	24 inches

62. Practical Limitations of Overhead Transmission-line Voltages.—From the foregoing review of the insulation problems to be met with on long-distance overhead transmissions, it will be clear that manufacturers are now in a position to provide insulation amply sufficient for present requirements. Power is actually being transmitted at 140,000 volts; and lines have been planned for a working pressure of 165,000 volts. The lines of the Au Sable Electric Co. in Michigan, transmitting power at 140,000 volts, consist of stranded copper conductors 3/8-in. diameter on 500-ft. spans, with a sag allowance of about 12 ft.

The shortest distance between conductors is 12 ft., this being the vertical height between the two conductors on one side of the steel supporting towers. There is pratically no visible corona, but a buzz or hum, due no doubt to brush discharge, can be heard in the neighborhood of the tranmission lines.

Although there are no insurmountable difficulties in providing ample insulation for these high voltages, it is the engineer's business to provide such insulation as will be justified by economic considerations. It is also his business to determine the voltage of transmission on the same basis, and resist the temptation to experiment in voltages higher than may be justified by commercial considerations.

It is well to bear in mind that the economical transmission voltage depends not only on the length of the line, but also on the amount of power to be transmitted; and although a 200,000-volt transmission offers no serious engineering difficulties, the conditions under which a transmission at so high a voltage would be a commercial success, are very seldom found.

63. Lightning Protection.—Although our knowledge of lightning phenomena is still far from complete, it is generally agreed that a single stroke of lightning is of short duration, frequently not exceeding the one-thousandth part of a second. If an overhead conductor receives a direct stroke of lightning, the potential value of the lightning charge is generally so enormously in excess of the working pressure on the conductors that the lightning leaps over the insulators down the pole to ground. Any charge on the line, which is not sufficiently high in potential above ground to jump over the insulators, will travel along the line in both directions until it is grounded through a lightning arrestor or dissipated because of the ohmic resistance of the conductors. The frequency of such travelling waves will depend upon the natural frequency of the line, and may be of the order of 1000 to 5000 cycles per second. These high frequencies tend to limit the discharge rate of lightning, as the inductance of the line, apart from any choke coils purposely connected thereto, will check the rise of current. If the resistance of an arrestor or the path through which a discharge occurs, were zero, the current passing would be a maximum. If C is the capacity in farads, and L the inductance, in henrys, of unit length of line, then $\sqrt{\frac{L}{C}}$ is the quantity that Dr. Steinmetz has called the natural impedence of the circuit; and

the maximum possible value of the current will be $I_{max.} = E \div \sqrt{\frac{L}{C}}$, where E is the impressed voltage, which may be considered as something less than the pressure which will cause a flash-over at the insulators.

The intense concentration of lightning disturbances is the cause of the difficulties experienced in protecting transmission lines by means of lightning arrestors; experience tends to show that an arrestor does not adequately protect apparatus at a greater distance than 500 ft., yet it is unusual to find arrestors on a transmission line at closer intervals than 2000 ft.

Disturbances are most likely to occur on exposed heights, and on open wet lowlands; special attention should therefore be paid to lightning protection at such places.

Apart from the effects of atmospheric electricity, it is necessary to guard against the abnormal pressure rises that will occur on long transmission lines through any cause, such as switching operations, or an intermittent "ground." Over-voltages up to 40 per cent. in excess of the normal line voltage can be produced by switching in a long line. High frequency impulses or surges are set up, which, in the special case of an arcing ground, may give rise to a destructive series of surges, a state of things which will continue until the fault is removed. An arrestor which may be suitable for dealing with transitory lightning effects may be quite inadequate to dissipate the charges built up by such continual surges.

64. Protection of Overhead Systems against Direct Lightning Strokes and Sudden Accumulations of High Potential Static Charges.—Under this heading the ordinary lightning rod and grounded guard wire will be briefly dealt with. If no guard wire is used, lightning rods should be provided at intervals along the line. They may be fixed to every pole or tower, but, in any case, they should not be spaced farther apart than 300 to 400 ft. unless the spacing of the supporting poles or towers has to be greater than this, for economic reasons. It is especially important to provide them on the poles or towers in exposed positions such as hill tops. They should project from 3 to 6 ft. or more above the topmost wire. A convenient form of lightning rod is a length of galvanized angle iron bolted to pole top or forming an extension to the structure of a steel tower. Long lines have been worked satisfactorily for extended periods without lightning

rods or guard wires, but these are extra high pressure transmissions which, on account of the better insulation throughout, are always less liable to trouble from lightning than the lower voltage systems.

Although engineers are still divided in opinion as to the value of the protection afforded by overhead grounded guard wires, carried the whole length of the line above the conductors, it is now generally recognized that this method of protection is efficient. The objections to the guard wire are the additional cost and the possibility of the grounded wire breaking, and falling across the conductors below, thus causing an interruption to continuous working. Trouble due to this cause is, however, exceedingly rare.

It has been suggested that the guard wire or wires should be of the same material as the conductors, in order that the "life" of all the wires may be the same; but there are other considerations in favor of using a galvanized stranded steel cable for the guard wire. This may be the ordinary cable, 5/16 to 7/16 in. in diameter, as used for guying poles; but, where great strength is required, the Siemens-Martin steel cable, with or without hemp core, is preferable. Bessemer steel wire has not been found satisfactory for this purpose. In the case of the "flexible" steel tower type of line, a strong steel guard wire joining the tops of the towers, adds greatly to the strength and stability of the line, and may even, on long lines, save its cost, by allowing the use of lighter structures and fewer intermediate (dead-ending) towers.

In regard to the position of overhead guard wires relatively to the conductors, it is obvious that a number of grounded wires surrounding the conductors will afford better protection than a single wire above the conductors; and two guard wires are sometimes provided; but the additional cost is rarely justified. Perfect protection cannot be obtained even with two guard wires, and cases have been reported of lightning missing the grounded wire and striking a conductor situated immediately below.

The best position for a single guard wire placed above the conductors is, according to Dr. Steinmetz[1] such that all the current carrying wires are included within an angle of 60 degrees below the guard wire. Additional wires can be installed in exposed positions, such as the summit, or very near the summit, of a range of hills, or by the shores of lakes or seas where the prevailing

[1]Discussion of the Committee on "Lightning Protection," of the National Electric Light Association, May, 1908.

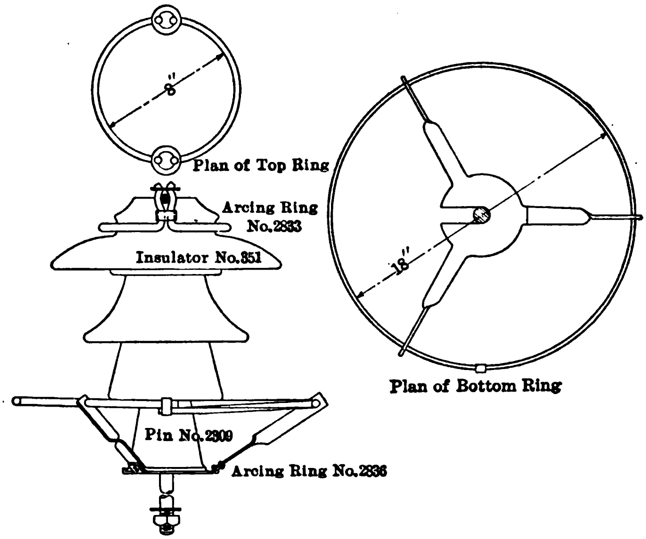

FIG. 40.—Arcing ring on pin type insulator.

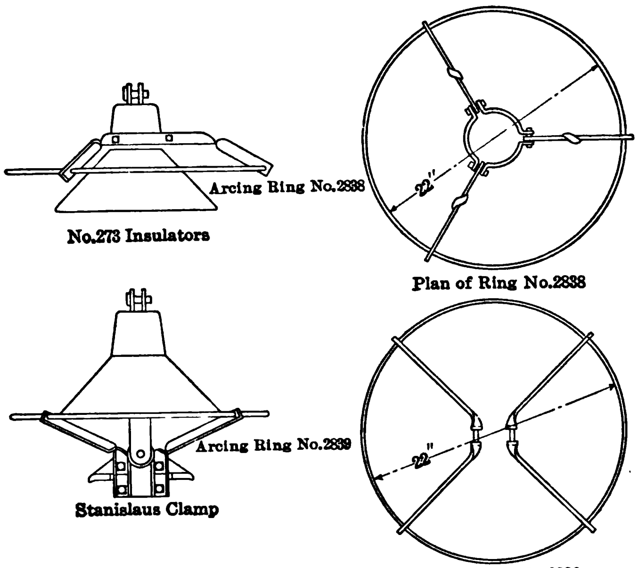

FIG. 41.—Arcing rings on suspension type insulator.

winds come over the water. In such positions, an additional guard wire on the side of the conductors may be useful. The guard wire should preferably be grounded at every pole, or at least every 500 ft.

As a special means of protecting insulators from the flash-over caused by lightning, or the power arc following a high potential discharge, the "arcing rings" first introduced by Mr. L. C. Nicholson, may be mentioned. These rings, which are grounded, are placed in such a position as to take the arc, and hold it at a sufficient distance from the porcelain of the insulator to prevent cracking or breakage by heat. The illustrations, Figs. 40 and 41, show the arrangement of the grounded arcing rings attached to standard types of insulator made by the Locke Insulator Manufacturing Company. It is not claimed that these rings will protect an insulator against a direct lightning stroke; but their utility on high-pressure lines transmitting large amounts of power has been proved without doubt.

Methods of Grounding.—The ground wire from lightning rod, guard wire, or arrestor, on high-tension transmission circuits, should be as short and as straight as possible; it need not be of very low resistence; but small reactance is of first importance. The ground connection should have as large a surface as possible, the material being of little importance, except that it is not wise to bury aluminum wires in the ground, because of possible electrolytic action. Galvanized iron is a good material. If the ground contact is made with one or more iron pipes buried or driven into the ground, these pipes may be from 1 to 1 1/2 in. in diameter, and a good connection should be made to the *top* of the pipe, as the inductive effect of an iron tube surrounding the ground wire might be considerable if a connection were made only at the bottom of the pipe. One or more pipes 8 to 10 ft. long, driven into the ground with 6-in. to 12-in. projecting above, will generally be found more effective than buried plates. A very low resistance ground is not essential on a high-tension system, and, generally speaking, the special forms of ground plate made of perforated copper, designed to hold, or to be in contact with, crushed charcoal, are unnecessary. If a plate is used, this should be not less than 12 in. square, and it may be of galvanized iron 3/16 in. or 1/4 in. thick, buried as deep as possible in the ground, and, in all cases, an effort should be made to secure permanently damp soil for ground plates or pipes.

66. Methods of relieving Conductors of High Potentials before Damage is done to Insulators or Machines and Apparatus connected to the Line.—Water Jet Arresters. By directing a stream of water from the nozzle of a grounded metal pipe on to the high-tension conductors, a high-resistance non-inductive path to ground is provided for the extra-high potential charges on the line; but there will be very little leakage of power current. Arresters constructed on this principle have been found useful in practice; but the employment of jets of water has its objections. It is usual to put the jets in action only at times when electric storms are pending; and the reliance on the "human element" renders the apparatus less valuable than an equally effective device which is always ready to act. Patents have been granted for various forms of water jet arresters, but they are not extensively used at the present time. The chief function of the water jet is to prevent the building up of static pressures on the line.

67. Spark Gaps.—Nearly all lightning arresters are designed on the principle of one or more spark gaps between the conductors and ground, the air space being so adjusted that the normal difference of potential between the line and ground is insufficient to jump the gap; but abnormally high pressures will break down the insulation of the gap, and so find a path to ground before the pressure is sufficiently high to damage the insulation of the line or the apparatus connected thereto.

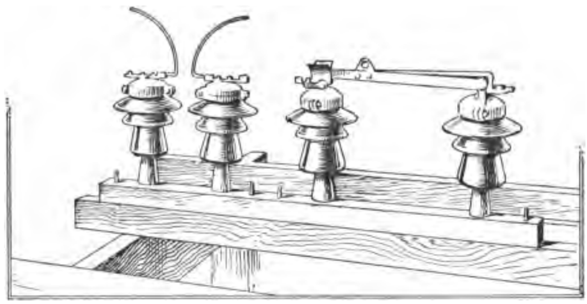

FIG. 42.—Horn arrester with disconnecting switch.

68. Horn Gap.—The ordinary horn gap arrester of the type shown in Fig. 42, is so well known that it requires no detailed description. When the potential reaches a value that can jump the gap at the base of the curved wires, the power arc will follow the discharge; but, owing partly to the upward tendency of the

heated air, and mainly to the magnetic field produced by the current itself, the arc is driven upward toward the ends of the "horns" where, after being sufficiently drawn out in length, it is finally ruptured. The horn gap is not effective when set to discharge at pressures below 13,000 volts, because, with a small gap (less than 1 in.), the arc may not rise and break properly. The usual settings for horn gaps are as follows; the figures referred to as "voltage" being the approximate breaking down potentials:

Voltage	Gap
25,000	1 in.
50,000	2 in.
75,000	4 1/4 in.
100,000	7 1/2 in.

A non-inductive resistance should be connected in the ground wire from the horn arrester. An ordinary wooden barrel filled with water, with a connecting plate at the bottom, and the upper terminal carried about 6 in. below the surface of the water, makes an effective resistance. If no resistance is provided in the ground connection, the momentary discharge of the power current may be excessive, and there is the possibility of synchronous machines being thrown out of step.

One serious disadvantage to the ordinary horn gap arrester is the liability of an intermittent arc setting up surges and high potential disturbances which may lead to more trouble than the original cause of the spark-over. Fairly satisfactory results have been obtained by providing a number of horn gaps on a high-tension transmission and "grading" these, by adjusting some of them to discharge with a very small rise of pressure through a high resistance; while other sets would have larger gaps and lower resistances in series; the very largest gap being such as to break down only rarely, under exceptionally high pressures, and this should have a very low resistance in series but may with advantage be protected by a fuse.

Horn arresters, if intellingently placed and properly connected and adjusted, are capable of affording good protection; but the multi-gap and "low equivalent" arresters, as originally suggested by Mr. P. H. Thomas, have some special features which have led to their frequent adoption on alternating current circuits for pressures up to about 40,000 volts.

69. Multiple-gap Low Equivalent Arrester.—In this type of arrester there are many air gaps in series between the line and ground. No single gap is greater than 1/32 in. or 1/16 in. and it occurs between the adjacent surfaces of small cylinders made of a so-called "non-arcing" metal as used in the earlier types of Wurtz arrester. The number of gaps in series depends upon

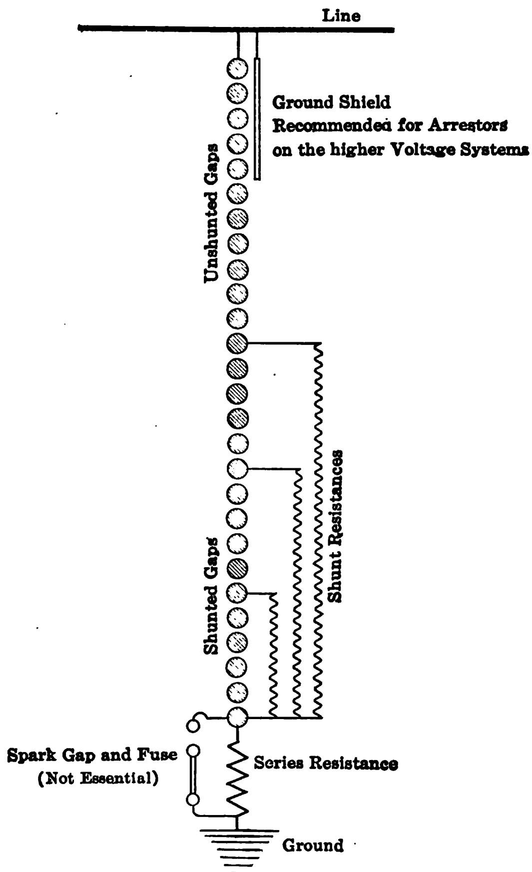

FIG. 43.—Diagram of multiple-gap low equivalent arrester.

the working voltage of the line, and the last of the metal cylinders is connected to ground (or to one of the return conductors, as the case may be) though a non-inductive resistance, which may, with advantage, be shunted by a fuse in series with a spark gap. Sometimes a portion of this resistance is bridged by a number of

spark gaps, all as shown in the diagram Fig. 43. These shunted gaps act as a sort of by-pass for heavy discharges, the amount of the series resistance, through which all discharges have to pass, being comparatively small. The theory of the low equivalent arrester has been ably discussed by other writers.[1] Its action is, breifly, as follows. There is a certain electrostatic capacity between consecutive cylinders, and between each one of these cylinders and ground; and the potential gradient is considerably steeper at the high voltage end of the arrester, with the result that, when the total voltage across the arresters reaches a certain critical value, the breakdown occurs between the first and second cylinders. The second cylinder is then connected to the first by an arc, so that its potential rises accordingly, until a breakdown occurs between the second and third cylinders; and so on. The line current then follows the discharge, and in so doing, tends to produce a uniform fall of potential along the line of cylinders, with the result that the maximum potential difference between cylinders is considerably less than that required for the initital breakdown, and the power arc is ruptured as the current passes through zero value. When a breakdown occurs between two cylinders, the potential of the lower cylinder of the series will depend upon the quantity of electricity which passes to it from the more highly charged cylinder. The initial current is really a capacity current, and it will therefore be greater at the higher frequencies; but, by a scientific proportioning of the shunted resistance, a very satisfactory arrester of this type can be made, for use on circuits up to about 13,000 volts: it is less effective on higher voltages, but is actually used on 20,000 volt, and even 35,000 volt transmissiom lines.

The multiple-gap arrester is essentially adapted for dealing with line oscillations: a heavy high frequency discharge will generally jump a straight gap over insulation several times greater than the equivalent insulation of the arrester; and, for these reasons, a judicious combination of the horn gap and multiple gap arrester is to be recommended.

One reason why the multiple-gap low equivalent arrester is not satisfactory on very high voltage systems is that the necessary increase in the number of gaps to prevent arcing over by the line voltage alone, is out of all proportion to the increase in voltage.

[1] See Dr. Steinmetz on the theory of this type of arrester in Vol. XXV (1906), of the Proceedings of the A. I. E. E.

There is also much uncertainty as to the number of gaps required, as this seems to depend on the position of the arrester relatively to surrounding grounded objects. With the ground potential brought very near to the arrester, the potential gradient at the end near the line frequently becomes high enough to ionize the air between the cylinders, thus carrying the line potential to lower cylinders, until the remaining gaps are so few that a discharge occurs. In order to obtain the more equal division of the total potential difference, and so allow of a reduction in the total number of gaps, such as would be obtained by removing the whole arrester to a considerable distance from grounded objects, a metal guard plate or shield is sometimes placed near the gaps at the high potential end of the arrester, and connected to the line wire as indicated in Fig. 43.

70. Spark-gap Arresters with Circuit Breakers or Re-setting Fuses.—If the resistance in series with a gap arrester is very small, a good path is provided to ground for taking a very heavy discharge; but there will be a large flow of power current in the arc following the discharge. This current may be interrupted by connecting some self-acting device such as a fuse or automatic circuit breaker in the ground connection; and arresters, whether of the horn type or with any other kind of spark gap, are now made with fuses so arranged that when one fuse blows, the dropping of a lever or an equivalent device, automatically inserts another fuse, so that the system is not left unprotected. Even without automatic replacement, if a number of gaps with fuses are connected in parallel, it will generally be found that one discharge will not blow all the fuses, and that during the passage of a single storm, the line will be adequately protected.

In the Garton-Daniels arrester, for use on alternating-current circuits up to 20,000 volts, the principle of the multiple gap is combined with a simple type of automatic circuit breaker connected as a shunt to some of the spark gaps, in order that the discharge path for the lightning shall remain unaltered even during the operation of the arrester. The arrester is built up of several unit parts connected in series; each unit being rated for 3300 volts. The illustration, Fig. 44, shows a complete single-phase arrester for 10,000 volts. On a 20,000 volt circuit, there would be eight units in series, the total air gap distance being $1\frac{1}{8}$ in., with a series resistance averaging 3800 ohms. The diagram, Fig. 45, refers to a single unit of the Garton-Daniels

arrester. The discharge follows the straight path through the two sets of air gaps and the resistance rod, as indicated by the round dots. The power current following the discharge will, after passing through the two upper gaps and the resistance rod, be shunted by the low resistance winding of the circuit breaker; and if this following current is too heavy to be ruptured by the combined action of these two gaps and the resistance rod, the

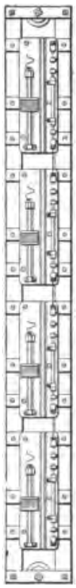

FIG. 44.—Garton-Daniels arrester for 10,000-volt circuit.

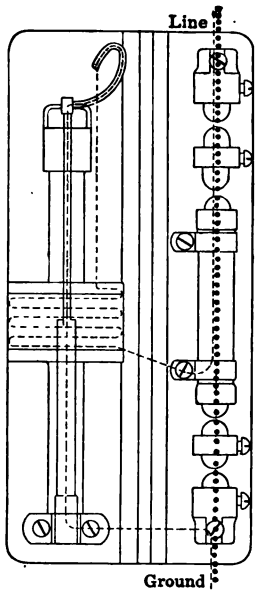

FIG. 45.—Diagram of Gartor Daniels arrester.

iron armature of the circuit breaker will be lifted by the action of the solenoid, thus throwing the two lower spark gaps in series, and extinguishing the arc.

Aluminum Cell Arrester.—When two aluminum electrodes are immersed in a suitable electrolyte, an insulating film of hydroxide of aluminum is formed on the surface of the metal: this effectually prevents the passage of any appreciable amount of current until a certain critical voltage is reached, when the film breaks down and the current is limited only by the resistance of the electrolyte. On lowering the voltage, the film is re-formed and the flow of current again limited to a very small amount.

With alternating currents, the critical potential difference per pair of plates is about 350 volts, and the practical construction of lightning arresters on this principle consists in stacking a large number of cone-shaped aluminum plates one within the other, with suitable separating washers of insulating material between them. In this manner a column is formed of a large number of cells in series, capable of withstanding high voltages. The whole is enclosed in a case containing oil, which improves the insulation and prevents the evaporation of the electrolyte which fills the spaces between adjacent trays within a short distance of the edge.

If cells built up in this manner are connected directly between line and ground, there will be an appreciable current passing through them, which is partly a leakage current, but chiefly a capacity current. It is therefore customary to insert a spark gap, usually of the horn type, in series with the aluminum cell arrester; the gap being set to break down with a pressure slightly in excess of the normal working voltage.

Although the film of hydroxide is formed on the plates at the factory before the arresters are installed, it is necessary to maintain it by periodic "charging" of the cells; this being done by closing, or nearly closing, the spark gap in series, so as to put the full line pressure across the arrester. It is generally recommended that this be done once every day.

In principle the aluminum cell arrester would appear to offer an ideal solution of the problem of lightning protection; because, once the critical voltage is exceeded, and the film broken down, a very large current—depending on the amount of separation and the area of the plates, and also the nature of the electrolyte—is allowed to pass to ground; and the device is capable of dealing with continual surges, such as will occur with an intermittent ground, for a period of about half an hour without excessive heating. In practice it has proved fairly satisfactory, especially on the higher voltages; but, apart from its large initial cost, it has frequently been found to be somewhat costly in upkeep, as the aluminum cells are liable to become damaged through frequent and heavy discharges, and have to be periodically reformed or replaced. Then again, the necessity of charging with the line current is an objection where there is not an operator constantly in attendance; and lastly, it must not be overlooked that the device suffers from the disadvantage common to all spark gap

devices, namely that high frequency surges are liable to be set up in the system when the spark gap discharges. In this particular case the trouble is liable to occur, not only when the horn gap breaks down while fulfilling its function of discharging an excess of pressure through the cells, but also when the spark is deliberately formed for the purpose of charging the cells. On very high pressure systems it is possible that the surges set up by spark gaps in series with the resistance of the cells are not likely to cause trouble; but this suggests the possibility of simpler and less costly devices such as the graded horn gaps previously referred to, being equally effective. On the other hand, when used on low voltage systems operating at about 11,000 volts (especially if the generators are directly connected to the transmission line, without the intervention of step-up transformers the operation of charging the aluminum cell arresters in the generating station has been known to break down the insulation of the generators.

The makers of this type of arrester appear to have recognized the danger of damage to apparatus arising from the operation of charging the cells, and they now recommend that the charging current be passed through a resistance in series with the arresters. Suitable resistances are provided in connection with the horn gaps so that the line pressure is not put directly across the aluminum arresters at the time of charging.

72. Condensers.—Although much has been done, and more good work will probably be done in the future by the intelligent "grading" of a number of spark gaps, to afford a path to ground and yet avoid the setting up of dangerous high-frequency surges, the objections to all spark gaps are (1) the necessity of an appreciable increase in pressure above normal line pressure to break down the resistance of the gap, and (2) the danger of an oscillatory current being set up in the network of conductors.

Consider any single spark gap, such as the horn type, with a resistance in the ground connection. If this resistance is less than twice the quantity which Dr. Steinmetz has named the natural impedance of the circuit, the interruption of the current passing to ground is liable to set up dangerous high-frequency oscillating currents in the wires and apparatus connected thereto; and, on the other hand, if the resistance is greater than this critical value, it may be too high to afford much relief in the event of suddenly applied electric impulses.

As stated in article 63, the natural impedance of a circuit is the square root of the ratio $\frac{\text{inductance}}{\text{capacity}}$, or $\sqrt{\frac{L}{C}}$ which, in the case of overhead transmission lines, will have a numerical value between 200 and 500 ohms. Assuming this to be 350 ohms, it will be seen that the resistance in the ground connection must be at least twice 350, or (say) 800 ohms on order to avoid the production of high-frequency oscillations.

If the spark gap can be avoided, this high series resistance in the ground connection becomes unnecessary. A non-inductive direct connection to ground can obviously not be made on a high-tension alternating-current overhead transmission line; but a path to ground may be provided either through a highly inductive choke coil, or through a condenser, or both, without the necessity of providing a spark gap in series. The inductive resistance may easily be designed to pass only an inappreciable current of normal or higher frequency, and it will therefore be useless for affording relief in the case of high-frequency surges; but it is capable of relieving the line of slowly accumulated static charges. The condenser, however, acts as an almost perfect insulator so far as direct currents are concerned; but it is pervious to high-frequency currents, and a suitably designed condenser, or rather battery of condensers, connected between line and ground without the intervention of any spark gap, is certainly an ideal device for dealing with the very high-frequency oscillations that accompany lightning phenomena. This is the chief function of the Mosciki condensers which, although not largely used on this continent, have found favor in Europe—where they have been in use for many years in almost every country—and also in South Africa and China.

As previously stated, the travelling waves induced by a lightning discharge on a transmission line have a frequency of the order of 1000 to 5000 cycles per second, and a set of condensers which will not pass any but a very small current at 25 or 60 cycles, will deal with much larger currents on these higher frequencies. As a matter of fact, an electric discharge between cloud and ground or between cloud and cloud may induce in the line travelling waves having frequencies considerably in excess of 100,000 cycles per second. This is proved by the fact that wireless telegraphic apparatus which responds only to frequencies ranging between about 100,000 and 1,000,000 cycles per second is inter-

fered with by atmospheric electric storms. A 40-amp. fuse in series with a condenser has been blown during atmospheric discharges, although the condenser could not possibly pass more than a hundredth part of this current on frequencies of 3000 or 4000.

It is perhaps not generally understood that high-frequency travelling waves may break down the insulation of generators or transformers even when the voltage of the induced charges is small as compared with the normal operating voltage. The trouble is that the wave is short; it has a steep front, and the point of zero potential may be only a few hundred feet behind the point of maximum potential. If, therefore, a travelling wave of this nature enters a piece of electrical machinery such as a generator or transformer, the full difference of potential, which may amount to only a few thousand volts, may be applied across adjacent layers of the coil winding, thus causing a puncture and ultimate breakdown of the insulation, even if the apparatus as a whole is insulated to withstand pressures of 100,000 to 200,000 volts to ground. As a protection against trouble of this sort from high-frequency induced charges, the condenser appears to offer a good solution.

It must not be understood from these notes on the uses of condensers as lightning arresters that the discharge is diverted to ground through the condenser and so dissipated, much as energy would be dissipated in a resistance; because the condenser cannot absorb or dissipate any but the smallest percentage of the energy passing through it. The energy is necessarily re-delivered to the line from which it originally came, and is ultimately dissipated through the ohmic resistance of the conductors; but the steepness of the wave front has been diminished. The function of the condenser is, in fact, somewhat analogous to that of an air chamber on a water pipe in which the rate of flow is subject to sudden variations.

73. Spacing of Lightning Arresters.—A reasonable distance must be allowed between the live metal parts of arresters placed side by side; the following limiting distances are suggested for guidance in installing lightning arresters.

Potential difference, (volts)	Separation, inches
11,000	9
22,000	18
33,000	25
44,000	32
66,000	42
88,000	52
110,000	60

74. **Choke Coils.**—When a lightning arrester of whatever type is connected between line and ground in or near generating or substations for the purpose of providing a path to ground for high-potential or high-frequency charges, an inductance is placed in series with the apparatus to be protected. This inductance must not be so great as to cause a serious drop in pressure when carrying the normal line current, neither must it be so small as to allow the induced charges travelling along the line to pass through it rather than jump the air gap of the lightning arrester. This inductance usually takes the form of an air-insulated coil of copper wire or rod, supported at each end on a suitable insulator. The "hour glass" form of coil, in which the diameter of the turns increases from the center toward both ends, is mechanically stiffer than a cylindrical coil, and any arc that might be started between adjacent turns has a greater tendency to clear itself. The air space between turns is usually from 1/4 in. to 3/8 in. Too little attention has been given in the past to the proper design and proportioning of choke coils for use in conjunction with lightning arresters. It has sometimes been argued that, except for the drop of pressure under working conditions and the higher cost, there is no objection to installing very large choke coils having a high inductance. This argument is, however, incorrect, except for the special case in which some protection against surges or resonance effects is provided on the machine side of the inductance in addition to the lightning arresters on the line side. A high inductance may be quite satisfactory if it is merely intended to hold back high-frequency currents travelling along the line; but surges may originate near the generators or transformers due to switching operations or other causes, and a very high inductance between the electrical plant and the line will tend to aggravate the effect of comparatively low-frequency surges which might otherwise be dissipated in

the line, or even through the lightning arrester. In fact, choke coils should be designed with due regard to the apparatus they are intended to protect, with a view to avoiding the building up of high voltages at the terminals of the generating plant in the event of surges being set up in or near the plant itself. When the lightning arrester discharges, it does not follow that high-frequency waves do not find their way through the choke coil to the machines; but the inductance of the choke coil will lower the frequency of such waves; or, in other words, will reduce the steepness of the wave front to such an extent that the insulation of the machines will not be injured. The first few turns of a transformer or generator winding will act as a choke coil and usually prevent damage to the turns farther removed from the terminals; but, as previously mentioned, they are liable themselves to suffer injury, as the charge will leap across the insulation and so get to ground. If it is assumed that the inductance of the first six turns of a transformer winding is sufficient to afford protection to the seventh and subsequent turns of the winding, then a choke coil having an inductance equal to that of the six turns of transformer winding will afford the necessary protection to the transformer. A higher inductance in series is unnecessary and may be dangerous.

The tendency among engineers appears to be toward the use of choke coils of too great inductance. As an example of what appears to be generally sufficient to afford reasonable protection to modern machinery, about 25 turns of copper rod wound into a coil 10 in. in diameter may be used on voltages from 10,000 to 25,000, while for pressures of the order of 100,000 volts, two such coils would be connected in series. The diameter of the copper rod would depend upon the current to be carried; but it is best to have it large enough in all cases to be self-supporting, although coils wound on insulating frames, with separating pieces between turns, are not necessarily objectionable.

It is possible that copper is generally used for choke coils because the calculation of the inductances at various frequencies is more easily made than in the case of a "magnetic" material, such as iron; but the cost can be reduced by using iron bar or strip in place of copper, and a peculiarity of the iron choke coil is its property of passing currents of normal frequency with comparatively small loss of pressure, while the choking effect with high-frequency currents is very much greater.

75. Arcing Ground Suppressor.—If any one conductor of a transmission system is connected to ground through an arc such as might occur over an insulator in the event of a rise of pressure due to any cause, there is the possibility of the arc continuing during an appreciable length of time, sufficient to do serious damage to the insulator, even if it should not totally destroy it. Apart from this danger, every intermittent arc is liable to set up dangerous high-frequency surges in the line, especially at the moment when it is finally interrupted. To protect a line against troubles due to this cause, a device known

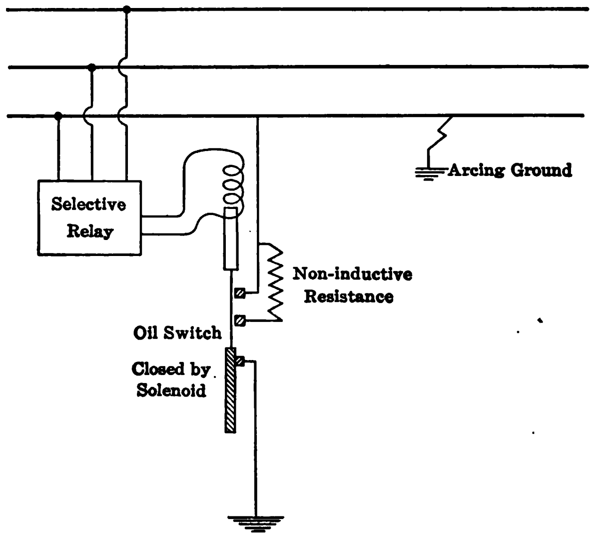

FIG. 46.—Diagram of arcing ground suppressor.

as the arcing ground suppressor has been introduced. This is an automatic device for momentarily short-circuiting the arc through a switch. By providing a metallic connection between the conductor and ground, the arc is suppressed, and it will usually not re-form when the switch is again opened, because the air in the path of the arc has had time to cool, and the line pressure, which was sufficiently high to maintain the arc once started, is not able to break down the insulation of the new layers of cooler air. The arcing ground suppressor is fully described in the *Proceedings A. I. E. E.* of March, 1911,[1] but the diagram, Fig.

[1] E. E .F. Creighton—Protection of Electric Transmission Lines; *Proc. A. I. E. E.*, Vol. XXX, p. 377.

46, will explain the principle of its action. Automatic switches are provided which will connect any one conductor to ground during the very short time necessary to allow the arc to clear itself. The principal feature of the device is the selective relay which will energize the solenoid operating the switch on the faulty line. On high-pressure systems, this relay may be of the electrostatic type, generally on the principle of the electrostatic ground indicator. On comparatively low-pressure transmissions, the forces would be too low to operate such a device satisfactorily, and recourse is then had to an electromagnetic relay worked through transformers. There are no difficulties or new principles involved in the design of such a relay. When the relay operates, the switch between line and ground is momentarily closed. On re-opening, a suitable resistance is inserted before the final break, to prevent the formation of oscillating currents in the line.

76. General Remarks on Lightning Protection.—The best means to adopt for the protection of any particular line or portion of a line against lightning disturbances is still largely a matter of conjecture, but by the exercise of sound judgment, an experienced engineer should be able to provide reasonable protection against discontinuity of service during atmospheric disturbances. There are many devices to choose from, each of which has a particular field of usefulness. It is probable that, in a few years' time, the additional information on this subject which is continually being accumulated, will lead to uniformity in the protective arrangements adopted under the various conditions arising in practice. In the meanwhile, however, a careful record of all accidents due to lightning or abnormal pressure rises should be kept in connection with each power system with overhead transmission, as this will generally lead, after careful investigation, to certain amplifications or modifications of the existing protective arrangements such as to prevent the repetition of similar accidents. In this manner, very fair protection can be afforded at the present day to almost any overhead transmission system; but it is doubtful if it will ever be possible to protect apparatus against a direct lightning stroke. Damage to machinery, due to this cause, is, however, very rare.

In regard to the protection of the line itself, it is obvious that protective devices, however complete or perfect they may be, provided at the two ends of a long transmission, afford no protec-

tion to the insulators along the line. The frequently grounded guard wire would appear to be a good protection to a line; but here again the engineer must use his judgment, because certain portions of a line may require far more protection than other portions, and even if the cost of guard wire protection be considered excessive for the entire length of a long-distance transmission, it may yet be a decided advantage to provide guard wire protection near the generating and transforming stations and on those parts of the line most likely to be affected by atmospheric disturbances.

In some cases, it may be wise to improve the insulation and to raise the voltage at which a "spill over" will occur; while under other circumstances it might be better to provide an easy path for a discharge over insulators, by means of suitably disposed arcing rings or equivalent arrangement. Mr. P. H. Thomas once explained the matter of line insulation by making use of a very simple analogy. Where a discharge strikes the line, a wave starts, and the potential of this wave will be such as can be allowed by the line itself; the energy of the discharge is limited by the static capacity of the line and the voltage at which a "spill over" will occur at the insulators. The energy of the travelling waves "grows less and less as they proceed. This action may be likeened to the formation of a wave in a long, narrow trough with high sides containing water and normally less than half full, by sudden flooding of the trough by a large quantity of water at some particular point; the excess water spills over and escapes from the trough at the point of the flooding, but there is still a wave started in each direction as high as the sides of the trough will permit; this passes along until the end is reached or the energy of the wave is gradually dissipated. It makes no difference how much water is thrown into the trough, there can be a wave only as high as the sides will permit."

One point that is sometimes overlooked is the effect of the line *current* on pressure disturbances. The disturbances that are set up by switching operations or by power arcs, following a lightning discharge, will be far more serious when the current is large than when it is small. This is one reason why extra high tension transmissions suffer less from lightning disturbances than moderate voltage systems on which the current is often larger. It is hardly an exaggeration to say that the handling of heavy currents on long distance transmissions presents more engineering difficulties than insulation problems on the high voltage schemes.

Very high voltage transmission lines may, indeed, operate satisfactorily without lightning protection, especially when working at pressures near the critical voltage of the corona formation, and some relief to high-pressure energy is afforded by the corona itself. Low-pressure lines, working at about 10,000 volts, are usually far less exposed than the high-pressure lines, and the low-pressure lightning arresters are rather more effective than those for the higher voltages. Such lines do not give so much trouble as those working at pressures between 30,000 and 80,000.

It will be gathered from the foregoing remarks that the power transmitted, and not only the pressure of transmission, is an important factor in the problem of lightning protection.

Without attempting a detailed analysis of the troubles due to switching operations, it may be stated that, as a general rule, it will be best to energize a dead line of considerable length by first connecting to the line the step-down transformers at the distant or receiving end, and then switching the step-up transformers at the generating end on to the low-tension bus-bars.

It is possible that the near future may see some developments in the matter of facilitating the dissipation of high frequency energy in the line itself, with the object of rapidly limiting the amplitude of the travelling waves and the distance from the center of disturbance at which their effects can be of practical account. It is obvious that what is required is a line that will transmit, without undue loss, the power currents at normal frequency, and yet afford means for the rapid dissipation of high frequency energy. Apart from the property peculiar to the corona, which leads to the dissipation of energy on over-voltages, there is a property common to all metallic circuits which leads to the more rapid dissipation of high frequency than of low frequency energy. The so-called "skin effect" which apparently increases the resistance of a conductor carrying alternating or fluctating currents, owing to the forcing of the current toward the outside portions of the wire at the higher frequencies, is clearly of value in limiting the distance over which high frequency disturbances are propagated. By covering the conductor with a thin layer of high-resistance metal, astonishing results can be obtained. Experiments made with wires having a coating of nickel only 0.07 mm. thick, showed that the resistance offered to currents of 300,000 cycles per second was four times the resistance offered by the same wires without the coating of high

resistance metal. This was referred to by Mr. Gino Campos at the Turin International Electrical Congress of 1911. He, also, by a judicious arrangement of choke coils shunted by non-inductive resistances, showed how a transmission line might be made to offer great resistance to high frequency currents, while currents, of normal frequency would be practically unaffected. The choke coil of comparatively low inductance will not interfere with the passage of the power current, but high frequency currents will prefer the shunt path to the choke coil and will be largely dissipated in the resistance.

Devices of this description are of undoubted value, but whether or not practical and commercial considerations will justify their general adoption, is a matter which will probably be decided in the course of the next few years.

CHAPTER VI

The Transmission of Energy by Continuous Currents.

77. General Description of the Thury System.—In the Thury system of electric power transmission by continuous currents, the current is constant in value and the pressure is made to vary with the load. All the generators and all the motors are connected in series on the one wire, which may be in the form of a closed loop serving a wide area, or it may consist merely of the outgoing and returning wires between a power station containing

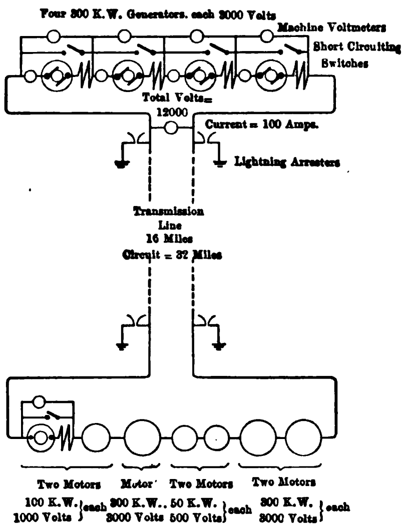

Fig. 47.—Diagram of connections—Thury series system.

all the generators, and one or more substations, with motors, at the end of a direct transmission. It is the latter arrangement (direct long-distance transmission) for which the Thury system is specially adapted. The diagram, Fig. 47, shows a typical arrangement of machines for a small installation on the Thury system. In this example there are four generators at one end of the line, and seven motors of various capacities at the other end

of the line, all the machines being connected in series and provided with short-circuiting switches. The connections are, however, so simple that the diagram is self-explanatory. The voltage at the terminals of any one dynamo is limited, as the necessity for a commutator renders it impossible to wind a continuous-current machine for so high a pressure as may be obtained from alternating-current machines. The limiting pressure per commutator on the existing Thury systems is 5000 volts, this being on the machines of the Metropolitan Electric Supply Company of London: the lowest is 1300 on a small two-generator plant in Russia.

In order to obtain the high pressures required for economical transmission over long distances, it is necessary to connect many generator units in series; the difficulty of insulation between machines and ground being overcome by mounting the dynamos and motors on insulators and providing an insulated coupling between the electric generators and the prime movers. An insulating floor is also provided.

When a machine, whether generator or motor, is not in use, it is short-circuited through a switch provided for this purpose. As the motor load varies, generators are switched in or out of circuit, thus varying the total voltage. When it is required to switch in an additional generator, the machine is brought up to speed until it gives the proper line current before the short-circuiting switch is opened to throw the machine in series with the line. To start up a motor, the short-circuiting switch is opened when the brushes are in the position of zero torque. The brush rocker is then gradually moved round, and the motor, starting from rest, increases in speed until the brushes are in the required position; the actual speed, for any particular position of the brushes, being dependent upon the load.

The motors may be distributed anywhere along the line, either on the premises of private users of power, where they may be directly coupled to the machinery to be driven, or in substations, coupled to constant pressure electric generators giving a secondary supply for lighting and power purposes. In most of the Thury undertakings in Europe, this secondary supply is three-phase alternating current.

A series wound dynamo machine, with a current of constant value passing through field-magnet and armature windings, is essentially a constant torque machine. In the case of a motor,

if the load is decreased, the motor will increase in speed and tend to "run away"; with increase of load, the motor will slow down, and in time come to a stand-still. In regard to the generators, the ideal prime mover is one which will give a constant torque, such as a steam engine with fixed cut-off and constant steam pressure; and a single generator so driven would be practically self-regulating, and maintain constant current regardless of load, as the speed—and therefore the pressure at terminals—would adjust itself to suit the motor load. The generators are, however, usually driven by prime movers which are far from fulfilling the ideal conditions. Most of the Thury stations are driven by water turbines, which are most efficient as constant speed machines; while the maximum torque at low speeds is generally about twice the torque under conditions of highest efficiency at normal speed.

Just as various devices are provided, when working on the parallel system, to maintain constant pressure of supply, so in the series system, it is necessary to provide regulating devices to maintain a constant current. Regulators controlled by the main current, or by a definite fraction of the main current, passing through a solenoid, can be made to act on mechanism designed to vary the speed of the prime movers. This method is quite practicable, but, where the type of engine—such as a water wheel under constant head—is not suited to variable speed running, the machines may be run at constant speed, and the automatic device made to alter the magnetic field cut by the armature conductors, this being the only alternative means of varying the voltage generated. The alteration of the effective magnetic flux may be effected:

(1) By shunting a portion of the main current so that it shall not all pass through the field winding, and

(2) By shifting the position of the brushes on the commutator.

A combination of both methods appears to give satisfactory results. The method (1) alone is liable to lead to sparking troubles because of the relatively greater armature reaction due to the weakening of the field; and, in practice, it is found inadvisable to shunt more than one-third of the total current. It is, of course, understood that the large variations of voltage are obtained by connecting more or fewer generators in series on the line.

The motors, whether connected directly to the machinery of

mills or factories, or used for driving sub-generators of the constant pressure type, are usually required to run at constant speed. Their regulation is effected by a small centrifugal governor which rocks the brushes by acting on intermediate mechanism driven by the motor itself. The breaking or reversal of a motor is most simply effected by shifting the brushes round, through the no-voltage position, until the current reverses in the armature coils.

A short-circuit on a motor merely removes that portion of the total load from the system, and the regulators on the generators will readjust the pressure accordingly. If a short-circuit occurs on a generator, the prime mover may be protected from the shock by a slipping coupling, which is commonly provided. If, owing to the failure of a prime mover, a generator tends to reverse and be driven as a motor, it may be short-circuited by a switch that can very easily be made to operate automatically on reversal of current.

78. Straight Long-distance Transmission by Continuous Currents.—Although high-pressure direct current may be used on the loop system with any number of motors or motor substations distributed along the line—and, if desired, with any number of generating stations at suitable points on the loop—it will generally be found that a parallel constant-pressure system is preferable for covering a large industrial area, the simple reason being that, with the series system having a load more or less uniformly distributed along the loop, the system is a high-tension transmission at the start only, since the required voltage decreases with the distance from the generating plant. It is true that the cost of the insulation may therefore be less than for a system on which the pressure is high throughout, but that can be said of any low-tension system. The point is that, in the case of the series loop serving a wide district with power taken off at intervals along the line, the average pressure at which power is supplied to the motors or substations is only about half that which is supplied to the line where it leaves the power station. It must not be concluded that the Thury system is not well adapted to supplying several motor substations. It is an easy matter, as previously mentioned, to connect any number of motors in series on the line, but in order to get the full benefit of the series system these substations should all serve a comparatively small district at the distant end of the transmission line.

79. Insulation of Line when Carrying Continuous Currents.—The question of sparking distances and the behavior of insulating materials when subjected to continuous-current pressures of high values is of the greatest importance when considering the relative values of the Thury system and the more common three-phase high-tension transmission. On the assumption of the theoretical sine wave, the maximum instantaneous value of an alternating e.m.f. is $\sqrt{2}$ times the root-mean-square value, and comparisons between alternating-current and direct-current transmissions are usually made on this basis, which makes the allowable continuous-current pressure to ground or between wires, for the same insulation and spacing, $\sqrt{2}$ times the working pressure of an alternating-current system. The ratio should, however, be based on experimental data, and, with a view to obtaining definite and conclusive information on this point, Mr. Thury conducted some years ago a very complete set of comparative tests with high voltages, both continuous and alternating. The results of these tests are probably more favorable to the alternating-current systems than would have been the case had they been conducted on existing high-pressure power-transmission systems, because the experimental alternator used in the tests gave a rather flat-topped e.m.f. wave without any irregularities. The tests conducted to determine the comparative pressures at which various insulating materials would be punctured all tend to show that with continuous currents something more than twice the alternating pressure is required to puncture the insulation; and, in regard to sparking distances, the direct-current voltage necessary to spark over a given distance is, on the average, double the alternating-current voltage. In fact, this very complete series of tests seems to indicate that any existing transmission line designed for a definite maximum working pressure with alternating currents is capable of being used to transmit continuous currents at twice this pressure. It is also interesting to note that insulators which become hot when subjected to high alternating-current voltages remain cool when tested with continuous currents. In fact, the leakage losses on the Thury transmissions are small. The total leakage loss over about 3000 insulators on the St. Maurice-Lausanne transmission (a distance of 35 miles), even in damp weather, is something of the order of 900 watts.

It is usual to employ two insulated wires for direct-current

high-pressure transmission, but under certain conditions it might be quite satisfactory to use the earth as the return conductor. The arrangement with two wires and the entire electric circuit insulated from earth is usual for pressures up to 25,000 volts. It has the advantage over any grounded system that any point on the circuit may become grounded without causing a stoppage, and repairs can readily be carried out by temporarily grounding two more points, one on each side of the fault. The facility and safety with which repairs on the high-tension system can be carried out by grounding the point where the work is being done is another advantage of this arrangement.

If a ground connection is made at both ends of the two-wire transmission, the ground wire being so situated as to balance the load as well as possible, an arrangement equivalent to the ordinary three-wire system is obtained. The pressure between wires may then safely be doubled because the potential difference between any one wire and earth can never exceed half the maximum pressure of transmission. On the other hand, some of the advantages of the ungrounded system are lost.

A direct-current transmission to any economic distance by means of a single wire, using the earth as the return conductor, is by no means an impossible scheme. The ground resistance is practically zero, the loss of pressure being almost entirely in the immediate neighborhood of the grounding plates. Tests made on the St. Maurice-Lauzanne line (35 miles) gave a total ground resistance of 0.5 ohm. Continuous currents of the order of 100 amp. returning through the earth do not appear to be objectionable in any way. By taking the ground connections to a considerable depth below the surface the current density at ground level would everywhere be so small that interference with opposing interests would hardly be possible.

80. Relative Cost of Conductors: Continuous Current and Three-Phase Transmissions.—In order to study the relative costs of conductor material required for the series direct-current system and the more common three-phase alternating-current transmission, a basis of comparison is necessary, and the following assumptions will be made:

(A) Same distance of transmission; no tapping of current at intermediate points.

(B) Same total amount of power transmitted.

(C) Same power loss in conductors (losses due to leakage or capacity of lines are neglected).

(D) Same insulation used on both systems.

This last condition is practically equivalent to stating that the maximum value of the voltage shall be the same. It is proposed to consider the following four conditions:

(a) Same maximum pressure above ground; the direct-current voltage being $\sqrt{2}$ times the alternating-current voltage (sine wave assumed).

$$\text{Ratio } \frac{E_a}{E}=\frac{1}{\sqrt{2}} \qquad (1)$$

where E and E_a stand respectively for the continuous and alternating voltages to ground. (See Figs. 48 and 49.)

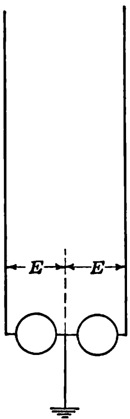

Fig. 48.

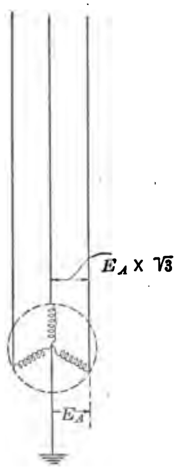

Fig. 49.

Figs. 48 and 49.—Comparison of voltages on direct-current and three-phase systems.

(b) Same as (a); but direct-current voltage double the alternating voltage.

$$\text{Ratio } \frac{E_a}{E}=\frac{1}{2} \qquad (2)$$

(c) Same pressure between wires; the allowable direct-current pressure being $\sqrt{2}$ times the alternating-current pressure.

$$\frac{\sqrt{3}E_a}{2E}=\frac{1}{\sqrt{2}}, \text{ or ratio } \frac{E_a}{E}=\frac{\sqrt{2}}{\sqrt{3}} \qquad (3)$$

(d) Same as (c); but direct-current pressure double the alternating-current pressure.

$$\frac{\sqrt{3}E_a}{2E}=\frac{1}{2}, \text{ or ratio } \frac{E_a}{E}=\frac{1}{\sqrt{3}} \tag{4}$$

To satisfy the condition of equal total power, the equation is,

$$2\ E\times I=3E_aI_a\cos\theta \tag{5}$$

and for equal line losses,

$$2\ I^2R=3\ I^2_aR_a \tag{6}$$

where I is the current per conductor in the direct-current transmission, and R the resistance per mile of single conductor; while I_a and R_a are the corresponding quantities for the three-phase transmission.

In either system the total weight (and cost) of the conductors is proportional to $\frac{\text{number of conductors}}{\text{resistance of each conductor}}$ which gives the relation,

$$\frac{\text{Cost of conductors, direct-current system}}{\text{Cost of conductors, three-phase system}}=\frac{2R_a}{3R} \tag{7}$$

but R_a can be expressed in terms of R thus·

By (6)

$$R_a=\frac{2I^2R}{3I_a^2} \tag{8}$$

and by (5)

$$I=\frac{3E_aI_a\cos\theta}{2E}$$

or

$$I^2=\frac{9E_a^2I_a^2\cos^2\theta}{4E^2}$$

which, when put for I^2 in formula (8), gives,

$$R_a=\frac{3E_a^2\cos^2\theta\times R}{2E^2} \tag{9}$$

Thus the equation given by (7) becomes,

$$\frac{\text{Cost of conductors, direct-current system}}{\text{Cost of conductors, three-phase system}}=\frac{E_a^2}{E^2}\cos^2\theta$$

Assuming the very common value of 0.8 for the power-factor of the three-phase system, the numerical ratio for the four conditions previously stated would be:

(a) For same maximum pressure to ground, with sine wave assumption,

$$\frac{\text{Direct-current cost}}{\text{Alternating-current cost}}=\frac{\cos^2\theta}{2}=0.32$$

(b) Same as (a), but allowable direct-current pressure assumed to be *double* the alternating-current pressure,

$$\frac{\text{Direct-current cost}}{\text{Alternating-current cost}}=\frac{\cos^2\theta}{4}=0.16$$

(c) For same maximum pressure between wires, with sine wave assumed,

$$\frac{\text{Direct-current cost}}{\text{Alternating-current cost}}=\frac{2}{3}\cos^2\theta=0.426$$

(d) Same as (c), but allowable direct-current pressure assumed double the alternating-current pressure,

$$\frac{\text{Direct-current cost}}{\text{Alternating-current cost}}=\frac{1}{3}\cos^2\theta=0.213$$

The transmission line, apart from the cost of conductors, would be cheaper for the direct-current than for the three-phase scheme because there are fewer insulators required and only two instead of three conductors to string; and if a grounded guard wire is erected above the conductors, it is more convenient to arrange this over the two direct-current conductors than over the three alternating-current wires, and it would not necessitate the same total height of tower. The important saving is, however, in the conductors themselves. Taking the figure most favorable to the direct-current scheme (b), the alternating-current conductors to transmit the same power with the same loss would cost six and a quarter times as much as if direct-current transmission were used, and even under the assumption (c), most favorable to the three-phase scheme, the cost would still be 2.35 times the cost of the direct-current conductors. For the purpose of getting out preliminary estimates, it is certainly safe to assume that, if the power factor of the three-phase load may be taken as 0.8. the cost of conductors on a long-distance direct-current transmission would be only one-quarter of the cost of conductors with the alternating-current scheme on the assumption of equal I^2R losses. The fact that it would probably be uneconomical to allow the same losses in both cases does not render the comparison less interesting or valuable.

81. Concluding Remarks on D. C. Transmission.—As an indication of what has been done in Europe since the introduction of the Thury system 23 years ago, it may be stated that there are at present about 15 separate transmissions in operation, in Switzerland, Italy, France, Hungary, Spain and Russia. The shortest length of loop is 12.4 miles (Batoum, Russia), with a line pressure of 2600 volts. The longest is 224 miles (112 miles straight transmission), this being the Moutiers-Lyons line at a maximum pressure of 57,600 volts, which, however, may be doubled in the future. The average of all the working lines is a 50-mile loop and a pressure of 14,500 volts.

In England, the direct-current series system has been adopted by the Metropolitan Electric Supply Co. of London on their Western section. The plant has been in operation since March of 1911 and given entire satisfaction. The current is about 100 amps., but can be varied from 70 to 120 amp. without causing trouble through sparking on the commutators. The commutators measure 5 ft. in diameter and 6 3/4 in. in length. They have 1439 segments and run sparklessly at 5000 volts between brushes. The machines have six poles and only two sets of brushes.

As an example of what might be done at the present time in the way of direct-current transmission on a large scale, it is clear that no difficulty need be experienced in building dynamos of a large size with 5000 volts on one commutator. Assuming a current of 400 amp., which would probably be transmitted by two or more conductors connected in parallel, the output of each machine would be 2000 kw. and two of these might be coupled to one prime mover. With twelve pairs of generators, the pressure between wires would be 120,000 volts and the maximum total output 48,000 kw. There would be very little new or experimental engineering work in connection with such a scheme.

Electrical engineers on the American continent are rather inclined to the belief that when energy has to be transmitted from one place to another the one and only course open to them is to adopt the three-phase alternating-current system. It is not suggested that at the present time this may not, in the majority of cases, be the best system available; but undoubtedly there are conditions under which the continuous-current series system would prove more economical. Of course, first cost of plant, operating charges and reliability have to be taken into account

when comparing different systems, and the most satisfactory way of doing this is to reduce all estimated costs to the common basis of annual charges. The cost of the direct-current generators must be set against the combined cost of alternators and exciters and step-up transformers with all intermediate switchgear.

In this connection the writer cannot refrain from quoting a paragraph which occurred in one of the leading articles in the *Electrical World* of New York, in which reference is made to the fact that transmission by continuous currents has received considerable attention in Europe.

"Any engineer who wanders through one of the large Thury stations and then calls to mind the usual long concrete catacombs bristling with high-tension insulators and filled with dozens of oil switches, scores of disconnecting switches, webbed with hundreds of feet of high-tension leads and spatted with automatic cut-outs, will stop and think a bit before he complacently sniffs at high-tension direct-current transmission."

In regard to reliability it is true that, on the Thury system, the generators have not the protection against lightning disturbances which the step-up transformers afford to the alternators on high-tension three-phase systems, and where thunderstorms are prevalent this must not be overlooked, as the cost of protective apparatus may prove excessive. In this connection it is interesting to note that the charges of electricity in the upper atmosphere are always positive, and the negative wire will therefore tend to draw a lightning discharge away from the positive wire or grounded guard wire, but to how great an extent this would affect the proper disposition of the wires it is difficult to say.

This chapter will be concluded with a brief summary of the important points in favor of, and unfavorable to, the employment of continuous currents on the series system for the purpose of transmitting energy at comparatively high voltages from one place to another.

Advantages of the Direct-Current Series System

(1) The power-factor is unity—a fact which alone accounts for considerable reduction of transmission losses.

(2) Higher pressures can be used than with alternating current, the conditions, as shown by actual tests, being more favor-

able to direct-current transmission than is generally supposed. Without any alteration to insulation or spacing of wires, approximately double the working pressure can be used if direct current is substituted for alternating current. Moreover, the insulation is subjected to the maximum pressure only at times of full load, whereas on the parallel system the insulation is subject to the full electrical stress at all times.

(3) There is no loss of power through "dielectric hysteresis" in the body of insulating materials.

(4) The necessity for two wires only, in place of three, effects a saving in the number of insulators required and allows cheaper line construction.

(5) Where it is necessary to transmit power by underground cables continuous currents have advantages over alternating current. Single-core cables can be made to work with continuous currents at 60,000 volts. By using two such cables and grounding the middle point of the system it is, therefore, quite feasible to transmit underground at 120,000 volts.

(6) The practicable distance of transmission, especially when the whole or a part is underground, is greater than with alternating currents.

(7) There are no induction or capacity troubles and no surges or abnormal pressure rises due to resonance and similar causes, such as have been experienced with alternating currents. This virtually makes the factor of safety on insulation greater than on alternating-current circuits, even when the working pressure is doubled.

(8) A number of generating stations can easily be operated in series, and when the demand for power increases, a new generating station can be put up on any part of the line if it is inconvenient to enlarge the original power station.

(9) The simplicity and relatively low cost of the switch gear is remarkable. A switch pillar with ammeter, voltmeter, and four-point switch is all the necessary equipment for a generator. The switch pillar for a motor includes, in addition, an automatic "by-pass" which bridges the motor terminals in the event of an excessive pressure rise. This compares very favorably with the ever-increasing—though in some cases unnecessary—complication and high cost of the switching arrangements in high-tension power stations on the parallel system.

(10) With the Thury system any class of supply can be given,

and the motors can be made to drive sub-generators capable of running in parallel with any local electric generating plant.

(11) In hydraulic generating stations where the variations of head are considerable, as will generally be the case if there is no storage reservoir, a greater all-round efficiency can be obtained than if the machines had to be driven at constant speed.

(12) For any industrial operation requiring a variable-speed drive at constant torque, the Thury motor, without constant-speed regulator, is admirably adapted. It might have a useful application in the driving of generators supplying constant current to electric furnaces in which the voltage across electrodes is continually varying.

Disadvantages of the Direct-current Series System

(1) The necessity of providing insulating floors and mounting all current-carrying machines and apparatus on insulators. The highly insulated coupling required to transmit, mechanically, large amounts of power between prime mover and electric generator is also objectionable.

(2) The smallness of the generators; the output of each generator being limited by the line current and the permissible voltage between the collecting brushes on the commutator. One prime mover is usually coupled to two or more direct-current generator units. This however, is necessarily more costly than if larger electric generators could be used; moreover, it practically limits the choice of hydraulic turbines to the horizontal type, since the coupling of several generators on the shaft of a vertical water-wheel would be difficult and unsatisfactory.

(3) With constant current on the line, the line losses are the same at all loads, and the percentage power loss in conductors is inversely proportional to the load. This is exactly the reverse of what occurs on the alternating-current parallel system, in which the percentage line loss is directly proportional to the load. It should, however, be mentioned that on the Thury system the line current may be reduced about 30 per cent. at times of light load, except when the circuit feeds motors of industrial undertakings requiring constant current day and night. It must not be overlooked that large percentage losses at times of light load are of serious moment only where steam engines are used or where storage reservoirs are provided for water-power generating stations.

In the case of water-power schemes without storage, the fact of the full-load line losses continuing during times of light load is not objectionable.

(4) The series system is less suitable than the parallel system for distribution of power in the neighborhood of the generating station.

(5) The water turbine working under constant head is not the ideal engine for driving constant-current machines.

(6) The necessity of special regulating devices to maintain constant speed on the motors.

(7) The impossibility of overloading the motor, even for short periods. This would be a very serious objection to the use of these motors in connection with electric traction systems.

(8) Greater liability to damage and interruption from the effects of lightning. It may be said that an overhead line, whether for alternating or direct current, is always liable to damage by lightning; but with the high-tension alternating-current system the transformers and automatic oil switches will usually protect the generators themselves from serious damage, while with the Thury system there is always a path for lightning discharges through the generators and motors, and the damage done may be very great. This simply means that particular attention should be given to the question of lightning protection on direct-current overhead lines, and the ease with which highly inductive choke coils can be introduced on a direct-current system, without opposing any obstacle (except ohmic resistance) to the passage of the line current, tends toward the attainment of increased safety.

CHAPTER VII

Mechanical Principles and Calculations

82. Introductory.—If a wire is stretched between two fixed points, A and B, lying in the same horizontal plane, and separated by a distance of l ft., there will be a certain *sag* of s ft. in the wire. This sag or deflection from the horizontal line $A\ B$, will be greatest at the center of the span, and its value, for a given length of span (l ft.), will depend upon the weight of the wire and the tension with which it has been drawn up. If the wire were perfectly uniform in cross-section and perfectly flexible, the curve $A\ D\ B$ (Fig. 50) would be a catenary. It should be observed, however, that in this and subsequent diagrams, the sag s is shown

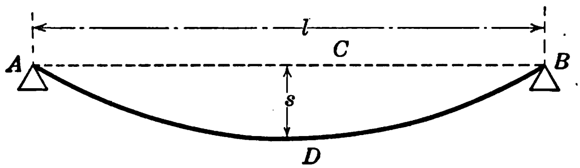

Fig. 50.—Wire hanging between two supports at the same elevation.

much larger relatively to the span l than it would be on most practical transmission lines; and as the span l is generally very little shorter than the length of the wire between the suspension points A and B, no appreciable error is introduced by assuming the weight of the wire to be distributed uniformly along the horizontal line $A\ C\ B$ instead of along the curve $A\ D\ B$. On this assumption the curve $A\ D\ B$ becomes a parabola, and as the calculations are more easily made on the assumption of a parabolic curve than with the possibly more correct catenary, it is customary to use the formulas relating to the parabola for the solution of sag-tension problems. On very long spans, even with fairly large conductors, the actual curves assumed by the wires hanging in still air under the influence of their own weight only, will approximate more nearly to the catenary than to the parabola, and if it is desired to introduce refinements of calculation, the catenary is the more correct curve to work to on long

spans,[1] but owing to the many more or less arbitrary assumptions that must necessarily be made in all calculations on the mechanical features of a transmission line, such refinements are, in the writer's opinion, unnecessary and only justifiable when they involve no additional time or labor in the calculations.

83. The Parabolic Curve.—The law of the parabola is:

$$y^2 = 2mx \qquad (1)$$

where m is a constant, equal to twice the distance between the directrix of the parabola (not shown in Fig. 51) and the vertex

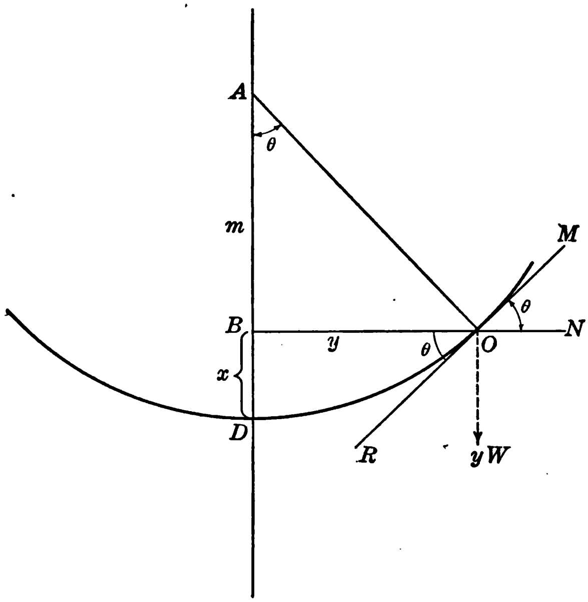

FIG. 51.—Parabolic curve.

D. This constant m is also the length of the subnormal $A\ B$ (Fig. 51). Thus, if the tangent $M\ R$ to the parabola be drawn at any point O, and the line $O\ A$ be drawn normal to the tangent at the point O, the angles $M\ O\ N$ and $B\ O\ R$ are each equal to the angle $O\ A\ B$ or θ. It follows that tan θ, which is the slope of the curve $\left(\frac{dx}{dy}\right)$ at the point O, is equal to $\frac{y}{m}$, that is to say, it is proportional to y since m is constant.

[1] In his paper read at the annual Convention of the A. I. E. E. at Chicago, June, 1911, Mr. W. Le Roy Robertson works out solutions of sag and span problems with the aid of the catenary.

Substituting the value of m as given by formula (1), the tangent of the angle which the tangent to the curve makes with the horizontal can be written:

$$\tan \theta = \frac{2x}{y}$$

For the particular case represented by Fig. 50, the value of m in formula (1) may be determined by substituting s for x and $\frac{l}{2}$ for y. This gives:

$$m = \frac{l^2}{8s}$$

and the tangent of the angle which the tangent to the curve makes with the horizontal can be written:

$$\tan \theta = \frac{y}{m} = y \times \frac{8s}{l^2} \tag{2}$$

At the points of support A and B (Fig. 50) this becomes:

$$\left.\begin{array}{c}\text{tan } \theta \text{ at points of support,}\\ \text{when both supports are on}\\ \text{same level}\end{array}\right\} = \frac{4s}{l}$$

A knowledge of this angle makes it an easy matter to calculate the tension in the wire at any point. Thus, in Fig. 52, consider

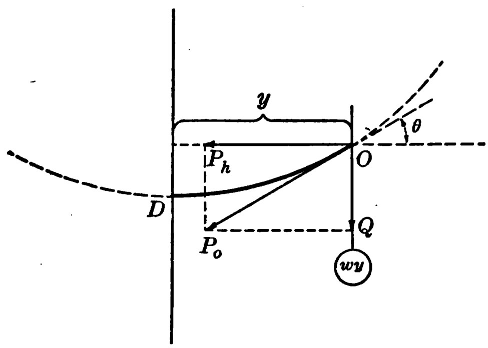

FIG. 52.—Vector diagram of forces due to weight of suspended wire.

the portion of the conductor included between the vertex D of the parabola and any point O at a distance y from the X axis. On the assumption previously made that the weight of the wire may be considered as being uniformly distributed horizontally, the weight of the section considered is y times w, where w is the

weight of the conductor per foot length; and this acts vertically downward at the point O. The wire must obviously take the stress in the direction of the tangent to the curve, and the tension in the wire at the point O is therefore,

$$P_o = wy \times \frac{OP_o}{OQ}$$
$$= wy \times \operatorname{cosec} \theta \tag{3}$$

The horizontal component is,

$$P_h = wy \times \frac{OP}{OQ}$$
$$= wy \times \operatorname{cotan} \theta \tag{4}$$

but, by formula (2)

$$\operatorname{cotan} \theta = \frac{l^2}{8sy}$$

therefore
$$P_h = \frac{wl^2}{8s} \tag{5}$$

which is the well-known formula as generally used for calculating the tension at the lowest point in the span (the point D in Fig. 50) of an overhead conductor hanging in still air under the influence of its own weight only.

The tension at any other point O, as given by formula (3), can readily be expressed in terms of l, s, and y, and it becomes:

$$P_o = \frac{w}{8s}\sqrt{l^4 + 64s^2y^2} \tag{6}$$

The maximum tension will be at the points of suspension A and B; it is,

$$P_{max} = \frac{wl}{8s}\sqrt{l^2 + 16s^2} \tag{7}$$

A simpler formula, which is a very close approximation to the correct formula, is,

$$P_{max} = P_h + ws$$
$$= \frac{wl^2}{8s} + ws \tag{8[1]}$$

[1] This formula is based on the fact that when the quantity a is small relatively to A, it is permissible to write,

$$\sqrt{A^2 + a^2} = A + \frac{a^2}{2A}$$

The ratio of the maximum tension (at point of support) to the tension at center of span is,

$$\frac{P_{max}}{P_h}=\frac{Formula(7)}{Formula(5)}=\frac{\sqrt{l^2+16\,s^2}}{l} \tag{9}$$

$$\text{which, approximately}=1+\frac{8s^2}{l^2} \tag{10}$$

Seeing that s is usually very small in relation to l, the quantity given by formula (9) or (10) is generally so nearly equal to unity that no serious error is introduced by using the simple formula (5) for calculating the tension in the conductors; in other words, the wire is so nearly horizontal throughout its entire length that the horizontal component of the tension, instead of the resultant, may be considered as the tension acting at all points of the wire.

The length of the parabolic curve $A\ D\ B$ (Fig. 50) is,

$$L=l\left(1+\frac{8}{3}\frac{s^2}{l^2}-\frac{32s^4}{5l^4}+,\text{ etc.}\right)$$

but it is usual to omit all except the first two terms of the series. This gives,

$$L=l+\frac{8s^2}{3l} \tag{11}$$

84. Effect of Temperature Variations on Overhead Wires.—If a is the temperature coefficient of the material of the conductor (see the table of constants in Chapter IV, page 65), then the length of the wire when the temperature is raised t degrees is,

$$L_t=L_o(1+at) \tag{12}$$

in which L_o is the original length of the wire.

This formula assumes that the wire is unstressed, or that the stress remains unaltered notwithstanding the increase in temperature. This, however, is not the case with an overhead conductor. As indicated by formula (5), the tension in a wire suspended

Thus, the total or resultant force at the point of support is,

$$\begin{aligned}P_{max}&=\sqrt{P_h^2+\left(\frac{wl}{2}\right)^2}\\&=\frac{wl^2}{8\,s}+\left(\frac{wl}{2}\right)^2\times\frac{8s}{2wl^2}\\&=\frac{wl^2}{8s}+ws.\end{aligned}$$

horizontally between two fixed supports is almost exactly proportional to the square of the span, and inversely proportional to the sag at center of span. The effect of temperature variation is to alter the length of wire and therefore the amount of sag and tension. With a reduction of temperature, the length of wire will decrease; this will cause an increase in the tension, but owing to the fact that the wire will stretch under the influence of the increased tension, the sag at the lower temperatures will be somewhat greater than it would be if there were no elongation of the wires with increase of stress. All sag-temperature calculations, whatever the method adopted, must therefore take into account not only the effect of elongation with increase of temperature, but also the effect of the elastic contraction of the wire with increase of sag.

If P is the tension in a wire of cross-section A, and if M is the elastic modulus (as given for various materials in Table I of Chapter IV), then the elongation of the wire due to the tension P is,

$$L_e = \frac{P \times L}{A \times M}$$

the original length of the wire being L.

If instead of $\frac{P}{A}$ the letter T be used to denote the stress in pounds per square inch of cross-section, the formula becomes

$$L_e = L \times \frac{T}{M} \tag{13}$$

It is customary to assume that the material of the conductors is perfectly elastic up to a certain critical stress known as the elastic limit; that is to say, if the application of a certain stress produces a strain represented by L_e, it is assumed that, on the removal of the stress, the conductor will contract to its original length L, and that this process of elongation and contraction follows a straight-line law. This is not scientifically correct, because, on removal of load, the amount of contraction is not directly proportional to the decrease of stress; but the departure from the straight-line law is not considerable, and no serious error is introduced by disregarding refinements of this nature.

A very simple and convenient diagram may be constructed, by which the elongation of a conductor owing to change of stress or temperature, acting either singly or jointly, can readily be

ascertained. Fig. 53 shows such a diagram, constructed for stranded conductors of either copper or aluminum. Referring to

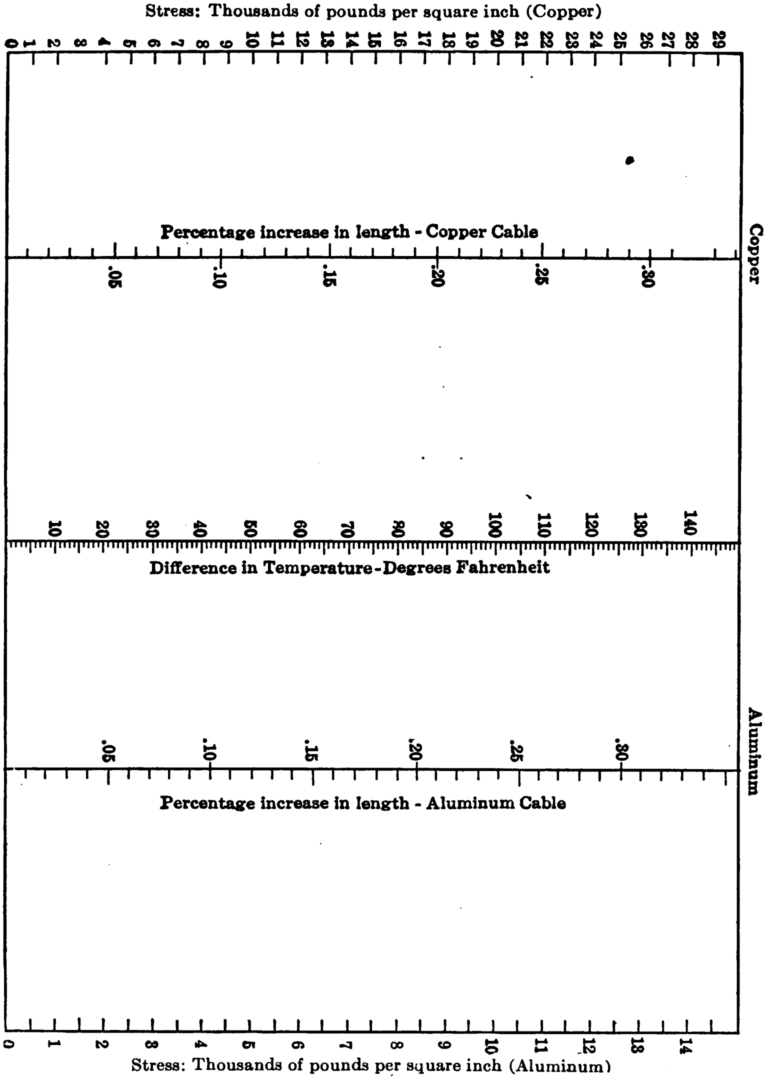

FIG. 53.—Chart for calculating changes in length of overhead conductors.

the vertical scales from left of diagram to center, if a straight line be drawn from a point on scale 1 corresponding to the stress

in a *copper* conductor, to a point on scale 3 corresponding to the temperature, the point at which this line crosses scale 2 will indicate the increase in length above the condition of zero stress and zero temperature. This increase in length is expressed as a percentage, such as feet per hundred feet of the conductor.

As an example, an increase in length of 0.1 per cent. of a copper conductor will occur

(a) If the stress remains constant and the temperature increases 104°.

(b) If the temperature remains constant and the stress increases 15,000 lb. per square inch.

(c) If the stress *increases* from 12,000 to 15,500 lb. per square inch at the same time as the temperature falls from 137 to 6° F.

The vertical scales 4 and 5 are used in a similar manner with the common temperature scale for determining the changes in length of stranded aluminum conductors.

85. Abnormal Stresses in Wires Due to Wind and Ice.—When a wire hangs between horizontal supports in still air under the influence of its own weight only, the tension at center of span for a definite distance between supports and a definite sag at center will be proportional to the weight per foot-length of the wire. When loaded with sleet or ice, or subject to wind pressures, or a combination of both these additional loads, the calculations of the proper sag to allow are based on the assumption that the weight per foot is no longer that of the wire only, but n times this amount; where n is a factor depending upon the character and amount of the abnormal loading that the wire is likely to be subjected to.

Ice and Sleet.—The effect of snow and sleet adhering to the wires, and forming an ice coating of variable thickness, is to add to the dead weight of the wires. Sleet will generally collect with slightly greater thickness near the lowest point of the span, but it is usual to assume that the extra vertical loading is uniformly distributed over the whole length of the span. Mr Max H. Collbohm[1] says that in the winter of 1908–9, in Wisconsin and neighboring States, the snowstorm of January 28 covered nearly all overhead wires with sleet and snow 2½ in. to 4 in. diameter. The temperature went down to 4° F. below zero, while a wind velocity of 40 miles per hour was recorded. These conditions

[1] *Electrical World*, New York, March 25, 1909, p. 734.

were, of course, exceptional; at the same time an average coating of sleet and snow weighing $\frac{1}{2}$ lb. per foot of wire is not unusual. A coating of ice $\frac{1}{2}$ in. thick on wires running through districts where sleet and low temperatures are common is generally allowed for in calculations. Sometimes this is increased to a radial thickness of $\frac{3}{4}$ in. Sleet deposits 6 in. in diameter have actually been observed on some of the steel conductors of the Central Colorado Power Company. The weather conditions in some of the passes through which this line is carried are, however, particularly severe.

Taking the weight of sleet at 57.5 lb. per cubic foot, the total weight per foot of the loaded conductor is $w+w_1$, or $w+1.254\,r\,(d+r)$ where w is the weight of the wire; r, the radial thickness of sleet (assumed to be of circular section), and d, the diameter of the uncoated wire; these dimensions being expressed in inches. Thus, in the case of a No. 0 B. & S. gage wire, d equals 0.325 in., and assuming r equals 0.5 in., w_1 equals $1.254 \times 0.5 \times 0.825 = 0.517$ lb., or about 1/2 lb. per foot run, which is a common allowance to make for ice loads on wires of average size.

Effect of Material of Conductor on Sleet Deposits.—It is frequently contended that sleet does not deposit readily on aluminum owing to the greasy character of the oxide which forms on the surface of aluminum conductors. The experience of many engineers does not, however, confirm this. Mr. W. T. Taylor says that copper collects a little more sleet than aluminum. He bases this statement on observations on long-distance telephone lines in California, where copper and aluminum conductors run side by side. Mr. E. H. Farrand[1] has observed copper wire accumulate snow to the extent of 3 in. in diameter in a few minutes, while iron wire took a coating 1/2 in. to 1 in. thick; but after a short time the wires of both materials had accumulated snow to the same diameter. Messrs. R. B. Matthews and C. T. Wilkinson[2] have made many observations in the sleet districts of the United States, and they are of opinion that an aluminum conductor will collect as much sleet and ice as copper or steel wires.

When wires are hung vertically one above the other, in the manner frequently adopted for double-circuit lines, there is

[1] *Journal of the Inst. E. E.*, May, 1911, No. 207, p. 659.

[2] "Extra high pressure transmission lines." Paper read before the Inst. of Electrical Engineers, Jan. 26, 1911.

the possibility of sleet falling suddenly from one of the lower wires, while the upper wire remains heavily loaded and with considerable sag. This might cause the lower wire to rise into contact with the upper wire. With this possibility in mind, the disposition or spacing can be made so that short-circuits due to this cause are not likely to occur.

Wind Pressures.—The pressures due to winds of high velocity acting on poles and wires in a direction approximately at right angles to the transmission line are of great importance, both where sleet formation is possible, and in districts where sleet cannot form. In the latter case, the velocity of the wind is frequently greater than in the colder districts; but, on the other hand, a moderate wind acting on the larger diameter of ice-coated wires will generally lead to the greatest stressing of the conductor material. The maximum wind velocities rarely occur at the lowest temperatures, but falling temperatures and rising wind are not unusual after sleet storms. In mountainous districts a transmission line may be subjected at certain points to gusts of wind blowing almost vertically downward; the pressure in such a case, being directly added to the weight of wire and the ice load, may lead to more serious results than an even stronger wind blowing horizontally across the line.

Numerous observations have been made on wind pressures, and it is found that the pressure exerted on small surfaces is proportional to the square of the wind velocity. It will also depend to some small extent upon the density of the air, or the barometric pressure; but the correction for barometric pressure is usually not worth making.

Wind Velocity.—It is well to distinguish between indicated and true wind velocities. The United States Weather Bureau observations are made with the cup anemometer, and wind velocities over short periods of time are calculated on the assumption that the velocity of the cups is one-third of the true velocity of the wind, for great and small velocities alike. This assumption is not justifiable, and a correction must therefore be made in order to convert the Weather Bureau recorded velocities into true velocities. The actual wind velocities corresponding to definite indicated velocities, as given by the U. S. Weather Reports, are as follows:

VELOCITY: MILES PER HOUR

Indicated	Actual
10	9.6
20	17.8
30	25.7
40	33.3
50	40.8
60	48.0
70	55.2
80	62.2
90	69.2
100	76.2

Unless otherwise stated, when wind velocity is referred to, this must be understood to be the true velocity.

Formulas for Wind Pressures.—The formula proposed by the U. S. Weather Bureau (Professor C. F. Marvin), giving pressure in pounds per square foot on small flat surfaces normal to the direction of the wind is:

$$F = 0.004 \frac{B}{30} V^2$$

where B is the barometric reading in inches, and V is the wind velocity in miles per hour. Other formulas are:

$$\begin{array}{ll} \text{Langley} \ldots\ldots & F = 0.0036\ V^2 \\ \left.\begin{array}{l}\text{Smeaton} \ldots\ldots \\ \text{Buck} \ldots\ldots\end{array}\right\} & F = 0.005\ V^2 \end{array}$$

In the case of cylindrical wires, the pressure per square foot of projected area is less than on flat surfaces. The coefficient by which the pressure on flat surfaces must be multiplied to obtain the pressure on the projected surface of a smooth cylinder is given variously by different authorities:

Newton	0.67
Borda	0.5
Ritter	0.45
Von Loessl	0.79
Irminger	0.57 (experimental)

The formula proposed by Mr. H. W. Buck, and generally conceded to be correct is:

$$F = 0.0025\ V^2$$

where F is the pressure per square foot of projected surface of a

cylindrical wire. A more convenient form of expression for this relation is:

$$F' = \frac{dV^2}{5000} \tag{14}$$

where F' is the pressure per foot length of the wire and d is the diameter of the wire in inches; and the denominator, although not exactly as obtained from Mr. Buck's equation, is an easily remembered round number, which is very close to the average of many experimental results. The upper curve in Fig. 54

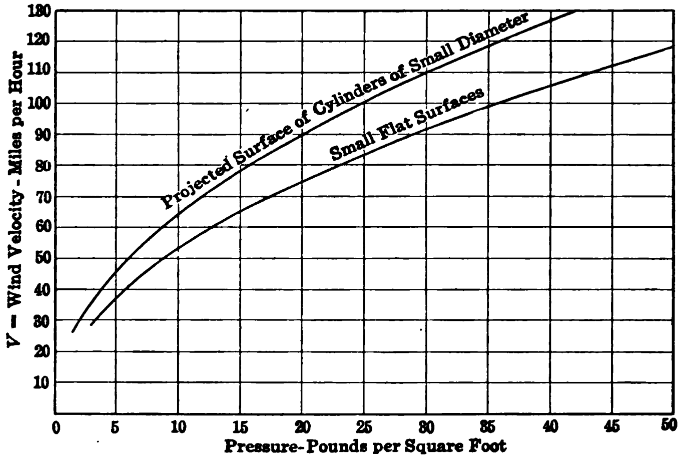

FIG. 54.—Curves giving relation between wind velocity and resulting pressure.

has been plotted from Mr. Buck's formula for cylindrical wires; while the lower curve for pressures on flat surfaces gives the relation between pressure and wind velocity according to Professor Langley's formula.

Relation between Wind Velocity and Height above Ground.—Owing to the resistance offered by the ground surface, the force of the wind is not so great near the ground as at higher altitudes, and greater maximum wind pressures on wires must be allowed for when the line is carried on high steel structures than in the case of the average wood pole line, as used for the lower voltage transmissions.

Mr. F. F. Fowle[1] has given valuable particulars based on

[1] "A study of sleet loads and wind velocities." *Electrical World*, New York, October 27, 1910.

maximum wind velocities at different elevations, observed in Chicago, from which the curve of Fig. 55 has been plotted. It will be seen that, at the average elevation of transmission line wires (25 to 45 ft.), the probable maximum wind velocity is less than half what it would be at 100 ft. above ground, and only about one-third of what may be expected at an elevation of 300 ft.

Mr. Fowle suggests that overhead line calculations might be based on a maximum wind velocity of 47 miles per hour for

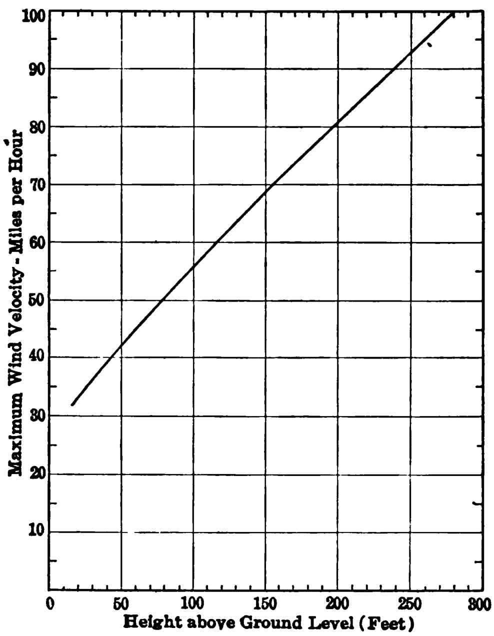

FIG. 55.—Maximum wind velocities at different elevations.

ordinary steel tower construction, and 40 miles for wood pole lines. These are probably safe limits, especially in climates where this maximum wind pressure is considered as acting on ice-coated wires. In exposed positions, and where the line runs through wide stretches of open country, it is well to allow a maximum of 60 for steel tower lines, and 50 for wood pole lines.

According to Mr. Fowle, the wind velocity very rarely exceeds 50 miles in Chicago, the maximum recorded during a period of thirty-six years being 84, while 90 miles per hour has been re-

corded in Buffalo. Wind velocities of 53, and on one occasion 60 miles, have been recorded during sleet storms; but such velocities are exceptionally high.

The British Board of Trade requirements for the calculation of pole lines are a limit of 30 lb. per square foot on flat surfaces, and $30 \times 0.6 = 18$ lb. on the projected surfaces of cylindrical poles or wires. The Committee on Overhead Line Construction, appointed by the National Electric Light Association of New York, assumes a half-inch ice coating for all sizes of conductors, and maximum wind velocities of 50 to 60 miles per hour. This committee states that 62 miles is a velocity not likely to be exceeded during the cold months.

Three classes of loading are considered by the Joint Committee on Overhead Crossings, particulars of which are as follows:

Class of loading	Vertical component	Horizontal component
A	dead	15 lb. per square foot.
B	dead + $\frac{1}{2}$-in. ice	8 lb. per square foot.
C	dead + $\frac{3}{4}$-in. ice	11 lb. per square foot.

For the class *B* loading the ordinary range of temperature is given as—20 to 120° F. For the calculation of pressures on supporting structures the requirements are 13 lb. per square foot on the projected area of closed or solid structures, or on $1\frac{1}{2}$ times the projected area of latticed structures. The same Joint Committee allows a safe stress on copper 50 per cent. of ultimate breaking stress—that is to say, the wires may be stressed to a point very near to the elastic limit.

The Board of Railway Commissioners for Canada specifies for H. T. wires crossing railways, a factor of safety of 2 when wires are coated with ice or sleet to a depth of 1 in., and subject to wind pressure of 100 miles per hour. This combination of abnormal loads being more or less imaginary, it is probable that the factor of safety is actually about 6, which appears to be unnecessarily high.

Swaying of Wires in Strong Winds.—If a transmission line is well designed and constructed, all the wires of one span will generally be found to swing synchronously in any wind. Under exceptional conditions, however, trouble is liable to occur through wires swinging together, even when all details of design and construction have received careful attention. Troubles of this

description are more likely to be met with when the spans are long and the sag in the wires necessarily large, and for this reason the spacing between wires must increase with increase of span length, irrespective of voltage considerations. Copper conductors are decidedly less likely to swing out of synchronism than aluminum conductors; not only because the latter have usually to be strung with a greater sag, but also because of the lightness of the material. Aluminum conductors of small section are easily shaken by sudden gusts of wind, and a little difference in sag will in all probability lead to non-synchronous swinging. It must not be overlooked that wires, after erection, do not always remain equally taut. This may be due to many causes, such as a slight slipping in the ties, straining of insulator pins on cross-arms, unequal ice loading, or local faults in the wires themselves. Again, it has been observed that during snow storms all the wires do not always become coated to an equal extent, and such a want of uniformity in the ice coating may well lead to wires being blown together in a strong wind.

On the high-tension transmission system of the Central Colorado Power Company, with spans averaging 730 ft., the lines cross some very exposed positions at the openings to canyons, and the excessively strong winds that occur at such points have been known to mix up the conductors. It was found necessary to dead-end the line at each tower, guy the towers, and increase the tension in the wires to a point near the elastic limit of the material, steel being used where necessary in lieu of copper.

Mr. R. Tscheutscher,[1] who has experienced no troubles through wires being blown together in high winds, cites the case of the installation over the Calumet River, of South Chicago, where the span is 400 ft., and there are six cables with 18 ft. sag, spaced about 30 in. apart, with supports on the same horizontal plane. Notwithstanding the high winds that blow over Lake Michigan, there has been no trouble through conductors being blown together.

86. Graphical Method for Determining Sags and Tensions in Overhead Conductors.—The formula (5), which gives the relation between tension, sag and weight of wire for a given span may be put in the form,

$$s=\frac{l^2W}{8P} \tag{15}$$

[1] Letter in *Electrical World*, New York, August 10, 1907.

from which the sag can readily be calculated for any assumed or known value of the tension P. The weight per foot run (W) will depend not only on the weight of the conductor itself (except under normal conditions when $W=w$; *i.e.*, the weight of the unloaded conductor), but also on the additional load due to wind pressure or ice loading, or both combined. The use of a multiplier n previously referred to, to take care of extra loads produced by abnormal weather conditions, will be found convenient.

If w = weight in pounds per foot length of the unloaded conductor, then $W = n \times w$, where n is a multiplier to take into consideration the extra load on the wire under the most severe weather conditions likely to be encountered in the district where the transmission line is located.

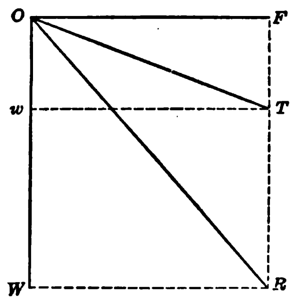

FIG. 56.—Vector diagram of forces acting on overhead wires.

It is usual to assume that the wind pressure acts in a horizontal direction and that the total load on a conductor is the resultant of two forces, one acting vertically downward due to weight of wire together with added weight of sleet or ice, if any, and one acting horizontally due to the wind pressure. These forces are indicated in Fig. 56 where OF represents the wind pressure, Ow the weight of the conductor, and wW the added weight of ice. The resultant pressure OR is equal to $\sqrt{F^2+W^2}$. If the line runs through a country where sleet does not form on the wires the maximum resultant pressure is OT instead of OR if the assumed maximum force due to wind is the same in both cases.

The diagram Fig. 57 gives values of the multiplier n (*i.e.*, the ratio $OT \div Ow$ of Fig. 56) corresponding to various wind velocities for standard sizes of copper conductors on the assumption that there can be no ice formation on the wires, while Fig. 58 gives values of n (*i.e.*, the ratio $OR \div Ow$) for similar conductors when the weight is increased by a coating of ice 0.5 in. thick with a correspondingly greater wind effect due to the increased diameter. The curves of Figs. 59 and 60 give similar relations but for conductors of aluminum instead of copper.

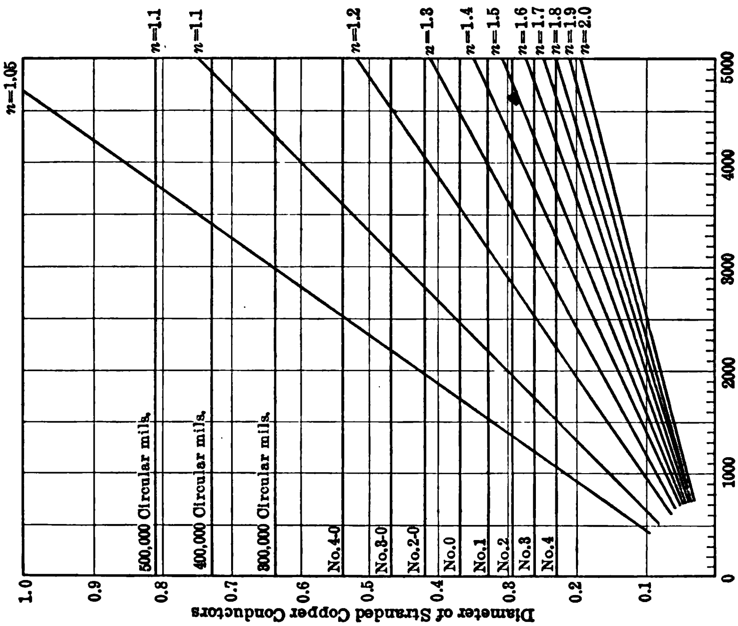

FIG. 57.—Chart giving factor n for copper conductors without ice loading.

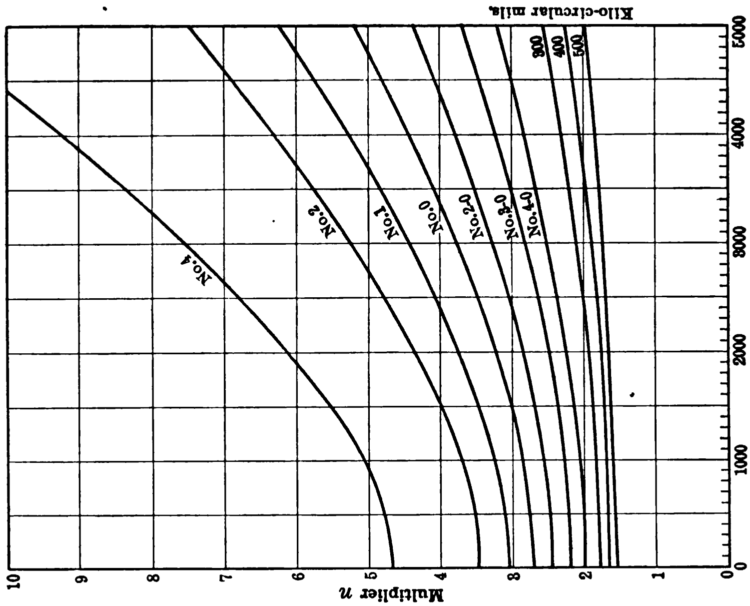

FIG. 58.—Chart giving factor n for ice-coated copper conductors.

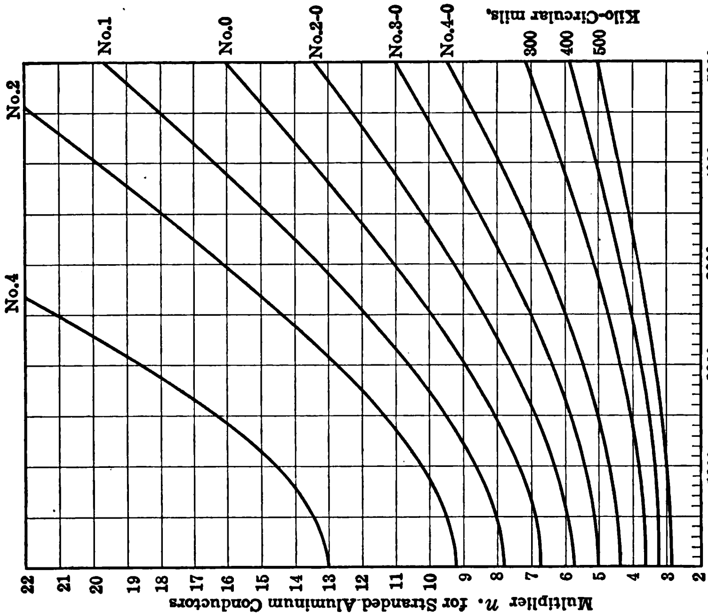

FIG. 60.—Chart giving factor n for ice-coated aluminum conductors.

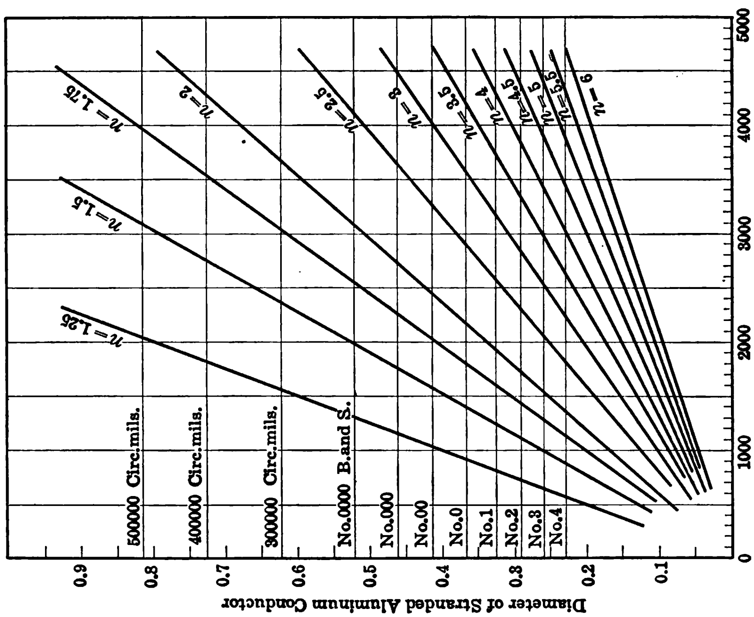

FIG. 59.—Chart giving factor n for aluminum conductors without ice loading.

The formula used for the calculation of wind pressure for use in drawing the diagrams is

$$F = dV^2 \div 4820$$

where d is the diameter in inches of the conductor or of the ice coating, as the case may be; V is the actual wind velocity in miles per hour, and F is the wind pressure in pounds per foot length of conductor.

This is the more correct form of the formula (14) already given. When using the diagrams, it should be noted that the distances plotted horizontally represent the squares of the wind velocities and the sizes of the conductors are expressed in equivalent B. & S. gage numbers or in circular mils for the larger sizes.

If the effects of wind and ice can be disregarded, the weight W in formula (15) is merely that of 1-ft. length of the conductor itself and, for a given material, it will be proportional to the area of cross-section of the wire. If the symbol T be used to denote the stress or tension in the wire *per square inch of cross-section*, the formula giving the relation between tension and sag may be written,

$$T = kn\frac{l^2}{s} \tag{16}$$

in which k is constant for a given material; it is equal to $\frac{w}{8A}$ where A is the cross-section of the conductor in square inches. The numerical value of k is therefore one-eighth of the weight in pounds of 12 cu. in. of conductor material, or 1.5 times the weight of a cubic inch. The values of k as given in Table I of Chapter IV (p. 65) are:

For copper..............................	$k = 0.485$
For aluminum..........................	$k = 0.146$
For iron.................................	$k = 0.430$

The length of wire between fixed supports of equal height bears a definite relation to the span and sag. This relation, as already given, is:–

$$L = l + \frac{8s^2}{3l} \tag{11}$$

where l is the distance between supports and s the sag at center, both expressed in feet.

The increase in length due to stress will be directly proportional to the tension per square inch (T) provided the elastic limit

of the material is not exceeded. The approximate elastic limit for conductor materials is stated in the table of Chapter IV. The formula for elastic stretching, as already given, is

$$L_e = LT \div M \tag{13}$$

where L is the length of wire when $T=0$, L_e is the elongation due to the stress T, and M is the elastic modulus.

The increase of length due to rise of temperature, on the assumption that stress remains unaltered, is:

$$L \times a \times t \tag{17}$$

where a is the temperature-elongation coefficient.

The values of M and a given below are taken from Table I of Chapter IV.

For copper.............	$M = 15{,}000{,}000$	$a = 0.0000096$
For aluminum..........	$M = 9{,}000{,}000$	$a = 0.0000128$
For steel guy wire.......	$M = 25{,}000{,}000$	$a = 0.0000065$
For copper-clad steel....	$M = 22{,}000{,}000$	$a = 0.0000067$

The purpose of the diagrams to be described hereafter is to simplify and expedite the calculation of sags and tensions required for the guidance of the men actually engaged on the erection of the line. It is a simple matter to calculate by means of formula (15) or the modified formula (16) the sag corresponding to any span (l), load (W) and tension (T), and by using either of these formulas the minimum allowable sag, for the safe limiting tension T_a when the wire is subject to the greatest expected load in the matter of ice and wind pressure, should be determined in the first instance. Having determined the amount of this sag—which should be that corresponding to the lowest expected temperature—the length of the wire in the span can be readily calculated by means of formula (11).

The calculation of the sags and corresponding tensions at other temperatures and under other conditions of loading is not by any means so simple a matter, because the alteration in the length of the conductor depends not only upon the temperature, but also on the tension. If the extra load on the conductor due to wind pressure and ice (if any) be removed, the sag will adjust itself until the formula (15) is again satisfied, the weight W being in this case that of the wire only. A further condition is that the length of wire shall be equal to the length as originally calculated for the loaded wire, less the elastic contraction due to the reduction in the tension. If the temperature be now

supposed to rise, the length of the wire will increase, but not in direct proportion to the temperature rise as indicated by formula (17) because so soon as there is any increase in length leading to an increased sag, the tension in the wire is immediately relieved, and since it is assumed that the elastic limit has not been exceeded there will be a reduction in length which could be calculated by formula (13) *if the amount by which the tension is relieved were known.*

A mathematical formula which expresses the required length or sag under normal conditions, in terms of the length corresponding to minimum temperature and heaviest loading, is very complex and difficult of solution. Mr. H. J. Glaubitz[1] has evolved an equation in which the first and third power of the unknown quantity (the deflection or sag) appear simultaneously. The solution of such an equation is tedious and is usually accomplished with the assistance of more or less scientific guesswork. The graphical method, which is probably in more general use, consists in plotting two curves, one showing the relation between sag and tension for the selected span when the wire hangs naturally under its own weight only, and another curve calculated for a definite constant temperature and giving the relation between sag and tension when a wire of definite known length under a known tension is subjected to various assumed changes in the tension. The point where the two curves cross will indicate the required conditions of sag and tension. This process is a lengthy and laborious one and has to be repeated for every assumed change of temperature.

The principles on which the writer's method is based are briefly these: Assuming the span to be known and constant, there is then a definite relation between length of wire and sag, as expressed by formula (11), and a definite relation between tension and sag when a wire of known material hangs in still air

[1] *Electrical World,* March 25, 1909, p. 731. The reader may also refer to the *Electrical World* of July 13, 1912, p. 101, where Mr. H. V. Carpenter evolves a formula containing the third and first powers of the unknown quantity, and proposes a chart to assist in arriving at the solution. An excellent article on Sags and Tensions in wire spans from the pen of Dr. Harold Pender appeared on page 604 of the *Electrical World* of Sept. 28, 1907. Whatever Dr. Pender may write on transmission-line problems, whether from the electrical or mechanical standpoint, is certain to be of interest to the engineer engaged on the design of overhead transmissions for dealing with large amounts of energy.

under its own weight only. This last relation is given by formula (16) when the multiplier n is unity. In accordance with the above, if there be drawn correctly to scale two curves, the one (A) (on transparent paper) giving the relation between length of wire and sag, and the other (B) giving the relation between tension and sag, and if the first be placed over the second so that the division corresponding to the known tension shall coincide with the division corresponding to the length of wire as calculated for that particular tension, then the length corresponding to any other value of the tension, if the temperature remains unaltered, can be read at a glance, and the particular length (and corresponding tension), together with the corresponding sag when the wire hangs naturally in still air, is directly indicated by the position of the point where the curve A crosses the curve B. In order to obtain this information for any temperature higher than the assumed minimum, a scale of temperature can be drawn on diagram B, the divisions being so spaced as to agree with the correct increase of length due to temperature rise when this scale is placed under the scale for length of wire of diagram A. This will enable the position of diagram A to be shifted so that the length corresponding to zero value of the tension, instead of being the length at minimum temperature, will be the length which the wire would have at the higher temperature if all stress were removed, and the point where the two curves cross will, therefore, indicate the required particulars in regard to sag and tension at the increased temperature.

If curves were plotted from formula (15) giving the relation between tension and sag, each curve would be a rectangular hyperbola, and, for different spans, P would be proportional to l^2. This would make the drawing of a number of curves very tedious; but by plotting the tension P and the value $l \div s$ a "straight-line" relation between the two quantities is obtained, and since, for different values of the span l, the tension P for a given value of $l \div s$ will be directly proportional to l, the drawing of a number of radiating straight lines giving the relation between P and $l \div s$ for various values of l becomes a very simple matter.

Method of Drawing the Diagrams

The diagram A, Fig. 61, gives the relation between the length of wire, expressed as a percentage of the span l, and the ratio $m = l \div s$. This relation is independent of the material of the wire

and of the load, provided the load is uniformly distributed. It is derived from the formula (11).

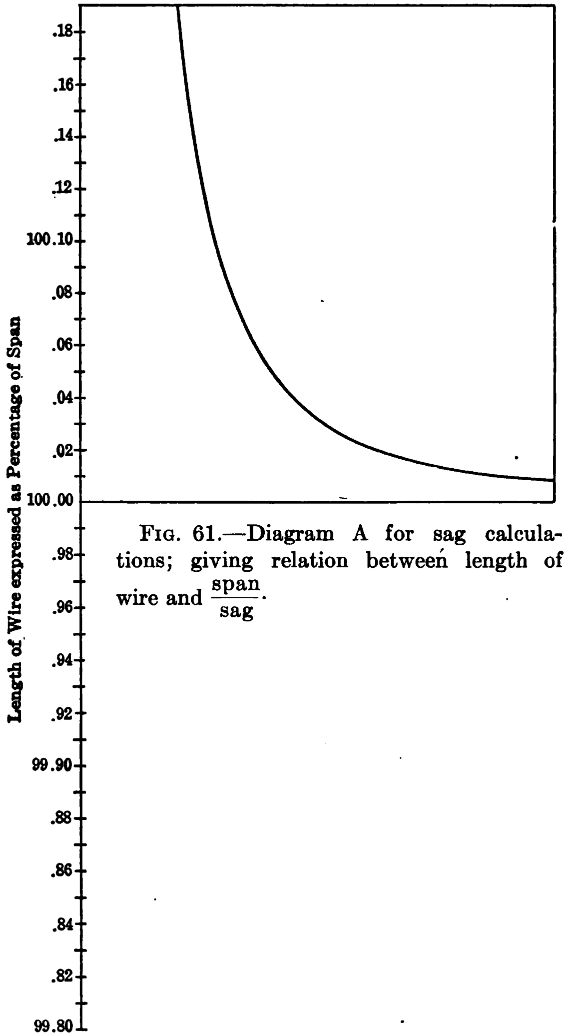

FIG. 61.—Diagram A for sag calculations; giving relation between length of wire and $\frac{\text{span}}{\text{sag}}$.

Let L_0 = percentage amount by which length of conductor differs from length of span, then

$$L_o = \frac{8s^2}{3l} \times \frac{100}{l}$$

and putting m for the value $l \div s$ the equation becomes

$$L_0 = 800 \div 3\, m^2$$

by means of which the curve A can readily be plotted with m measured horizontally and L_0 or $100+L_0$ vertically. The vertical scale indicating the length should include values of L less than

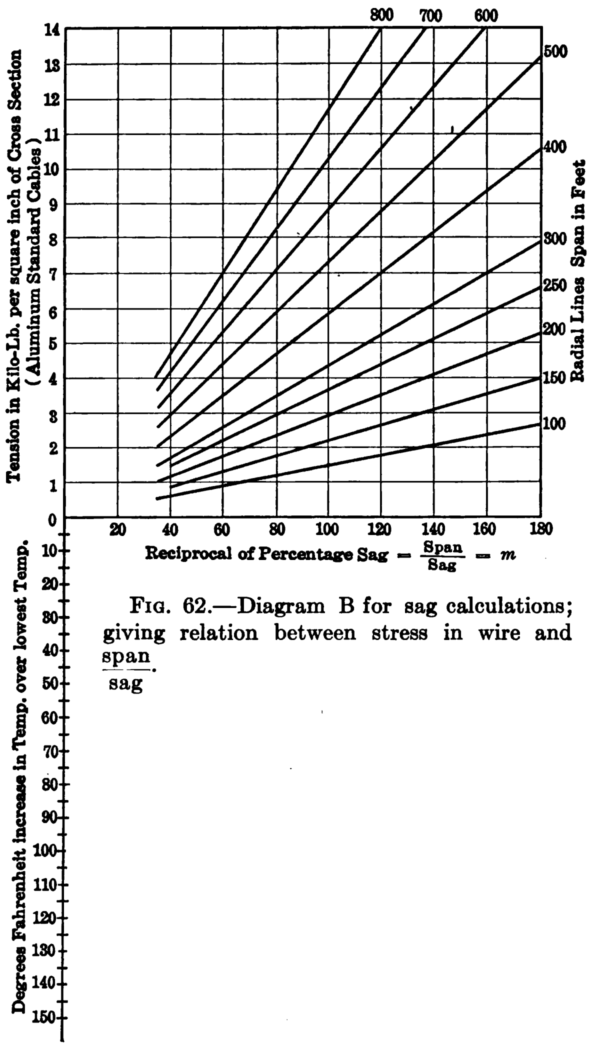

FIG. 62.—Diagram B for sag calculations; giving relation between stress in wire and $\frac{\text{span}}{\text{sag}}$.

the span l because the length of wire if all tension is supposed to be removed will frequently be less than the length of span. This curve (A) should actually be drawn on transparent paper so that it can be placed over the second diagram.

In plotting diagram B, Fig. 62, the horizontal divisions representing the ratio m must obviously correspond exactly with the scale adopted for diagram A.

To divide correctly the vertical scale representing tension in pounds per square inch of cross-section, use is made of formula (13) for determining the increase in length due to elastic stretching; but the percentage increase is $100\ T \div M$, and the scale of the stress T will depend upon the elastic modulus M, and, therefore, on the material.[1]

In order to plot the curves for various spans (all of which are straight lines radiating from a common center) use is made of the relation $T = kl^2 \div s$, which is the original formula (16), where $n = 1$, and if m be substituted for $l \div s$ the equation becomes,

$$T = m \times k \times l$$

In regard to the scale for temperature rise, which may conveniently form a continuation of the tension scale below the datum line, use can be made of formula (17); but the percentage increase is $100 \times a \times t$, and the scale for temperature rise will depend upon the coefficient of expansion a and, therefore, on the material. For aluminum, a rise of temperature of 55° F. will produce an increase in length of approximately 0.07 per cent.

Method of Using the Diagrams

The use of the diagrams is best illustrated by an example. Let particulars of line be as follows:

Span = 200 ft.

Conductors, No. 3-0 B. & S. gage stranded aluminum.

Lowest expected winter temperature = −20° F.

Ice coating, 0.5 in. thick.

Limiting safe stress, $T = 13{,}000$.

Maximum wind velocity = 60 miles per hour (which may occur simultaneously with the maximum ice load).

[1] In the diagrams Figs. 61 and 62, the scale of tensions of Fig. 62 relatively to the scale of lengths of Fig. 61 is based on a lower value of M for stranded aluminum cables than the average value given in Table I of Chapter IV. The value used for drawing the curves is $M = 7{,}500{,}000$. It is, however, suggested that the diagrams of Figs. 61 and 62 are too small for practical use: they are intended to illustrate the principle only of these graphic calculations, the supposition being that the reader who cares to adopt this method for practical calculations would construct his own diagrams to a larger scale.

It is required to determine the sag and tension at a temperature of 60° F. when no wind is blowing and the conductor is subject to its own weight only.

From the diagram, Fig. 60, it is found that the multiplier n for $V^2=3600$ is 8.7, and by transposing formula (16), the value of m is seen to be

$$m=l\div s=T\div(k\times n\times l)$$
$$=\frac{13{,}000}{0.147\times 8.7\times 200}=50.8$$

which corresponds to a length of 100.104 per cent. as read off curve (A) or calculated by fcrmula (11). Place 100.104 of the (transparent) diagram A over the mark corresponding to a tension of 13,000 lb. on diagram B and under the zero point of the tension scale will be found the length 99.93, which is the length (expressed as percentage of span) which the conductor would have at the lowest temperature if all tension were removed. At a temperature of 60° F. the rise of temperature over lowest winter temperature is $60+20=80°$. Place the division for length 99.93 over the mark for temperature 80, and the point on the length scale of diagram A which lies on the datum line of diagram B indicates the length which the wire would have at 80° F. if there were no mechanical stress. (In this example the length would be 100.032.) Observe now where the curve on diagram A crosses the span line marked 200 on diagram B and read off the value $m=68$ (nearly), giving sag $=200\div 68=2.94$ ft. and $T=2000$, giving $P=2000\times 0.1318=263.6$ lb., where 0.1318 is the cross-sectional area of a No. 3-0 conductor. This tension of 263.6 lb. is approximately the pull which should be indicated on a dynamometer if the conductors are being strung at a temperature of 60° F. and there is no wind blowing. If there is only a slight wind blowing at the time of erection the increased tension required is negligible; if the wind amounts to what in nautical terms would be described as a "strong breeze" (34 miles per hour), the multiplier n as read off diagram Fig. 59 is only 1.25 for a No. 3-0 aluminum cable. If, however, the stringing of the conductors were to be carried on in so strong a wind blowing across the length of the line, the required tension and deflection could easily be read off the diagrams without moving their relative positions. It is merely necessary to read the tension and sag values corresponding to an assumed span of $200\times 1.25=$

250 ft. instead of the actual span of 200 ft. The results for the same temperature rise (80°) with an estimated wind velocity of 34 miles per hour would be $m=65$, giving a deflection $=200 \div 65=3.08$ ft., and $T=2400$, making the required pull on a dynamometer $P=2400 \times 0.1318=316$ lb.

The reason for this method of correcting for wind pressure on wires will be clear when it is recognized that the law connecting tension T and ratio m is no longer

$$T=m \times k \times l$$

but

$$T=m \times k \times l \times n$$

and the effect of multiplying T by n is obtained by assuming a value for l which is n times as large as the actual span.

The deflection from the horizontal line at the center of the span, as calculated with the aid of the charts when the multiplier n is greater than unity, is no longer equivalent to the vertical sag. It is sometimes useful to know the sag or vertical component of the total deflection at center of span when the horizontal component of the total load (*i.e.*, the wind pressure) is considerable. If F is the horizontal component of the total load on the wire, and W_v the vertical component—which must include ice loading if any—then

$$\text{vertical sag} = \text{deflection} \times \frac{W_v}{\sqrt{W_v^2+F^2}} \tag{18}$$

87. Analytical Method for Determining Sags and Tensions in Overhead Conductors.—The method of calculation about to be described requires the knowledge of the sag and tension corresponding to one particular temperature. A different sag is then assumed, and the temperature at which this sag will occur is calculated by means of a simple formula. The manner of obtaining the preliminary data will be explained later.

It is assumed that the conductor is strung between two fixed supports on the same level, and that the material of the conductor is not strained beyond the elastic limit.

The meaning of the symbols used is as follows:

l = the length of span, or horizontal distance between points of support, in feet.

S = the vertical sag at center of span, in feet, when wire hangs in still air under the influence of its own weight only.

P = the tension in the conductor at the lowest point of span, in pounds.

T = the stress in the conductor at the lowest point of span, in pounds per square inch of cross-section.

L = the length of conductor measured between the two points of support.

t = temperature, in degrees Fahrenheit.

S_c, T_c, L_c = values of sag, stress and length corresponding to a definite temperature t_c.

a = the coefficient of linear expansion of the conductor per degree Fahrenheit.

M = the modulus, or coefficient, of elasticity of the conductor, being the ratio of stress in pounds per square inch to extension per unit length.

w = the weight of conductor in pounds per foot of length.

W = the resultant or total load in pounds, per foot, including wind pressure and ice (if any).

n = a multiplier depending on the material of the conductor and weather conditions, being the ratio W/w.

k = a constant depending upon the material of the conductor, being 1.5 times the weight in pounds of a cubic inch of the conductor material.

The well-known formula giving the relation between sag, length of span, horizontal load, and tension is:

$$S = \frac{l^2 w}{8P} \tag{15}$$

The approximate formula for the length of a parabolic curve (which is quite sufficiently accurate for all practical purposes) is:

$$L = l + \frac{8S^2}{3l} \tag{11}$$

It is assumed that the sag S_c, and therefore the corresponding stress T_c and length L_c are known for the particular temperature t_c, which may be fairly high so that another value, S, of the sag, arbitrarily chosen, shall be smaller than S_c; and it is proposed to calculate the temperature t which will correspond to this assumed sag, S.

With the reduction in the amount of sag, there must of necessity be a reduction in the length L_c of the conductor and an increase in the tension P_c or stress T_c. The amount by which the

length has decreased is not directly proportional to the reduction in temperature, because the increase in tension causes an elastic elongation of the conductor, and the reduction in length is actually the difference between the amount by which the wire would contract with the lowering of the temperature if the tension were to remain constant, and the amount by which the wire would be extended, due to the increased tension, if the temperature were to remain constant; although, from a strictly scientific point of view, the argument may be inaccurate. The decrease in length due to temperature reduction is:

$$L_c \times a \times (t_c - t) \tag{19}$$

and the increase in length due to additional tension is:

$$\frac{L_c}{M}\left(\frac{S_c}{S}T_c - T_c\right),$$

or

$$\frac{L_c}{M}T_c\left(\frac{S_c}{S} - 1\right) \tag{20}$$

Therefore,

$$L_c - L = L_c \times a \times (t_c - t) - \frac{L_c}{M}T_c\left(\frac{S_c}{S} - 1\right)$$

The lengths L and L_c can be eliminated by substituting their values in terms of sag and length of span, as given by formula (11). This leads to the equation:

$$t_c - t = \frac{8S_c^2 - 8S^2}{(3l^2 + 8S_c^2) \times a} + \frac{T_c}{M \times a}\left(\frac{S_c}{S} - 1\right) \tag{21}$$

This formula is very simple to use, because, for any particular material and size of wire, it can be written:

$$t_c - t = \frac{C_1 - 8S^2}{C_2} + C_3\left(\frac{S_c}{S} - 1\right) \tag{22}$$

where C_1, C_2 and C_3 are constants, the values of which are:

$$C_1 = 8S_c^2 \tag{23}$$

$$C_2 = (3l^2 + 8S_c^2)a \tag{24}$$

$$C_3 = \frac{T_c}{M \times a} \tag{25}$$

Moreover, since $8S_c^2$ is always very small in comparison with $3\ l^2$, the constant C_2 may, for nearly all practical purposes, be written:

$$C_2 = 3l^2 \times a \tag{26}$$

This value of C_2 may be used for spans up to 500 ft. if the multiplier n does not exceed 12, and for spans up to 1000 ft. if n does not exceed 6. In the case of larger spans, in which the sag is relatively large, or if a closer approximation is required, the more exact expression (24) should be used. The only unknown quantities in equations (21) or (22) being the sag S and temperature t, it follows that, by inserting any numerical value for S in the equation, the change of temperature, and therefore the actual temperature t corresponding to the assumed value of the sag, can be readily calculated. In order that this method of calculation may be of practical utility, it is necessary that the sag S_c and the stress T_c at the particular temperature t_c shall be known. The fundamental data on which all line calculations are based must include the limiting or maximum allowable value of the stress and the conditions of maximum loading under the most severe weather conditions. The maximum load per foot being W and the weight of the unloaded wire being w, it follows that the ratio $\frac{W}{w} = n$ will be greater as the wind conditions, either without ice or combined with a coating of ice on the wires, are the more severe. The wires will generally be subject to the greatest stress at times when strong winds, with or without a coating of ice or sleet, occur at a low temperature, because the lowness of the temperature alone will account for a considerable increase in the tension.

If the extra load on the wire due to wind and ice combined is great in proportion to the weight of the wire, the maximum deflection will usually occur under winter conditions; but there will be a higher temperature at which the sag of the unloaded wire hanging in still air, subject to only its own weight, will be exactly the same as the deflection under winter conditions when subject to wind pressure and extra load of ice (if the line runs through a district where sleet and ice formation is possible). This temperature, which may be called the critical temperature for the material of the conductor when the maximum winter loading has been determined, is easily calculated; and its numerical value, together with the known value of the sag under conditions of maximum load, and of the tension corresponding to this sag, may be used in equation (21) or (22) for the known quantities t_c, S_c and T_c.

Calculation of Critical Temperature t_c

Let T_m be the stress in the wire under the most severe conditions of load, and t_o the temperature at which this stress occurs. The tension T_c will be equal to T_m divided by n, because, at the critical temperature t_c, the sag is the same as the maximum deflection of the loaded conductor, but the weight per foot of length is in the ratio $\frac{w}{W}$ or $\frac{1}{n}$. (Refer to formula (15), bearing in mind that, for any given size of conductor, the stress T is proportional to the tension P). With an increase of temperature from t_o to t_c the reduction in stress is

$$T_m - T_c = T_m - \frac{T_m}{n} = T_m\left(1 - \frac{1}{n}\right)$$

and the reduction in length of the wire due to this difference of tension is

$$\frac{L_c}{M}T_m\left(1 - \frac{1}{n}\right)$$

It is required to calculate the temperature rise $(t_c - t_o)$ which will produce an *extension* exactly equal to this elastic contraction, in order that the length L_c of the wire, and consequently the sag S_c, shall remain as before.

The extension due to temperature rise is

$$L_c \times a \times (t_c - t_o)$$

and the required equation is:

$$L_c \times a \times (t_c - t_o) = \frac{L_c}{M}T_m\left(1 - \frac{1}{n}\right)$$

or

$$(t_c - t_o) = \frac{T_m}{M \times a}\left(1 - \frac{1}{n}\right) \qquad (27)$$

The curves in Fig. 63 give the relation between the ratio $\frac{w}{W}$ (being the reciprocal of n) and the temperature rise $(t_c - t_o)$ for stranded cables of different materials. The values of M and a, which have been adopted for the purpose of drawing the diagram, are:

For hard-drawn copper,	$M = 15 \times 10^6$, $a = 9.6 \times 10^{-6}$
For hard-drawn aluminum,	$M = 9 \times 10^6$, $a = 1.28 \times 10^{-5}$
For galvanized steel,	$M = 25 \times 10^6$, $a = 6.5 \times 10^{-6}$

Having determined the critical temperature t_c at which—it is interesting to note—the tension in the wires, if correctly strung, will be the same whatever the length of span, the sag S_c can be

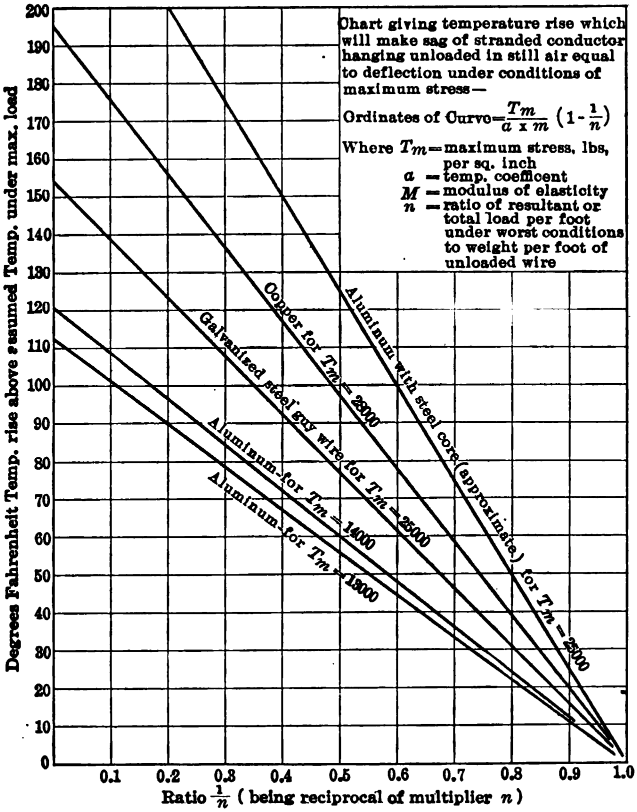

FIG. 63.—Chart for determining "critical temperature."

calculated by the formula (15), or by the more convenient formula:

$$S_c = \frac{kl^2}{T_c} \qquad (28)$$

or

$$S_c = \frac{knl^2}{T_m} \qquad (29)$$

which can readily be deduced therefrom. In this manner the numerical values of the quantities t_c, S_c, and T_c for use in formula (21) or (22), are obtained.

Example.—Consider a span 485 ft. long, with rigid supports. Conductors of aluminum, size No. 2-0 B. & S. Maximum loading, 0.5-in. coating of ice, combined with a wind velocity of 47 miles per hour at a temperature $t_o = -20°$ F.

$$n = 8 \quad \frac{1}{n} = 0.125$$

$$T_m = 14{,}000 \quad k = 0.146$$

$$t_c - t_o = 106° \text{ (read off Fig. 63).}$$

Hence

$$t_c = 86°.$$

$$Tc = \frac{14{,}000}{8} = 1750, \text{ and, by formula (28),}$$

$$S_c = \frac{0.146 \times 485 \times 485}{1750} = 19.6 \text{ ft.}$$

By formula (23), $C_1 = 8 \times 19.6 \times 19.6 = 3070$

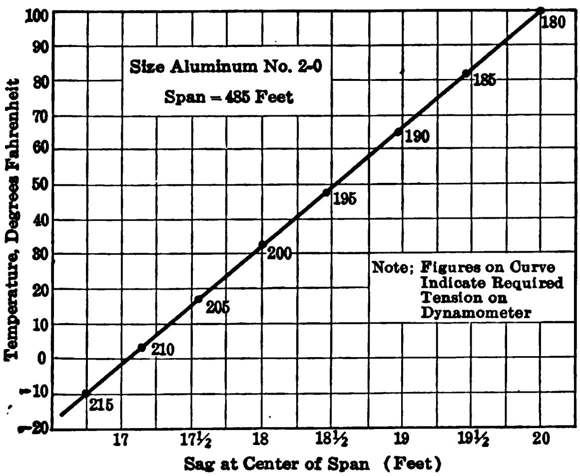

FIG. 64.—Sag-temperature curve for use when stringing wires.

By formula (26), $C_2 = 3 \times 485 \times 485 \times 1.28 \times 10^{-5} = 9$

By formula (25), $C_3 = \dfrac{1750 \times 10^5}{9 \times 10^6 \times 1.28} = 15.2$

By formula (22), $(86 - t) = \dfrac{3070 - 8S^2}{9} + 15.2\left(\dfrac{19.6}{S} - 1\right)$

By choosing values for S not very different from S_c the corresponding temperatures are easily calculated. Thus

when $S = 16\ 1/2$ ft., $t = -15.8°$ F.
when $S = 17\ 1/2$ ft., $t = +15.4°$ F.
when $S = 18\ 1/2$ ft., $t = +48.4°$ F.
when $S = 20\ 1/2$ ft., $t = +120°$ F.

With the aid of these figures a curve such as Fig. 64 is readily plotted. This curve gives the men in the field all necessary information for the correct stringing of the conductors, whatever may be the temperature when the work is carried out.

88. Tensions in Conductors when Spans are of Different Lengths.—It is well to keep the consecutive spans in a transmission line as nearly as possible of the same length, because, although it is possible to string the wires so that the tension shall be the same in all spans at the time of stringing or under specified conditions of load and temperature, there will be an unbalancing of the tensions in adjoining spans with every change of temperature. If properly strung, the wires in long and short spans should be subjected to the same maximum tension under the severest conditions of loading, and the condition of equal tensions will repeat itself at the higher temperature—previously referred to as the critical temperature—when the sag of the unloaded conductor is the same as the deflection of the loaded conductor at the lower temperature; but, except when the deflection at center of span remains unaltered,[1] the pull on each side of a supporting insulator will be unbalanced. It is partly for this reason that extra long spans are usually "dead-ended" on guyed poles or strain towers. When calculating sags in spans of different lengths, it is therefore not correct to assume that the sag is always directly proportional to the square of the length of span; because when the wire hangs in still air subject to its own weight only, this proportionality exists for no other temperature but the critical temperature as determined for use in the sag-temperature calculations.

[1] The condition is that the quantity $\frac{n(t_1 - t_o)}{n - n_1}$ shall remain constant. This expression is derived in a similar manner to the formula (27): there must obviously be some particular value of the wind or ice loading (corresponding to the factor n_1) which, in conjunction with a rise of temperature $(t_1 - t_o)$ will cause the deflection to be the same as when the temperature is t_o and the (maximum) loading is n times that due to the weight of the wire acting alone. The necessary relation between t_1 and n_1 is given by the above formula.

89. Tension in Different Sized Wires on the Same Span.—It may be questioned whether, having calculated the sag-temperature conditions for a conductor of diameter d_1, there is not a short cut by which similar relations can be arrived at for a wire of diameter d_2 when the length of the span, l, remains unaltered. There does not appear to be a quick way of obtaining the required results; but there is one condition that holds:

Let n be the load coefficient $\left(\frac{W}{w}\right)$ for conductor d_1, and n_2 the load coefficient for conductor d_2 then, since l^2 is assumed constant, it follows from the formula (16) that

$$S \propto \frac{n}{T}$$

and

$$S_2 = S_1 \times \frac{n_2}{n_1}$$

which is true only when the stress (T) per square inch of cross-section is the same for both sizes of conductor (the material of the conductors being assumed the same).

90. Supports at Different Levels.—For any given horizontal tension in a suspended wire of a particular material, hanging

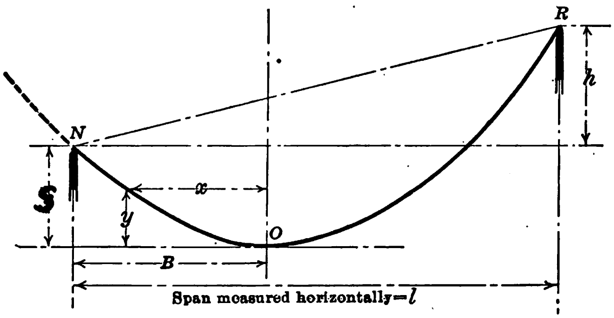

FIG. 65.—Wire hung between supports at different elevations.

between supports under the influence of its own weight only, there is a definite curve such as the parabola drawn in Fig. 65, which may be considered to extend indefinitely in both directions The suspended wire may be secured to rigid supports at any two points, such as N and R, lying on this curve, without altering the tension in the wire. The law of this parabola is:

$$x^2 = Cy \qquad (30)$$

and in the case of a suspended wire the multiplier C will be directly proportional to the tension P, and inversely proportional to the density of the conductor material. The value of C in terms of the tension and weight of conductor is readily obtained by inserting for x and y the corresponding span and sag values as given by formulas (15) and (28). Thus

$$C=\frac{2P}{w} \tag{31}$$

or

$$C=\frac{T}{4k} \tag{32}$$

If S = sag below level of lower support,

B = horizontal distance of lowest point of wire from lower support,

h = difference in level of the two supports,

l = length of span measured horizontally; all as indicated on Fig. 65, then, by inserting the required values in equation (30), the following equations are derived therefrom:

$$B^2=C\times S,$$

$$(l-B)^2=C(S+h),$$

or

$$l^2+B^2-2lB=CS+Ch,$$

from which B² on the one side and its equivalent CS on the other side cancel out, leaving

$$l^2-2lB=Ch.$$

Therefore

$$B=\frac{l^2-Ch}{2l} \tag{33}$$

and

$$S=\frac{B^2}{C} \tag{34}$$

From an inspection of formula (33) it is seen that if $Ch=l^2$, the lowest point of the wire coincides with the lower support N, while if Ch is greater than l^2 the distance B is negative, and there may be a resultant upward pull on the lower insulator N, a point to bear in mind when considering an abrupt change in the grade of a transmission line.

The length of wire between the two supports N and R may be considered as the sum of two distinct portions of the parabolic curve NOR; the one, NO, the length of which, according to formula (11), is

$$B+\frac{4S^2}{3\times 2B}$$

and the other, *OR*, of length

$$(l-B)+\frac{4(S+h)^2}{3\times 2(l-B)}$$

The sum of these two quantities is

$$L=l+\frac{2}{3}\left[\frac{S^2}{B}+\frac{(S+h)^2}{(l-B)}\right] \tag{35}$$

For the calculation of the sags and tensions at different temperatures; the formula (22) is no longer applicable; but the calculation can be proceeded with on the same general lines. An example will best illustrate the writer's method.

Example.—Calculation of sags and tensions at different temperatures with supports not on the same level. All data and constants are assumed to be the same as in previous example, the horizontal length of span being 485 ft., as before, but there is a difference of levels between supports of $h=40$ ft. Four numerical values of the stress T, other than the known value ($T_c=1750$) at the critical temperature, have been arbitrarily chosen, and the corresponding temperature t has been determined in the manner indicated in the accompanying table.[1]

NUMERICAL EXAMPLE—SUPPORTS AT DIFFERENT LEVELS

Stress T, in pounds per square inch	1700	1750	1800	1900	2000
$C=T\div 4k$	2910	2990	3080	3250	3420
$B=\frac{l^2-Ch}{2l}$	122.4	119.1	115.2	108.3	101.2
$S=B^2\div C$	5.15	4.75	4.32	3.61	2.99
$L=l+\frac{2}{3}\left[\frac{S^2}{B}+\frac{(S+h)^2}{l-B}\right]$					
$=485+$	3.89	3.77	3.66	3.45	3.27
$L-L_c$	+0.12	0	−0.11	−0.32	−0.50
Difference in length due to stress variation $=(T-T_c)L\div M$	−0.00272	0	+0.00272	+0.00816	+0.0136
Elongation due to change of temperature, being difference between total elongation and elongation due to stress variation	+0.1227	0	−0.1127	−0.3332	−0.5136
Difference of temperature necessary to cause the above elongation (degs. Fahr.) $t_c-t=\frac{\text{temperature elongation}}{a\times L}$	−19.7	0	+18.1	+53.5	+82
Actual temperature, t, being the "critical" temperature less the above differences	105.7	86	67.9	32.5	4

[1] The numerical values as given by the various formulas in this and the previous example have been worked out on the slide rule and are approximate only.

The tension in the conductor is ascertained by multiplying the stress T by the cross-sectional area (in this example 0.1048 sq. in.), and the curve in Fig. 66 shows the relation between tension and temperature as calculated in the manner indicated by the above table of results. If the points on the curve of Fig. 64—which indicate tensions in a similar span, but with supports on the same level—are transferred to Fig. 65, they will be found to agree very closely with the plotted curve. This

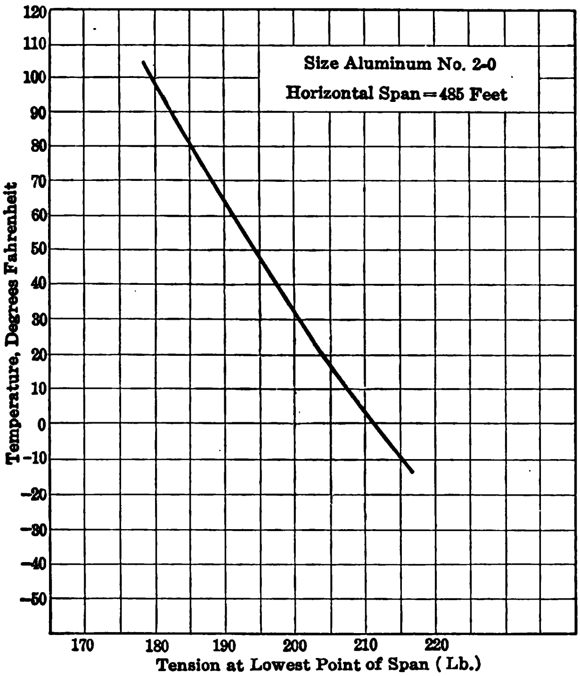

FIG. 66.—Tension-temperature curve.

may be explained by the fact that the actual length of wire in the span differs very little in the two cases considered.

It is possible, and sometimes convenient to express the formulas (33), (34), and (35) in terms of the equivalent sag, (S_e), of the same wire, subjected to the same tension when the horizontal span (l) is unaltered, but the supports are on the same level.

It is merely necessary to substitute for C in formulas (33) and

(34) its equivalent value $C=\frac{l^2}{4\,S_e}$. This leads to the following set of formulas, in which B, S, l, and h are all as indicated in Fig. 65.

$$B=\frac{l}{2}\left(1-\frac{h}{4S_e}\right) \tag{36}$$

$$\text{or } l-B=\frac{l}{2}\left(1+\frac{h}{4S_e}\right) \tag{37}$$

$$S=S_e\left(1-\frac{h}{4S_e}\right)^2 \tag{38}$$

$$\text{or } S+h=S_e\left(1+\frac{h}{4S_e}\right)^2 \tag{39}$$

$$L=l+\frac{4S_e^2}{3l}\left[\left(1-\frac{h}{4S_e}\right)^3+\left(1+\frac{h}{4S_e}\right)^3\right] \tag{40}$$

91. Further Examples of Temperature-sag Calculations.—Although the two following problems may not be of great practical utility, they will serve as a further illustration of the methods of calculation explained in articles (86) and (87).

(a) *Determine the temperature rise which will produce a sag in the unloaded wire equal to the deflection of the same wire at a lower temperature when subject to extra load of wind and ice.*

This condition requires the *length* of the wire to be the same in both cases.

The difference in the stress under loaded and unloaded conditions is obtained from formula (16) which can be written,

$$\frac{T}{n}=k\frac{l^2}{S} \tag{16a}$$

Thus, if T is the maximum stress in the wire under loaded conditions, the stress when $n=1$, that is to say when the wire is hanging in still air subject to its own weight only, will be $\frac{T}{n}$ because the quantities in the right-hand member of the equation (16a) are all constant under the assumed conditions.

The reduction in length due to the lower value of the stress, if temperature remained constant, would be,

$$\left(T-\frac{T}{n}\right)\frac{L}{M}$$

The elongation due to increase of temperature, if stress remained constant, would be,

$$L \times a \times t$$

The equation of these two quantities, when simplified, gives the condition,

$$t = \frac{T}{a \times M}\left(1 - \frac{1}{n}\right)$$

from which the required temperature rise can be calculated.

(b) *Determine the reduction of temperature which will make the tension in the unloaded conductor the same as in the loaded conductor when the resultant load is* n *times the weight of the wire.*

In this problem the *stress* is the same in both cases, and the alteration in length of the conductor corresponding to the (necessarily) smaller sag is caused by reduction of temperature and not by elastic contraction.

If s = sag under loaded condition, and

s_1 = sag of the unloaded wire when the temperature has fallen t degrees F. and the stress T is again equal to what it was under the loaded condition, then,

$$s_1 = \frac{s}{n}$$
$$= \frac{k l^2}{nT}$$

The reduction in length is,

$$L_1 \times a \times t = L - L_1$$
$$= l + \frac{8S^2}{3l} - l - \frac{8S_1^2}{3l}$$

which resolves itself into

$$L_1 \times t = \frac{8S^2(n^2-1)}{3n^2 \times a \times l}$$

No appreciable error will be introduced, when dealing with spans of moderate length, if the length of wire L_1 is replaced by the length of span l. Hence,

$$t = \frac{8S^2(n^2-1)}{3n^2 \times a \times l^2} \qquad (41)$$

which, if it is desired to eliminate s, can be written

$$t = \frac{8(n^2-1)k^2l^2}{3 \times a \times T} \qquad (42)$$

It may be observed that, for a given material and limiting tension, the required reduction in temperature is proportional to $(n^2-1)\times l^2$, or $t=\frac{(n^2-1)l^2}{K}$

By using the data for materials previously given and assuming maximum allowable tensions corresponding to $T=28{,}000$ for copper; 13,000 for aluminum; and 25,000 for steel guy wire, the calculated value of K is,

For copper	$K=12{,}000$
For aluminum	$K=38{,}000$
For steel	$K=\ 8{,}250$

92. Length of Spans. Copper and Aluminum.—When the wires of a transmission line are supported on single wood poles, the span is about 150 ft. It may be anything from 120 to 250 or even 300 ft.; but extra long poles, specially selected to withstand the greater stresses, are required when the span is appreciably in excess of 170 ft.

On steel tower lines, very much greater spans can be used. The length of span is determined not only by strength considerations, but also by considerations of economy (see articles (14) and (15) in Chapter III). The requirements in the matter of supporting poles or structures (which depend largely on length of span) will be referred to in the following chapter.

Whether copper or aluminum should be used on a given transmission line cannot be determined on general principles. Conductor materials were discussed in Chapter IV, and, apart from the physical properties of these materials, the relative cost, which is a variable quantity, must be taken account of when deciding upon the material best suited for the work.

Generally speaking, the deflections or sags of aluminum conductors on spans of moderate length will be about 30 per cent. or 35 per cent. greater than with copper conductors. The difference will be more marked with the smaller sizes of wires and the shorter spans. With extra long spans in the neighborhood of 1000 ft., it will be found that the maximum sag of aluminum and copper conductors will be about the same, *if storm and abnormal winter conditions are neglected;* that is to say when the factor n is unity. Under this condition it will even be found that copper has a *greater* sag than aluminum on the very long spans. The reason of this is that the greater temperature elongation of

aluminum is inappreciable in the case of long spans with necessarily large sags, while it is an important factor in the comparison of the two metals when the spans are short. The above statement is made rather as being of scientific interest than of practical utility, because, under storm conditions (when n has a large value), aluminum will be found to be an unsatisfactory material to use on long spans.

Although the tension in a conductor can always be kept reasonably small by allowing sufficient sag, it is obvious that a large sag involves higher and more costly supporting structures, if the clearance between ground and lowest point of the conductor is to remain the same. When crossing open country, the clearance above ground need not be very great (it is usually greater than would, in the writer's opinion, appear to be necessary); but in crossing roads, a clearance of 21 feet should be allowed, and over foot paths the clearance should never be less than 15 ft. For mechanical reasons, it is a general rule that no conductor for the transmission of electric energy shall have a lower ultimate strength than a No. 6 B. & S. gage hard-drawn copper wire.

Although it is convenient in sag calculations to assume rigid supports at each end of the span, the deflection, under heavy load, of pole or steel mast as used on the shorter spans, may be regarded to some extent as a factor of safety. Refinements of calculation are out of place when figuring on short span lines: the tendency is to string wires of short-span transmission lines too slack rather than too tight. On spans not exceeding 150 ft., if the wires are strung at comparatively low temperatures, it is almost impossible to draw them up too tight.

93. Factors of Safety.—In America, the factor of safety for conductors is about 2: this means that, under the worst assumed conditions of loading, the material of the conductor may be stressed to very nearly its elastic limit. It is usual to assume a wind velocity of 70 miles per hour combined with a sleet deposit 1/2 in. thick at a temperature of 0° F. For guy wires, a factor of safety of 3 to 3 1/2 is generally allowed.

In Great Britain, where higher factors of safety are used, the Board of Trade calls for a maximum tension not exceeding one-fifth of the breaking load, on the assumption of a temperature of 22° F. and a wind velocity corresponding to a pressure of 18 lb. per square foot of the projected surface of the wire. Possible accumulations of snow and ice are ignored.

On the Continent of Europe, the net factor of safety for wires under the worst conditions of loading is about 2 1/2.

When the "flexible" type of tower construction is used, it is customary to allow a somewhat higher factor of safety for the conductors than when the towers are of the so-called rigid type.

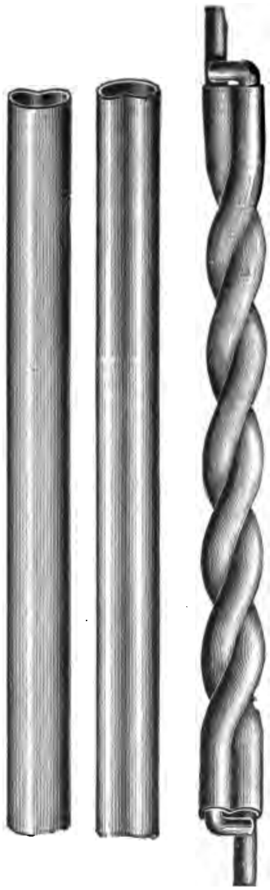

Fig. 67.—Type of joint for overhead conductors.

Joints in wires can and should be made of the same strength as the wire itself. A discussion of the various types of joints as used in practice would be out of place in these pages. A very common type is the McIntyre joint, illustrated in Fig. 67. It is said that when the sleeve is long enough to allow of three complete turns, the strength of this joint is equal to that of the cable itself.

94. Ties.—The tie wire secures the conductor to the insulator, usually in such a way as to prevent as far as possible the creeping of the conductor from one span to another. In some cases, however, it is advisable to allow the conductor to slip, or the tie wire to break, before the tension in the cable is great enough to damage insulator or cross-arm. Soft or semi-hard wire is generally used for ties, and the size should not be less than No. 4 B. & S. copper, or No. 2 B. & S. aluminum. The tie wire should be of the same material as the conductor. The accompanying illustrations of ties on pin type insulators are from blocks kindly lent by the Aluminum Company of America.

A top tie on a 15,000-volt insulator is shown in Fig. 68. The middle of the tie wire is looped round the front side of the neck of the insulator and the ends are given one complete turn around the cable close to and at each end of the insulator. They then cross in the neck of the insulator at the back and are ended in three close wraps around the cable at both ends.

The top tie shown in Fig. 69 is made by laying the middle of the tie wire diagonally across the top of the cable, passing the ends under the cable and in opposite directions around the neck of the insulator, and serving them out on the cable. The tie

FIG. 68.—Example of tie.

FIG. 69.—Example of tie.

shown in Fig. 70 has the advantage that it completely covers the cable with a layer of the tie wire to any desired distance on each side of the insulator. This affords some protection in the event of an arc striking over the insulator. The illustration

shows a sheet metal shield between conductor and insulator; but if it is desired to provide some protection to the cable against possible chafing in the groove, it would appear simpler, and indeed preferable, to continue the serving of the wire right across the portion of the conductor resting in the groove.

A long sleeve over line wires at point of support is objectionable. It is true that breakages of wires caused by too great rigidity at

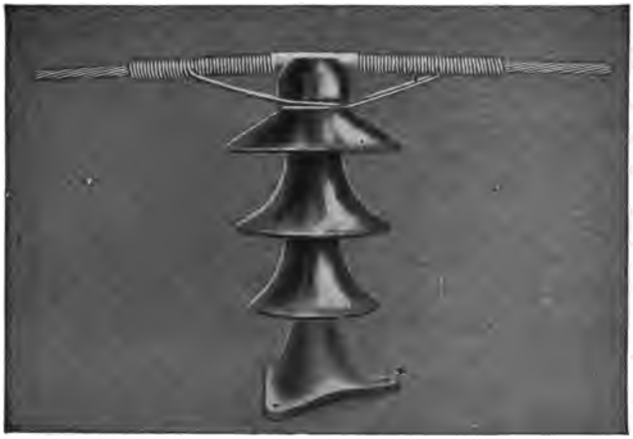

FIG. 70.—Example of tie.

points of support are of rare occurrence; but where mechanical forms of cable clamps are used, as with the suspension type of insulator, and in many of the larger pin type insulators, the method of clamping the conductor to avoid mechanical injury or weakening due to vibration or to swaying of the wires in a wind, should receive careful attention.

CHAPTER VIII

Transmission Line Supports

The supporting poles or structures for overhead electric power transmission lines are of various kinds. Where the ordinary wood telegraph pole or a larger single pole of similar type is not suitable, double poles of the "A" or "H" type, or even braced wooden towers of considerable height and strength, may sometimes be used with advantage. Under certain conditions it may be economical to use steel poles of the tubular type, or light masts of latticed steel, even for comparatively short spans; and poles of reinforced concrete have much to recommend them. But for long spans, and the wide spacing of wires necessary with the higher pressures, steel towers, either of the rigid or flexible type, will generally be required.

The use of wood poles is therefore limited to the lower pressures, and spans of moderate length. It is probable that, in rough country where suitable timber is plentiful, and the cost of transporting steel towers would be high, wooden supports of the "A" or "H" type might be used economically for voltages up to 44,000, even if two three-phase circuits were carried on one set of poles; but for higher voltages the steel tower construction with fairly long spans will, in almost every case, be preferable.

The decision as to the best type of line to adopt is not easily or quickly arrived at. The problem is mainly an economic one, and the decision will depend, not only on the first cost of the various types of line construction, but also on the probable life of the line and the cost of maintenance.

It is necessary to make up many preliminary estimates of the completed line, and these must obviously include, not only the cost of the various types of supporting structure delivered at points along the line, but also the cost of foundations and erection. Again, even if a suitable kind of wood is readily available in the district to be traversed by the transmission line, it is possible that the cost of seasoning the poles, and treating them with preservative compounds to ensure a reasonably long life, may render the

use of steel structures more economical even for comparatively low pressures.

Wood Pole Lines

95. Kinds of Wood Used for Poles.—Among the varieties of straight-growing timber used for pole lines on the American continent may be mentioned cedar, chestnut, oak, cypress, juniper, pine, tamarack, fir, redwood, spruce, and locust. In England the wood poles are usually of Baltic pine or red fir from Sweden, Norway, and Russia. The woods used for the cross-arms carrying the insulators include Norway pine, yellow pine, cypress or Douglas fir, oak, chestnut, and locust.

Probably the best wood for poles is cedar; but chestnut also makes excellent durable poles. Much depends, however, on the nature of the soil, and, generally speaking, poles cut from native timber will be more durable than poles of otherwise equally good quality grown under different conditions of soil and climate.

The redwood poles, as used on the Pacific coast, are sawn from large trees: this renders the supply of poles all of one size a comparatively easy matter; but apart from this advantage, the sawing of the timber naturally increases the price of the finished pole.

With the more extended adoption of preservative treatments (to be referred to later), the inferior kinds of timber which under ordinary conditions would decay rapidly, will become of relatively greater value, and with the growing scarcity of the better kinds of timber, it is probable that poles of yellow pine, tamarack, and Douglas fir will be used more extensively in the future.

The trees should be felled during the winter months, and after being peeled and trimmed should be allowed to season for a period of at least twelve months.

96. Weight of Wood Poles.—For the purpose of estimating the costs of transport and handling of poles, the weight may be calculated on the assumption that the pole is of circular section and of uniform taper, such that the diameter D at the bottom is equal to the diameter d at the top *plus* a quantity tH, of which H is the distance between the two sections considered, and t is a constant depending on the taper and therefore on the kind of wood. Some approximate (average) values of t together with average weight per cubic foot of various kinds of *dry* timber will be found in the accompanying table, from which the value of t

for cedar is seen to be 0.0165 (the height H being understood to be expressed in *inches*), and the weight per cubic foot 35 lb.:

CONSTANTS FOR WOOD POLES

Kind of wood	Wt. per cubic foot (approx.)	Natural taper t (average)	Modulus of rupture T[1]	Modulus of elasticity M[2]
	lbs.			
Juniper	....	....	3700	....
American Eastern white cedar.	35	0.0165	4000	700,000
Spruce	27	....	4500	1,300,000
White pine	26	....	4500	1,000,000
Red pine	34	....	5000	....
Douglas fir[3]	34	....	7000	1,400,000
Norway pine	....	....	7000	1,400,000
Redwood	....	....	7000	700,000
Idaho cedar	23	0.01	6000	....
Chestnut	42	0.016	7000	900,000

The *volume* of a frustrum of right circular cone is:

$$Volume = \frac{H}{3} \times \frac{\pi}{4}(D^2+Dd+d^2)$$

but $D = d+tH$ and the formula becomes:

$$Volume = \frac{\pi H}{12}(3d^2+t^2H^2+3dtH)$$

By using this formula and putting for H the value $65 \times 12 = 780$ in., and for d the value 7 in., the weight of a pole of American eastern white cedar measuring 7 in. diameter at top and 65 ft. over all length works out at 2410 lb.

97. Life of Wood Poles.—It is not easy to estimate the probable life of poles, because this will depend not only on the kind of timber, but also on the nature of the soil, climatic conditions, the time of seasoning, whether or not the poles have received treatment with preservative compounds, and the nature of such treatment.

[1] Being stress in pounds per square inch at moment of rupture under bending conditions.

[2] Inch units. Average figures, which must be considered approximate only.

[3] This name is intended to cover yellow fir, red fir, Western fir, Washington fir, Oregon fir, North-west and West-coast fir.

In England the life of well-seasoned, creosoted poles may be about 35 years in good soil, and from 18 to 20 years in poor soil. On the American continent, where untreated poles have been, and are still, used in large numbers, the average life will probably be about 12 years. The better woods, such as cedar and chestnut, might last on the average 14 to 16 years, while juniper and pine might have to be replaced in 6 to 10 years. On certain lines where untreated poles of unsuitable timber have been erected in poor soil, or where destructive insects are particularly active, the poles have had to be replaced in less than 4 years. The creosoted poles, as used in England, will usually stand best in moist or clayey ground; there is a tendency for the creosote to run out and be absorbed into the ground when the soil is loose and sandy, with the result that the poles deteriorate rapidly just below the ground level. Marshy soil is generally bad for wood poles, also ground that is alternately wet and dry.

98. Preservative Treatment of Poles.—Many chemical solutions and methods of forcing them into the wood have been tried and used with varying success; but it is generally conceded that treatment with coal-tar creosote oil gives the best protection against decay; and its cost is probably lower than that of any other satisfactory treatment.

There are three recognized methods of applying the oil:

(*a*) The high-pressure treatment (Bethel system).
(*b*) The open-tank treatment.
(*c*) Brush treatments.

In Europe the treatment known as the Rüping process is largely used; it is less costly than the Bethel system. In France copper sulphate is used extensively as a preservative (Boucherie process), but the results are not entirely satisfactory.

High-pressure Treatment

This is undoubtedly the best, but it is also the most costly. The poles, after being trimmed and framed, are placed in large treating cylinders capable of being hermetically closed. If the poles are green or wet they are first subjected, in these cylinders, to a steaming process from three to eight hours, the steam being admitted under a pressure of 12 to 20 lb. The steam is then blown off, and the treating cylinder is exhausted, the vacuum

being maintained for a period of one to two hours. Immediately afterward the creosote is forced in under pressure at a temperature of 140° to 200° F. Seasoned timber is not subjected to the steaming process, but the temperature inside the treating cylinder is raised by means of heating coils to about 150° F. prior to the filling process.

The poles will absorb from a minimum of 10 lb. to a maximum of 15 lb. of oil per cubic foot. The softer and more porous woods will absorb the most oil; but, on the other hand, the benefit such woods derive from the treatment in the matter of increased life is more marked than in the case of the closer grained timber.

Open-tank Treatment

The butts of the poles are placed in the creosote oil, which is preferably heated to a temperature of 200° F. to 220° F. They are maintained in this bath for a period of one to three hours, after which they are placed in cold oil for another period of one to three hours. This process will permit of a complete penetration of the sapwood to a height of about 2 ft. above ground level. When properly carried out it is capable of giving very satisfactory results. The open-tank process is specially applicable to the treatment of the more durable kinds of timber, such as cedar and chestnut.

Brush Treatment

The oil is applied hot with hard brushes, a second coat being applied after the first has soaked in. The temperature of the oil should be about 200° F. This method of application of the oil is the cheapest and the least effective, but it affords some protection when the wood is well seasoned and dry. There is little advantage to be gained by the external application of preservative compounds to green timber; indeed, the sealing up of the surface of such timber, by enclosing the fermentative juices, may even lead to more rapid decay. The brush treatment cannot be applied to poles which are set in the winter months in cold climates, as the frost would so harden the surface of the poles that there would be no absorption of the preservative liquid.

The quality of oil used, whatever the method of application, is a matter of importance. In circular No. 98 of the United States Forest Service, Department of Agriculture, issued May 9,

1907, there is an important concluding statement to the effect that light oils boiling below 400° F. will not remain in the timber; but the heavy oils, containing a high percentage of anthracene oil, will remain almost indefinitely, and afford excellent protection against decay and boring animals.

Inflammability of Treated Timber

Poles and cross-arms treated with creosote oil are less liable to destruction by fire than untreated timber of the same kind. This appears to be due to the fact that the free carbon deposited by the burning oil on the surface of the timber affords some protection from the action of the fire. A committee appointed by the National Electric Light Association of New York City conducted a series of experiments on similar specimens of treated and untreated short-leaf pine, and proved conclusively that the latter suffered considerably more damage from the effects of fire than the specimens that had been impregnated with creosote oil.

99. Reinforcing Pole Butts.—Wood poles usually decay most rapidly at or near the ground surface. It is not always necessary to replace poles which may be otherwise sound, but which have been weakened locally by decay just below the ground surface. They may be reinforced by means of steel rods about ½ in. diameter, pointed at both ends and driven into the pole above and below the damaged portion. Concrete is then filled in around the pole, extending at least 12 in. above the ground level. This and similar methods of prolonging the life of poles have proved satisfactory. The cost may be from $3 to $4 per pole, and the life may be prolonged 5 to 10 years.

100. Typical Wood Pole Lines.—For a single three-phase line transmitting power at 20,000 to 22,000 volts single poles having a top measurement of about 8 in. would be suitable. The distance between wires would be about 3 ft.; the arrangement being as shown on the sketch, Fig. 71, with pole-top details as in Fig. 72. This shows an arrangement without overhead guard wire, but with some or all poles protected by a grounded lightning rod. In exposed positions, and at angles, pieces of bent flat iron may be fitted with advantage on the cross-arm near the insulators, as shown by the dotted lines in Fig. 72. These pieces serve the double purpose of hook guard in case of the wire slipping off insulator, and of additional protection against light-

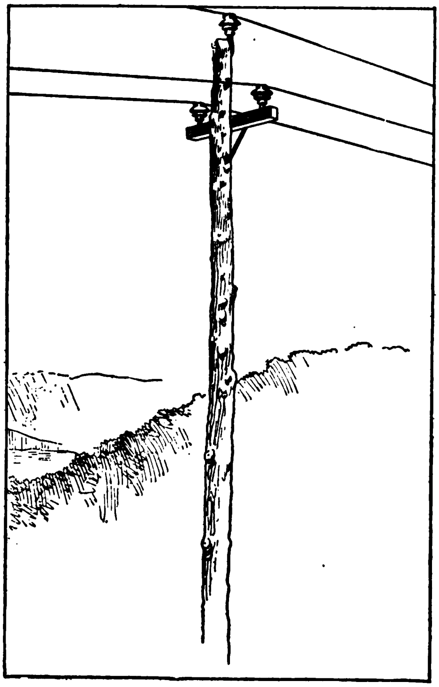

FIG. 71.—Typical pole line for 20,000-volt three-phase transmission.

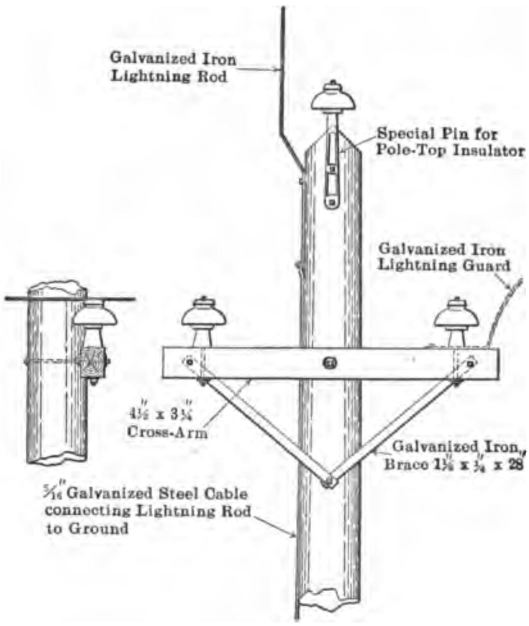

FIG. 72.—Pole-top details.

ning. A discharge from the line tends to leap across to this grounded metal horn over the surface of the insulator, thus frequently preventing the piercing or shattering of insulators.

Fig. 73 shows a simple "A" frame construction for a duplicate three-phase line at 10,000 to 11,000 volts.

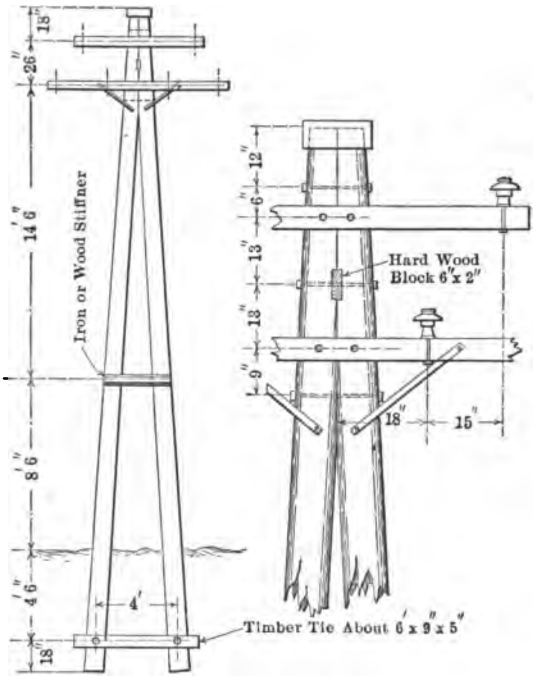

FIG. 73.—Typical "A" frame construction for duplicate three-phase line.

101. Insulating Qualities of Wooden Poles.—One advantage which may be claimed for wooden poles is the possibility of working on live wires with little risk of life when the conditions are favorable. In the Black Hills in the mining district of South Dakota, where the climate is dry, it is usual to work on live wires at 24,000 volts, supported on ungrounded poles, without using any insulating devices. In this instance, the separation between wires is unusually large, being 5 ft., which affords additional safety. On the other hand, it may be argued that accidental contact with a wood pole carrying high-tension conductors may be a danger to the public, which is practically absent in the case of well-grounded steel poles or towers. Tests have been made to

determine, if possible, the nature of charges likely to pass to ground through a person touching ungrounded wooden poles of a high-tension transmission. These tests showed that it is possible for poles to become dangerously charged, but not probable. A grounded metal ring or wire placed round the pole from 6 to 7 ft. above ground surface eliminates all possibility of accidents from this cause.

102. Strength and Elasticity of Wood Poles.—Apart from the dead weight to be supported by the poles of a transmission line—which will include not only the fixtures and the conductors themselves, but also the added weight of sleet or ice in climates where ice formation is possible—the stresses to be withstood include the resultant pull of the wires in adjoining spans, and the wind pressure on poles and wires. It is customary to disregard the dead weight or column loading, except when the spans are large and the conductors numerous and heavy. A formula for approximate calculation of loads carried by poles when acting as struts or columns will be given later. The pull due to the conductors on corner poles is usually met by guying these poles, by which means the pull tending to bend the pole is largely converted into an increased vertical downward pressure; but even on straight runs there may be stresses due to unequal lengths of span which would cause a difference in the tensions on each side of the pole. The most important stresses to which the poles are subjected, apart from such accidents as are due to falling trees or the severing of all wires in one span, are those caused by strong winds blowing across the line. The resulting pressure at pole-top due to strong winds acting on long spans of ice-coated wires may be very great, and the poles must be strong enough to resist this.[1]

For the purpose of making strength and deflection calculations, the pole may be considered as a truncated cone of circular section, firmly fixed in the ground at the thick end, and with a load near the small end in the form of a single concentrated resultant horizontal pull. The calculation is therefore exactly the same as for a beam fixed at one end and loaded at the other. Such a beam, if it exceeds a certain length depending upon the amount of taper, will not break at the point where the bending moment is greatest (*i.e.*, at the ground level), because, in a beam of circular section and uniform taper, the stresses in the material are not

[1] See article 85 in Chapter VII.

necessarily greatest at this point, as will be shown later. The ordinary telegraph or electric lighting pole usually breaks at a point about 5 ft. above ground unless the butt has been weakened by decay.

Calculations on strengths and deflections of wood poles cannot be made with the same accuracy as in the case of steel structures; and the constants in the table previously referred to are averages only for approximate calculations. The factor of safety generally used on the American Continent is 6, both for poles and cross-arms. The maximum wind pressure is taken at 30 lb. per square foot of flat surface, or 18 lb. per square foot of projected surface of smooth cylinders of not very large diameter. In England the factor of safety for telegraph poles is 8, and for power lines 10. The latter figure would seem to be unnecessarily high: it suggests a want of confidence either in the strength calculations or in the tests and load assumptions on which the calculations are based.

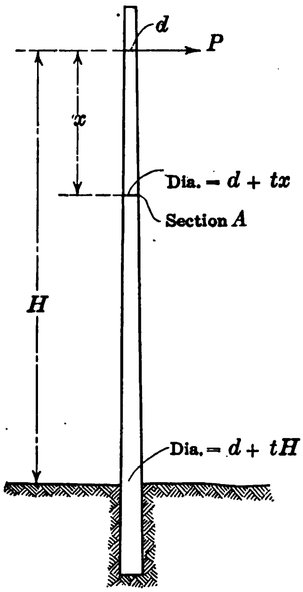

FIG. 74.—Wood pole with horizontal load near top.

103. Calculation of Pole Strengths.—The relation between the externally applied load and the stresses in the fibers of the wood is:

Bending moment = stress in fibers most remote from neutral axis × section factor, or $M_B = T \times Z$.

If P is the force in pounds applied at a point distant x in. from the cross-section A (see Fig. 74), then:

$$M_B = Px \text{ lb.-in.}$$

And if the stress T is expressed in pounds per square inch, and the section is assumed circular:

$$Px = T \times \frac{\pi d^3}{32}$$

But it is assumed that the diameter at any point x in. below the section of diameter d is $d + tx$, therefore:

$$T = \frac{32P}{\pi} \times \frac{x}{(d+tx)^3} \qquad (1)$$

In order to find the position of the cross-section at which the pole is most likely to break—that is to say, where the fiber stress is a maximum—it is necessary to differentiate the last equation with respect to x, and find the value of x which makes this differential equal to zero. This gives

$$x=\frac{d}{2t}$$

for the point where the stress T is a maximum. The position of this cross-section is evidently not necessarily at ground level. If this value of x is greater than H, then the maximum fiber stress will be at ground level, and it is calculated by substituting H for x in formula (1).

The diameter of the pole at the weakest point is:

$$\begin{aligned} d_w &= d+tx \\ &= d+t\left(\frac{d}{2t}\right) \\ &= 1.5\,d \end{aligned}$$

and it is only when the diameter at ground level is greater than one and a half times the diameter where the pull is applied that the pole may be expected to break above ground level.

If the stress T, the taper t, and the pole-top diameter d are known, the load P is readily calculated as follows:—

$$\text{Bending moment} = P\times x$$

$$\text{Resisting moment} = T\times\frac{\pi d^3_w}{32}$$

But $x=\dfrac{d}{2t}$ and $d_w=1.5d$

therefore

$$\frac{Pd}{2t}=T\times\frac{\pi\times(1.5d)^3}{32}$$

$$\text{or}\quad P=\frac{2t\ T\pi\times3.375\times d^3}{32\times d}$$

$$=0.662\times T\times t\times d^2 \qquad (2)$$

Similarly, if the pull P is known, the pole-top diameter should be:

$$d=\sqrt{\frac{P}{0.662\times T\times t}} \qquad (3)$$

Example of Strength Calculation

Consider a pole of Eastern white cedar designed to sustain a pull of 500 lb. applied 26 ft. above ground level. The average breaking stress (from table of constants) is 4000 lb. per square inch, and assuming a factor of safety of 6 the safe working stress is $T=660$ lb. per square in. The other numerical values are:

$$P=500 \text{ lb.}$$
$$H=26 \text{ ft.}$$
$$t(\text{from table})=0.0165$$

By formula (3):

$$d=\sqrt{\frac{500}{0.662\times660\times0.0165}}$$
$$=8.33 \text{ in.}$$
$$d_w=1.5\times8.33=12.5 \text{ in.}$$

The distance below point of application of load of the section where fibre stress is a maximum is:

$$x=\frac{d}{2t}=252 \text{ in.}=21 \text{ ft.}$$

Therefore this pole, if subject to a load about six times greater than the maximum working load, may be expected to break $26-21=5$ ft. above ground level.

Double pole supports of the type illustrated in Fig. 73 will be twice as strong as each of the component poles in resisting stresses applied in the direction of the line; but they will be able to withstand about four times as great a load as the single pole when the stresses are in a direction at right angles to the direction of the line. When loaded in this manner up to the breaking point these double poles of the "A" type usually fail through the buckling of the member in compression due to initial want of straightness. The strength of both the "A" and the "H" type of pole structure can to some extent be increased by judicious and rigid bracing.

104. Deflection of Wood Poles.—It is now generally recognized that there are advantages in having transmission-line supports with flexible or elastic properties. The ordinary single wood pole is very elastic, and will return very nearly to its original form after having been deflected very considerably by abnormal stresses. The figures given for the elastic modulus in the table previously

referred to are subject to correction for different qualities and samples of the same timber. It is well to make a few experiments on the actual poles to be used if accuracy in calculated result is desired. The double-pole structures of the "A" or "H" type will have about half the deflection of the single poles in the direction of the line, and, of course, very much less in a direction at right angles to the line. An "A" pole of usual construction with the two poles subtending an angle of 6½ degrees will only deflect about one-fiftieth of the amount of the single-pole deflection under the same transverse loading. The movement is usually dependent upon the amount of slip between the two poles at top, which again depends upon the angle subtended by the poles.[1] If this angle is as much as 10 degrees there will be practically no likelihood of the poles slipping at the top joint; but this large angle is unsightly, and probably makes a less economical structure than the more usual angle of about 6½ degrees.

105. Calculation of Pole Deflections.—Assume the pole to be fixed firmly in the ground, and that there is no yielding of foundations. The load P being applied in a horizontal direction at the top end, as indicated in Fig. 75, the pole may be considered as a simple cantilever, the deflection of which, *if the section were uniform throughout the entire length,* would be:

$$\delta = \frac{1}{3}\frac{PH^3}{MI}$$

where d and H are in inches; I is the moment of inertia of the section, and M is Young's modulus (pounds per square inch).

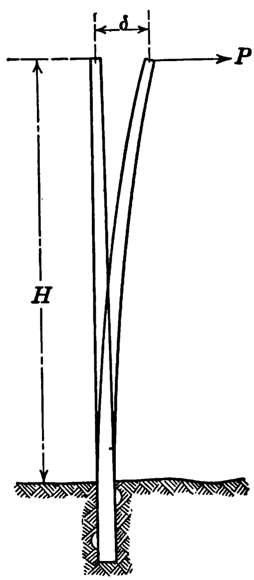

Fig. 75.—Deflection of wood pole.

For a circular section $I = \frac{\pi d^4}{64}$ where d is the diameter of the (cylindrical) pole in inches. The formula then becomes:

$$\delta = 6.78\frac{PH^3}{Md^4} \qquad (4)$$

[1] Much useful information on the behavior of "A" poles under test is to be found in Mr. C. Wade's paper read before the Institution of Electrical Engineers on May 2, 1907.

If P is evenly distributed, as would be the case if a uniform wind pressure on the pole surface had alone to be considered, the deflection would be:

$$\delta = \frac{1}{8}\frac{PH^3}{MI}$$

but it is best to consider the wind pressure on pole surface as a single equivalent load concentrated at pole-top and added to the load due to wind pressure on the wires. When estimating the probable value of this equivalent load, it should be remembered that the wind pressure is not evenly distributed along the length of the pole, since the wind velocity at ground level is comparatively small and increases with the height above ground surface.

The formula (4) assumes a constant diameter throughout length of pole, and the question therefore arises as to where the measurement of diameter should be made on an actual pole. Mr. S. M. Powell has shown that, on the assumption of a uniform taper, the quantity d^4 in formula (4) should be replaced by $(d_g^3 \times d_1)$ where d_g is the diameter at ground level and d_1 is the diameter where the force P is applied.

Example of Calculation of Pole-Top Deflection

Using the same figures as in the example of strength calculations:

$$\begin{aligned} P &= 500 \text{ lb.} & H &= 26 \times 12 = 312 \text{ in.} \\ d_1 &= 8.33 \text{ in.} & t &= 0.0165 \\ d_g &= 8.33 + (0.0165 \times 312) = 13.48 \text{ in.} \\ M &= 700{,}000 \end{aligned}$$

then

$$\delta = \frac{6.78 \times P \times H^3}{M\,(d^3_g \times d_1)}$$

$$= \frac{6.78 \times 500 \times (312)^3}{700{,}000 \times (13.48)^3 \times 8.33}$$

$$= 7.\ 2 \text{ in.}$$

When possible it is well to make tests on a few actual poles; then for similar poles of the same material subject to the same loading:

$$\delta \propto \frac{H^3}{d^3_g \times d_1}$$

106. Yielding of Foundations.—A permanent deflection of the pole when the stresses are abnormal may occur owing to the yielding of the earth foundation; but this is unusual if the poles are properly set in good ground.

The diagram Fig. 76 has been drawn to show the depth to which poles of various heights are usually set. These depths are such as would be adopted on a well-designed pole line, and need not be exceeded except in special cases. In marshy or otherwise unsatisfactory ground, special means must be adopted to provide a reasonably good setting for the pole butts.

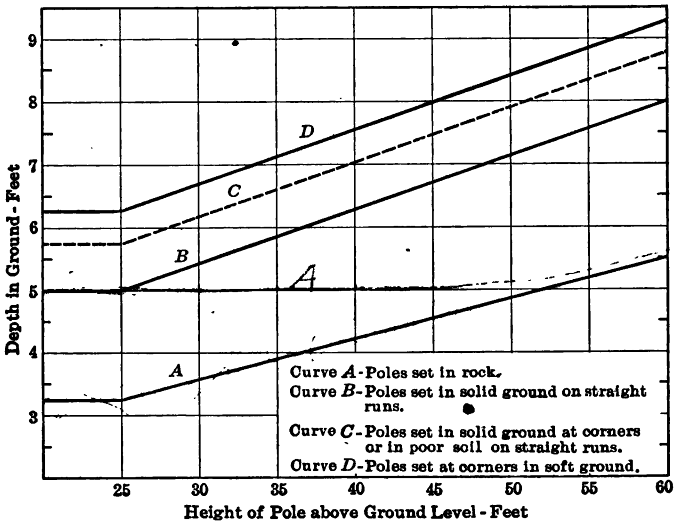

FIG. 76.—Chart giving depth of holes for wood poles.

Loam and gravel, and even sand, or a mixture of these, provides a firm foundation for poles. A pole that is properly set should break before the foundations will yield to any appreciable extent. Even if there should be a movement of the pole butt in the ground with excessive horizontal load at pole-top, this will result in a firmer packing of the earth, which will then be better fitted to resist any further movement.

Firm sand, gravel, or loam will withstand a pressure of about 4 tons per square foot; but only half this resistance should be reckoned on in the case of damp sand, moist loam, or loose gravel.

107. Spacing of Poles at Corners—Guy Wires.—In order to reduce the stresses, not only on the pole itself, but also on the

insulator pins and cross-arms, it is usual to shorten up the spans on each side of the corner pole. The reduction in length of span will depend upon the amount by which the direction of the wires departs from the straight run. A rough and ready rule is to reduce the span length 1 1/2 per cent. for each degree of deviation from the straight line. For angles less than 5 degrees, it is not necessary to alter the span.

It is not advisable to turn more than 25 degrees on one pole, and whenever the side strain is likely to be excessive, double cross-arms and insulators should be used. By giving proper attention to the matter of guying and to the mechanical construction generally, it is not difficult to meet all requirements at points where a change of direction occurs.

A safe plan is to assume that a corner pole must carry the full load without breaking if the guy wire or wires should fail to take their proper share of the load: but all corner poles should be propped or guyed for extra safety, and to avoid the unsightly appearance of poles bent under heavy side stresses or set at an angle with the vertical.

Sometimes when sharp corners have to be turned, the spans on each side are "dead-ended" on poles with double fixtures. Such poles are head-guyed, and the span adjoining the guyed pole is usually shortened, being not more than three-fifths of the average spacing. For further particulars of common practice in guying poles in special positions, the reader is referred to the sample specification for wooden pole line in Appendix F.

108. Load to be carried by Corner Poles.—If T is the total tension in pounds of all the wires on each side of the corner pole,

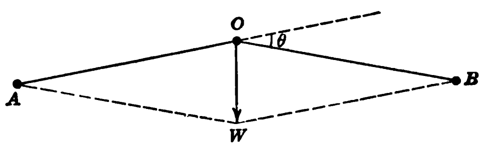

FIG. 77.—Diagram of stresses at corner pole.

and if θ is the angle of deviation as indicated in Fig. 77, then the resultant pull in the direction $O\ W$ at the pole top will be,

$$W \text{ (lb.)} = 2\ T \sin\left(\frac{\theta}{2}\right) \tag{5}$$

The stress in the guy wire is readily calculated when the angle α (Fig. 78) which the wire makes with the vertical is known.

If W is the side pull as calculated by formula (5), then

$$\text{Tension in guy wire} = \frac{W}{\sin \alpha}$$

$$= \frac{W \times OC}{CD} \qquad (6)$$

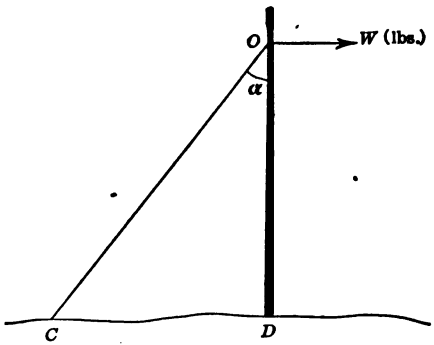

FIG. 78.—Diagram of stress in guy wire.

109. Props or Struts—Wood Poles in Compression.—Sometimes it is difficult or impossible to provide guy wires in certain locations; or impurities in the atmosphere may render the use of props or push braces preferable to guy wires. In such cases it is necessary to know approximately what load a wooden pole will support in compression, that is to say when used or considered as a column. Assuming a factor of safety of 8, the working load W (in pounds) that can be supported by a pole of average quality and circular cross-section may be arrived at approximately by using the formula $W = \frac{1000\, d^4}{H^2}$ (7)
where d is the average diameter of the pole in inches, and H is the length in feet.

CONCRETE POLES.

110. General Remarks: Weights and Costs.—As a substitute for wood poles supporting overhead wires, steel poles of the tubular form and latticed steel masts are used. The full advantage of the galvanized or painted steel structure is best realized in the high towers with extra wide spacing, such as are used for the transmission of electric energy at high pressures. The use of Portland cement for molded poles of moderate height, to be used

in lieu of wood poles on the shorter spans, is by no means new; the experimental stage has long ago been passed, and with the deplorable but no less rapid depletion of our forests and the incomparably longer life of the concrete poles, these will probably be used in largely increasing numbers during the next few years.

There is much to be said in favor of the wood pole when the right kind of timber, properly seasoned and treated, is used; but, apart from the general unsightliness of wood poles in urban districts, their life is uncertain and always comparatively short. In Switzerland the experiment has been tried of covering the ordinary wood pole with concrete mortar about 1 in. thick. The strength, and especially the life, are greatly increased thereby, as the decay which so frequently occurs at ground level will be largely, if not entirely, prevented; but it is doubtful whether the system will in the long run prove satisfactory or economical. The ideal material to use for reinforcing concrete is undoubtedly steel or iron. Longitudinal rods or bars of iron can be placed exactly where required to strengthen those parts of the pole section that will be in tension, and the concrete, filling up the spaces between the reinforcing rods, takes the place of all bracing and stiffening members of the ordinary steel structure in an almost perfect manner. It is probably at this time generally admitted that iron embedded in cement will last almost indefinitely without suffering any deterioration. When excavating for the foundations of the new General Post Office in London, England, some old Roman brickwork was discovered in which the hoop-iron bonds were still bright and in perfect condition. The life of a concrete pole is, in fact, almost unlimited, a consideration which should not be overlooked when estimating the relative costs of different kinds of supporting structures. It requires no painting and practically no attention once it is erected. If any small cracks should at any time develop, they can readily be filled with cement.

While referring to the advantages of the cement pole it may be added that every pole is virtually a lightning rod, an advantage which it shares with the steel pole or tower. On lines where both timber and concrete poles have been used and where many wood poles have been shattered by lightning, the concrete poles have rarely been struck. There is an instance of a concrete pole of the Marseilles (Ill.) Land & Water Company having been struck, but the only damage done was the chipping out of a small piece at the

top of the pole and one at the bottom where the current entered the ground after following down the steel reinforcing bars inside the pole.

The weight of concrete poles is necessarily considerable, and unless the poles are made near the site where they will be erected the cost of transportation will generally be prohibitive. Some data given by Mr. George Gibbs in a paper read before the American Society of Civil Engineers and abstracted in the *Electrical World* in the issue of Sept. 2, 1911, may be of interest. The concrete poles referred to are erected on the Meadows division of the Pennsylvania Railroad, the average spacing being 120 ft. The total (over-all) lengths vary between 35 ft. and 65 ft. The specification called for poles to withstand a transverse loading of 6000 lb. applied 6.5 ft. below the top. The cross-section of the poles is a square with chamfered corners, the taper being 1 in 120. The weight of a 35-ft. pole without fixtures is 5300 lb., a 40-ft. pole 7600 lb., while that of the 65-ft. pole is 17,300 lb. These weights are in excess of what would ordinarily be required because, the foundations being poor, the portion of the pole buried in the ground is abnormally long. The weight of a solid pole 30 ft. long will in many cases not exceed 2500 lb., while hollow poles might weigh 2000 lb. for a 35-ft. overall length, and 3800 lb. for 40-ft. length.

It is probable that the concrete poles of cross-country transmission lines are usually made somewhat heavier than the strength requirements necessitate because, being molded on the site, not always with the best and most convenient appliances, they are made solid throughout or through a large part of their length, whereas a hollow construction would have been adopted had suitable collapsible cores been available.

Poles up to 35 ft. in length are usually molded in a horizontal position, the forms being removed after three or four days. After a period of seasoning lasting from two to three weeks they are erected in the same manner as wood poles.

Poles longer than 35 ft. are best molded in a vertical position; in fact, it is possible that this method may be found advantageous even in the case of shorter poles. The forms are set up immediately over the hole previously prepared for the pole base. They are set truly vertical and temporarily guyed, the reinforcing inside the form being held together and in position by whatever means of tying or bracing may be adopted. Sometimes iron wire is

used, but more uniform results are obtained by using specially designed iron distance pieces with the required spacing between them. The concrete is raised to the top of the mold by any suitable and economic means (preferably direct from the concrete mixer by an arrangement equivalent to the ordinary grain elevator) and is dropped in. By this means the hole in the ground is entirely filled with concrete. No tamping is required, a firm hold being obtained, since the ground immediately surrounding the concrete base has not been disturbed.

The best quality of crushed stone and sand should be used, the usual proportions being: cement, one part; sand, two parts; crushed stone, three or four parts, not too large to pass through a 3/4-in. screen. The mixture used for the poles on the Pennsylvania Railroad is 1.5 : 2 : 4. When gravel is used the mixture may be one part of Portland cement to five parts of gravel, provided that the latter is graded, including sand, and with the largest pieces of a size to pass through a 3/4-in. screen.

The cost of concrete poles, when the long life and other advantages are taken into account, does not compare unfavorably with that of other types; but it must not be overlooked that the cost of materials and labor required to manufacture the poles do not represent the first cost of the finished pole erected in position. Much valuable information on the costs of manufacture and handling of concrete poles, together with practical details relating to methods of manufacture, will be found in Mr. R. A. Lundquist's book on Transmission Line Construction.[1]

As a rough guide to probable cost; it may be stated that the cost of reinforced concrete poles at the place where they are molded may be anything between 35 cents and $1 per 100 lb. weight.

As an example of a concrete-pole line, the transmission line of the Northern Illinois Light and Traction Company, of Marseilles, Ill., may be mentioned. This company transmits three-phase energy at from 30,000 volts to 33,000 volts. Most of the poles used are about 30 ft. high, spaced from 125 ft. to 132 ft. apart. The section is square, with 6-in. sides at the top of the pole and 9 in. at the base. The reinforcing consists of six ½-in.-square steel bars through the entire length of the pole. Many of the concrete poles on this line have now been in position over four years, and they have given entire satisfaction.

[1] McGraw-Hill Book Co., 1912.

111. Strength and Stiffness of Concrete Poles.—When designing a concrete pole to withstand a definite maximum horizontal load applied near the top, the pole is treated as a beam fixed at one end and loaded at the other. The calculations are very simple if certain assumptions are made, these being as follows:

(1) Every plane section remains a plane section after bending.

(2) The tension is taken by the reinforcing rods.

(3) The concrete adheres perfectly to the steel rods.

(4) The modulus of elasticity of concrete is constant within the usual limits of stress.

The ultimate crushing stress of the concrete may be taken at from 2000to 2200 lb. per square inch. The reinforcing bars should be covered with concrete to a depth of not less than 1 in. The effect of keeping the reinforcing bars under tension while the concrete is poured in the mold and until it has hardened sufficiently to support the strain itself has been tried and found to improve the performance of the poles, but it is doubtful whether the extra apparatus and labor required are justifiable on economic grounds. When subjected to excessive load a concrete pole will generally yield by the crushing of the material in the base near ground level; but, unless it is pulled out of its foundations, it will not fall to the ground.

The comparative rigidity of concrete poles cannot be said to be a point in their favor, as the flexibility and elasticity of wood poles and some forms of steel structures are features of undoubted advantage under certain conditions. On the other hand, the degree of deflection of concrete poles before breaking is remarkable. The elastic limit is variable, and no exact figure can be given for the elastic modulus of cement concrete; but for a 1 : 2 : 4 mixture 2,000,000 may be taken as a good average figure for approximate calculations. For cinder concrete this coefficient may be as low as 900,000.

Some tests made on 30-ft. concrete poles gave deflections of from 3 in. to 4 in. at a point near the top of pole, when subjected to a test load equal to about double the maximum working load.[1] Another series of tests made recently in England on some 44-ft. poles of hollow section, 17 in. square at the base and 8 in. square at the top (inside dimensions 13 in. and 4 in. respectively), with

[1] These poles were probably of large cross-section. Some tests made on poles measuring 10 in. square at the base and 32 ft. high gave a deflection of just over 2 ft. with a horizontal load of 2000 lb. applied near the top.

loads applied 38.5 ft. above ground level, gave a deflection of 66 in. under a horizontal load of 10,500 lb., and the permanent set on removal of load was 21 in. The pole did not fail completely until the deflection was 78 in.

The illustration Fig. 79 shows a typical concrete pole of hollow section suitable for carrying six transmission wires on two wooden cross-arms. The pole is 35 ft. long over all, about 6 ft. being buried in the ground. With a top measurement of 7 in. square and a taper to give an increase of 1 in. width for every 5 ft. of length, the size at the bottom will be 14 in. square. The drawing shows a section through the hollow pole taken at a point about 4 ft. above the ground level. Iron spacing pieces, as here shown, or their equivalent, must be placed at intervals to hold the longitudinal steel reinforcing bars in the proper position. The number of rods will vary with the distance below the point of application of the load. The bending moment to be resisted at every point being known and the taper of the pole decided upon, the amount of reinforcing required at any given section is easily calculated. The weight of a pole as illustrated would be about 2700 lb. without fixtures. The reinforcing rods and spacing rings would account for approximately one-seventh of the total weight. A factor of safety of four is generally employed in strength calculations of reinforced concrete poles. In some cases the calculations have been based on a safety factor of 5; but there appears to be no justification for using so large a factor.

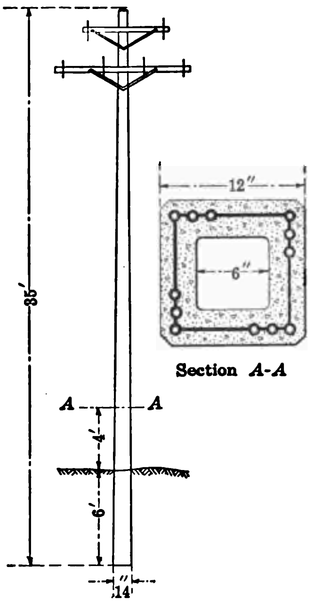

Fig. 79.—Concrete pole of hollow section.

Steel Towers

112. General Remarks: Types of Towers.—It cannot be said that there is at the present time a standard type of steel structure for supporting the conductors of overhead transmission lines;

neither is it likely that one particular design will ever be found suitable for all countries, climates and voltages. Any kind of supporting structure which will economically fulfil the necessary requirements will answer the purpose of the transmission line engineer, who merely requires a durable mechanical structure to carry a variable number of insulators at a height above ground, and with a spacing between them, depending upon the voltage of transmission and the length of span.

As a substitute for wood poles, steel tubes have been used, either in one piece, or built up of a number of pieces of different sizes in order to economize material and give a large diameter at the bottom where the bending moment is greatest, and a small diameter at the top where the bending moment is negligible. Steel poles of considerable height, suitable for longer spans, may be built up of three or four verical tubes of comparatively small diameter jointed and braced together at suitable intervals to give stiffness to the structure. It is doubtful whether, in the long run, such composite tubular structures will hold their own against the latticed steel masts built up of standard sections of rolled steel, as used extensively on the continent of Europe, and, to a relatively smaller extent, in America. The term "tower" is applied mainly to the light steel structures in which the spacing between the main upright members, at ground level, is large compared with the height of the structure; the usual proportion—which will generally be found to be the most economical in material—being 1 to 4; that is to say, if the base is square, the side of this square will be about one-quarter of the distance from the point of measurement to the top of the tower. If the towers are large, the footings are usually separate pieces which are correctly set in the ground by means of a templet, and to which the legs of the tower proper are afterward bolted. A good example of large steel towers is to be found in the 100,000-volt transmission line of the Great Western Power Co. of California. Two three-phase circuits are carried on these towers, the vertical spacing between the cross-arms being 10 ft. There are three cross-arms, each carrying two conductors—one at each end. The horizontal spacing between wires is 17 ft. on the two upper cross-arms and 18 ft. on the lower cross-arm, which is 51 ft. above ground level. No conductor is closer than 6 ft. 5 in. to the steel structures, this being the minimum clearance in the horizontal direction. The average distance between towers is 750 ft., and

they are joined at the top by a grounded guard wire 5 ft. above the bottom of the highest cross-arm. The base of the tower measures 17 ft. square, the parts under ground being separate pieces of steel, buried to a depth of 6 ft. to which the tower proper is bolted after being assembled and erected on site.

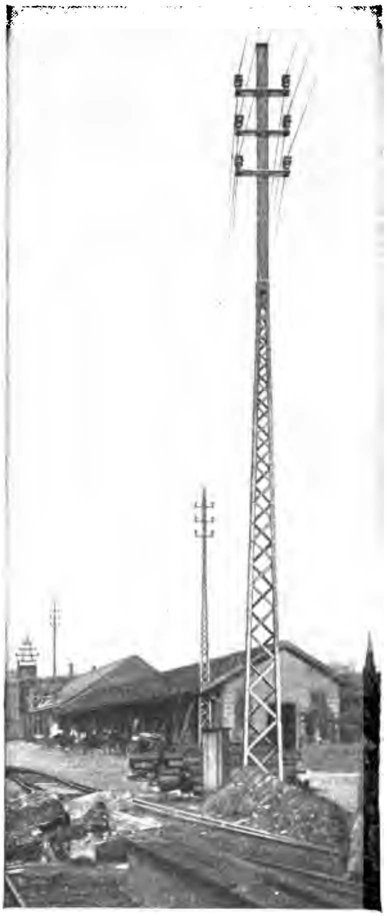

FIG. 80.—Steel mast.

FIG. 81.—Rigid steel tower.

Although the larger towers are nearly all built of the square type as used for windmills, there is a notable exception in the case of the 140,000 volt line in Michigan, where the towers are of a special three-legged type, built up entirely of angle sections.

Fig. 80 shows a typical form of latticed steel mast, while Fig. 81 is a corner tower on the same transmission line; it is generally similar to the standard towers with wide base, as used on straight runs for moderately long spans, except for the special arrangement carrying the larger number of insulators.

113. Flexible Towers.—Although calculations of stresses in transmission lines are usually based on the assumption that the ends of each span are firmly secured to rigid supports; this condition is rarely fulfilled in practice; there is some "give" about the poles or towers, especially when the line is not absolutely straight, and the insulator pins will bend slightly and relieve the stress when this tends to reach the point at which the elastic elongation of the wires will be exceeded. Then, again, the wires will usually slip in the ties at the insulators, even if these ties are not specially designed to yield or break before damage is done to the insulators or supporting structures. The use of the suspension type of insulator, which is now becoming customary for the higher voltages, adds considerably to the flexibility of the line.

In regard to the towers themselves, all steel structures for dead-ending lines or sections of lines are necessarily rigid, and the usual light windmill type of tower with wide base is also without any appreciable flexibility. The latticed steel masts, as used more generally in Europe than in America, are slightly more flexible, and the elastic properties of the ordinary wooden pole are well known. The deflection of a wooden pole may be considerable, and yet the pole will resume its normal shape when the extra stress is removed.

The present-day tendency is undoubtedly toward the increased use of the so-called flexible steel structures; that is to say, of steel supports designed to have flexibility in the direction of the line, without great strength to resist stresses in this direction; but with the requisite strength in a direction normal to the line, to resist the side stresses due to wind pressures on the wires and the supports themselves.

Such a design of support has the important advantage of being cheaper than the rigid tower construction, in addition to which it gives flexibility where this is advantageous, with the necessary strength and stiffness where required. The economy is not only in the cost of the tower itself but in the greater ease of transport over rough country, the preparation of the ground, and erection.

The advantages of flexibility in the direction of the line are

considerable. Probably the most severe stresses which a transmission line should be capable of withstanding are those due to the breakages of wires. Such breakages may be caused by abnormal wind pressures, by trees falling across the line, or by a burn-out due to any cause. Suddenly applied stresses such as are caused by the breaking of some or all of the wires in one span are best met by being absorbed gradually into a flexible system. The supports on each side of the wrecked span will bend toward the adjoining spans because the combined pull of all the wires in the adjoining spans is greater than the pull of the remaining wires, if any, in the wrecked span. This movement of the pole top results in a reduction of tension in the wires of the adjoining span owing to the increased sag of these wires; there will be an appreciable deflection of the second and third poles beyond the break, but the amount of these successive deflections will decrease at a very rapid rate and will rarely be noticeable beyond the fourth or fifth pole.

It is obvious that, as the remaining wires in the faulty span tighten up, the stress increases; but the combined pull of these wires on the pole top is smaller than it was before the accident, since it is assisted by the pull of the deflected poles, and these joint forces are balanced by the combined pull of all the wires in the adjoining sound span, which pull, as previously mentioned, is smaller than it was under normal conditions.

The greater the flexibility of the supports in the direction of the line, the smaller will be the extra load which any one support will be called upon to withstand; on the other hand, it is usual to provide anchoring towers of rigid design about every mile or three-quarter mile on straight runs, and also at angles, in addition to which every fifth or sixth flexible tower may be head-guyed in both directions. In the writer's opinion, too much stress is usually laid on the necessity for providing rigid strain towers at frequent intervals to prevent the effect of a break in the wires, or the failure of a single support travelling along the line and causing injury to an indefinite number of consecutive spans. The semi-flexible structures referred to are not designed, or should not be designed, without very careful consideration of the conditions they have to fulfil; and there appear to be no scientific reasons, and no records of injury to actual lines, which would justify the assumption that transmission lines of this type are liable to be wrecked in the same wholesale or cumulative manner

as a row of card houses. The strain towers are undoubtedly helpful at the time when the wires are strung; but it is possible that they are used at more frequent intervals than the economies of sound engineering require.

In level country, a modified clamp in the form of a sleeve with flared ends may be used in conjunction with the lighter (and cheaper) flexible type of supporting structures; and a compromise between the loose sleeve and the rigid clamp or tie can be used on all lines with flexible supports. Clamps of this type are designed to allow the wire to slip before the combined pull of all the wires exceeds the load that will permanently deform the supporting structure; and although it is almost impossible to ensure that such devices will remain for any length of time in the same condition as when they are installed, yet they will generally afford a reasonable degree of protection in the event of the simultaneous breaking of all the wires in one span. It is not unusual to carry a galvanized Siemens-Martin steel strand cable above the high-tension conductors on the tops of the steel structures. This has the double advantage of securely, but not rigidly, tying together the supports, and of providing considerable protection against the effects of lightning. The disadvantages are increased cost and possible—but not probable—danger of the grounded wire falling on to the conductors and causing interruption of supply.

The dead-end towers should be capable of withstanding the combined pull of all the wires on one side only, when these are loaded to the expected maximum limit, without the foundations yielding or the structure being stressed beyond the elastic limit. The flexible supports must withstand, with a reasonable factor of safety, the dead weight of conductors, etc., and the expected maximum side pressures; but in the direction of the line their strength must necessarily be small, otherwise the condition of flexibility cannot be satisfied.

It is easy to design braced A-frame or H-frame steel structures of sufficient strength to withstand the dead load and lateral pressure and yet have great flexibility, with correspondingly reduced strength, in the direction of the line. Great care must be used in designing a line of this type so that strength and durability shall not be sacrificed to lightness and flexibility without very carefully considering the problem in all its aspects. As an approximate indication of present practice, it may be stated that

a load of from one-twentieth to one-tenth of the total load for which the rigid-strain towers are designed should not stress the intermediate flexible structures beyond the elastic limit. It is well to bear in mind that at the moment of rupture of one or more wires on a "flexible" transmission line the resulting stresses in the structures and remaining wires will be in the nature of waves or surges until the new condition of equilibrium is attained, and the maximum stresses immediately following a rupture will generally exceed the final value. The advocates of rigid tower construction contend that the so-called flexible type of support, and even the want of rigidity introduced by the use of suspension insulators, are liable to cause trouble when used on steep grades; the argument being that there is a tendency to transfer the stresses to the supports on the hill-top tower. The writer has been unable to understand the line of reasoning. In the first place, the dead weight of the wire is a small percentage of the total stress; but there is no necessity for the highest tower to take more than its proper share even of this vertical load. Provided the footings of the intermediate supports on the hill-side are good, there appears no reason why these supports should not take their share of the dead weight, even if they are entirely free to move horizontally in the direction of the line at the points of attachment to the wires. In order that the dead weight per tower shall not be greater on a steep grade than on the level, the spans should be measured along the ground: that is to say, the horizontal spacing of supports may be $l \times \cos \alpha$, where α is the average angle which the line makes with the horizontal, and l is the length of span on level ground. So close a spacing of the supports on a steep grade would, however, generally be uneconomical; almost any type of supporting pole or structure is usually of ample strength to carry the maximum dead load.[1]

The mathematics required for the exact determination of stresses and deflections in a transmission line consisting of a series of flexible poles is of a very high order, even when many assumptions are made which practical conditions may not justify; but the limiting steady values of these stresses and deflections can be calculated in the manner described in Appendix D, and as the range between these limits will usually not be very great, the

[1] The reader who is interested in studying the mechanical aspects of lines on steep grades, is referred to Appendix E, where a simple method of treating these problems is explained.

probable maximum stresses under given conditions can be estimated with a reasonable degree of accuracy. The illustration Fig. 82 kindly supplied by Messrs. Archbold Brady and Co., shows a common form of "flexible" high voltage transmission line following a railway.

FIG. 82.—Flexible steel tower line.

114. Loads to be Resisted by Towers.—The maximum load which a tower must be designed to withstand will depend upon the number and size of wires to be carried and the estimated ice coating and wind velocity. Apart from the wind pressure on the structure itself, the loading in a direction transverse to the line will be equal to the resultant wind pressure on all the wires (which may or may not be ice coated, depending on the climate);

the effective length of each wire being the distance between supports.

In the direction of the line the forces are normally very nearly balanced, but in the event of one or more wires breaking, the unbalanced load may be considerable, and it is well to design the towers, if possible, so as to withstand the stresses imposed upon them if two-thirds of all the conductors in one span are severed. It must not be overlooked that if the wires break in one span only, the cross-arm, if pin type insulators are used, will be subjected to a twisting moment; and if the break in the wires is at one end only of the cross-arm, the whole tower is subjected to torsional strain.

The vertical or dead loads consist of the weight of the tower itself and the wires of one span, with possible increase in weight due to sleet or ice. The cross-arms must be of ample strength to take all vertical loads including weight of insulators, with a margin to cover the extra weight of one or more men working on the tower. Particulars of pressure exerted by wind were given in Article 85 of Chapter VII. A good average figure on which to base calculations is 30 lb. per sq. ft. of exposed surface of the tower and 15 lb. per sq. ft. of projected area of wires. Where ice coating is to be expected it will usually be sufficient to allow 10 lb. per sq. ft. of the projected area of ice-coated wires. The transverse loading is obviously dependent upon the length of span, which must be determined with due regard to economic considerations. In regard to the possibilities of unbalanced loads in the direction of the line the conditions are modified somewhat by the type of insulator used. With the suspension type, if all the wires in one span break, the whole of the suspension length is thrown into the adjoining span, and the reduced stress due to increased sag may be calculated by assuming that the span into which the extra length is thrown is fixed firmly at both ends. The actual reduction in stress will, however, generally be from 40 per cent. to 60 per cent. of that calculated in this manner, depending on length of span and length of the string of insulators. The sudden throwing into the span of this increment of length does not always act beneficially, because the momentary stresses due to the jerk may counteract the advantages gained by the ultimate reduction in tension.

115. Steel and Wood Supports Compared.—The life of a steel tower line depends somewhat on climatic conditions. In Great

Britain the dampness of the climate, together with the impurities in the atmosphere in the neighborhood of manufacturing and populous districts, render light steel structures less durable than in America (except, perhaps, on the Pacific coast, where special precautions are required to guard against rapid corrosion due to the prevalence of fogs and moisture). Not only has the iron work protected by paint to be repainted on the average every three years, but the spans must usually be short, as the private ownership of valuable property renders the construction of a straight transmission line with long equal spans almost impossible in the United Kingdom. These conditions are all in favor of the employment of selected and well creosoted wood poles, the life of which may be 30 years or more.

When making comparisons between wood and steel for transmission line supports, it is not only the matter of first cost that has to be considered. Steel structures have the advantage of being invulnerable to prairie and forest fires; moreover, owing to the longer spans rendered possible by the stronger and taller supports, there is less chance of stoppages owing to broken insulators, and less leakage loss over the surface of insulators. A fact that is often overlooked is that the size of conductor limits the practical length of span; for instance, with a small conductor such as a No. 4 B. & S., it would not be wise to have spans much above 250 or 300 ft. This suggests what is frequently found to be the case, namely, that the total cost of a line may be reduced by using a conductor of rather larger section than the electrical calcalculations would indicate as being necessary, because the stronger cable permits of a wider spacing of the supporting towers.

The normal length of span on steel tower lines usually lies between 400 and 600 ft., but very much longer spans can be used where the character of the country would render their use economical or where rivers have to be crossed. On the transmission system supplying Dunedin City, New Zealand, with electric energy at 35,000 volts, there is a span 1700 ft. long where the line crosses the ravine near the power station. The peculiarity of this span is the great difference in level between the two supports, the upper tower, which is a special steel structure, being 650 ft. above the lower tower.

If the temptation to use very light sections of structural steel is avoided, and if towers are regularly inspected and painted when necessary, their life should be 50 years or more. Galvanized

towers are usually not painted; but it is not safe to rely upon the thin coating of zinc to prevent corrosion for more than a few years at or near the ground level. A casing of concrete extending about 12 in. above ground level will afford protection; or the parts that are buried may be painted instead of being galvanized, and if the anchor stubs are made in two lengths, the upper length can at any time be replaced without taking down the tower or interrupting the service.

116. Design of Steel Towers.—Although details of design and the proportioning of parts are matters best left to the manufacturer, the general type of supporting structure to be used under given conditions should receive careful attention on the part of the transmission line engineer. The most economical design of tower to withstand the probable loads that it will be subject to, and to satisfy local conditions, including such considerations as transport and erection facilities, is a problem deserving close attention on the part of the engineer responsible for the design of the transmission line. A study of the probable loads to be resisted under the worst weather conditions will enable the designing engineer to specify certain test loads which will ensure that the finished structure will be strong enough to fulfil the practical requirements. The proper value of these test loads and their distribution or point of application should be determined only after mature consideration. The cost of a tower—apart from the height, which is a function of the length of span—is determined largely by the specifications of test loads. A specification calling for tests that are unnecessarily severe, is just as true an indication of incompetence on the part of the designing engineer as a specification giving test conditions that will result in a tower too weak for the actual requirements.

The calculation of stresses in the various members of so simple a structure as a transmission line tower is not a difficult matter, especially if graphical or semi-graphical methods are adopted. If the designing engineer will make sketches of two or three alternative designs likely to fulfil the required conditions, he should be able quickly to calculate the approximate value of the stresses in the chief members, and so obtain a rough idea of the relative weights and costs of alternative designs. The danger of leaving the problem entirely in the hands of the manufacturer is that the latter is always tempted to put forward a design of which he has perhaps made a specialty, and which may have given entire

satisfaction in practice without necessarily being the best type of structure for the purpose, or being entirely suitable for use under different conditions.

117. Stresses in Compression Members.—The failure of steel towers under excessive loads is usually due to buckling of one of the main leg angles in compression. There are many empirical formulas in use for determining the loads that struts or columns will withstand. Rankine's formula for determining the unit stress to be used in the calculations of compression members is,

Unit stress $= \dfrac{\text{Load}}{\text{Area of section}} =$ pounds per square inch of cross-section of compression member

$$= \frac{T}{1 + \dfrac{l^2}{Cr^2}} \qquad (8)$$

where T = maximum stress of steel in tension (or in *short* column in compression) expressed in pounds per square inch,

l = length in inches of unsupported portion of compression member,

r = least radius of gyration, in inches,

$= \sqrt{\dfrac{\text{moment of inertia}}{\text{area of section}}}$

C = a constant.

For mild steel as generally used in tower construction, with an ultimate tensile strength of about 60,000 lb. per square inch, a factor of safety of 3 (which is usual) would give 20,000 as a safe working stress. The value taken for T in formula (8) usually lies between 18,000 and 30,000. The figure decided upon appears to depend largely on the margin of safety included in the maximum load assumptions. For preliminary calculations the following numerical values are suggested,

$$T = 25{,}000$$
$$C = 18{,}000$$

The "straight line" formula, as suggested by Burr, is quite satisfactory provided the ratio $l \div r$ lies between 40 and 200; this last figure corresponds to a length of compression member not exceeding about twenty times the width of flange. This formula is

$$T_c = T - k(l/r) \qquad (9)$$

where T_c is the allowable stress in the compression member,

T is the unit stress in short columns,
k is a coefficient based on experimental results.

If the strut may be stressed to the elastic limit (about half the ultimate load), the formula can be written,

$$T_c = 26{,}000 - 90\ l/r \qquad (10)$$

For a factor of safety of 3 and a working stress not exceeding 20,000 lb., the formula becomes,

$$T_c = 20{,}000 - 70\ l/r \qquad (11)$$

It is recommended that l/r shall not exceed 100 for main members and 120 for lateral or secondary members. The fact that, for a given cross-sectional area, the *shape* of the section is an important factor in determining the stiffness and ultimate strength of the members in compression, suggests that, where lightness and economy of material are of great importance, the section of structural steel having the greatest moment of inertia per square inch of cross-section should be chosen. The standard sections of rolled angles or tees do not necessarily make the most satisfactory or economical struts or columns, and it is for this reason that tubular sections of steel are sometimes used in place of angle sections in transmission line towers.

As an example of the relative economy of the tubular form and other forms of section, when used as comparatively long struts, a steel tube 7 in. internal diameter, 1/8 in. thick, weighing 10 lb. per foot, will be as efficient in resisting compression as a steel angle 7 1/2 in. by 7 1/2 in. by 1/2 in. thick, weighing 25 lb. per foot, or as an I beam 8 in. by 6 in. by 1/2 in. thick, weighing 35 lb. per foot. So large a tube would not be required except in very high towers: a tube from 4 to 5 in. diameter would generally be large enough for the main members of a transmission line tower up to 100 ft. high. The illustration, Fig. 83, is from a drawing kindly supplied by Messrs. Stewarts and Lloyds, Limited, of Glasgow, Scotland; it represents a tower 146 feet high as supplied for a power transmission line in the South of England.

118. Outline of Usual Method for Calculating Stresses in Tower Members.—The illustration, Fig. 84, which is reproduced by kind permission of the Shawinigan Water and Power Co., and the Canadian Bridge Co., Limited, shows a typical square base galvanized

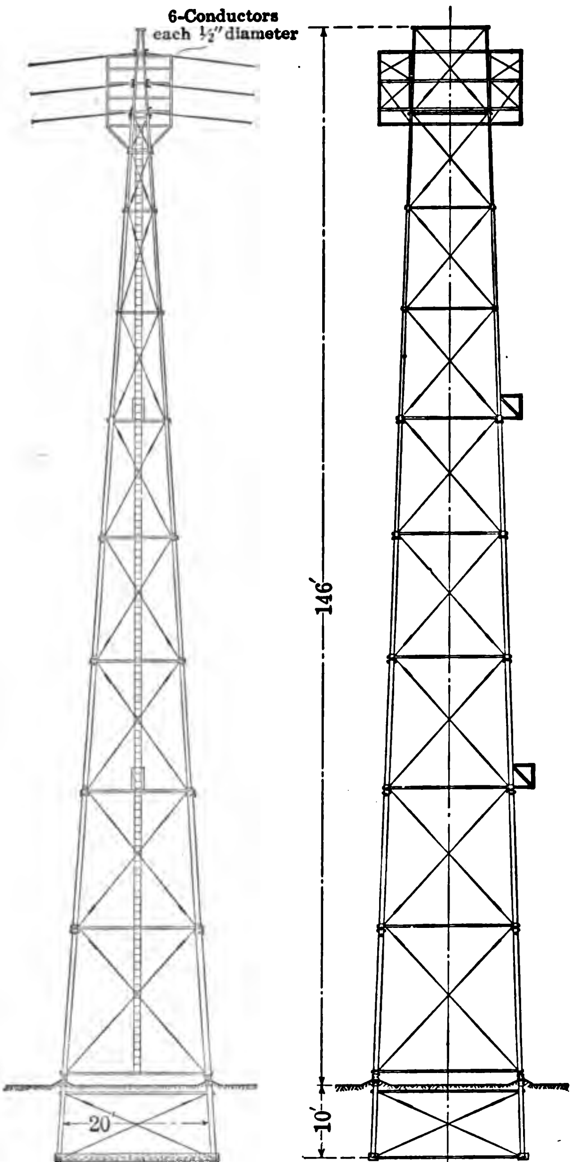

FIG. 83.—Steel tower with members of tubular section.

steel tower as used on the Three Rivers line of the Shawinigan Water and Power Co. of Montreal. These towers are designed to carry six aluminum conductors of nineteen strand 200,000 circular mil cable, each being supported by seven suspension disks of

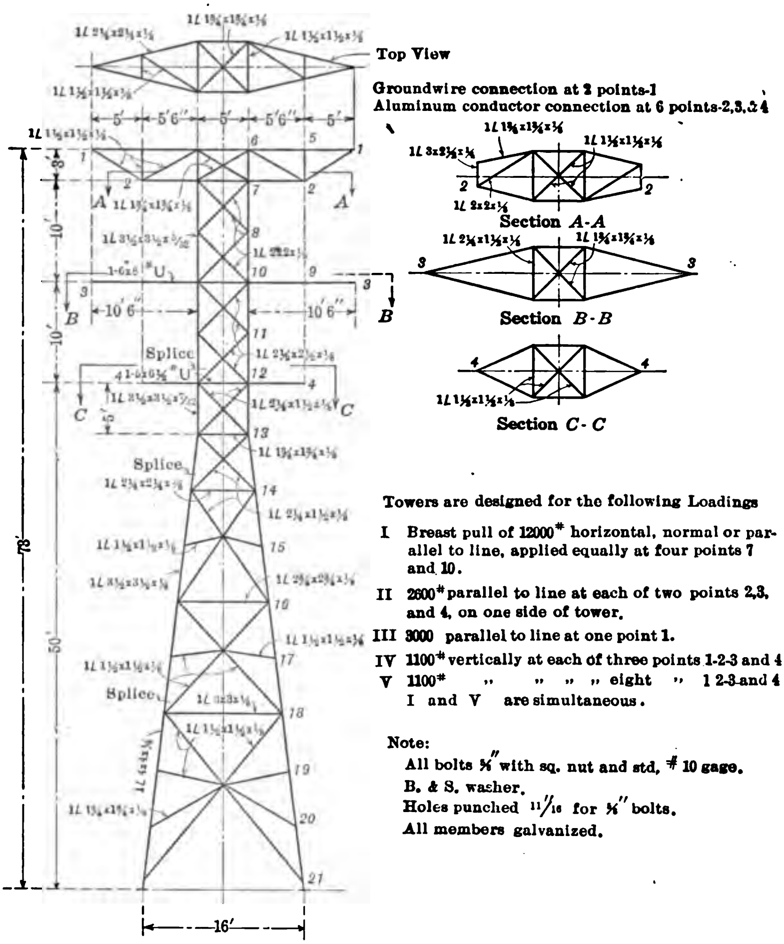

FIG. 84.—Steel tower with members of angle section.

the Ohio Brass Co.'s standard type. In addition to the conductors, there are two ground wires of 3/8 in. stranded Siemens Martin steel cable attached to the points (1) at each end of the upper cross arm. The line is built for 100,000 volts.

The method of procedure in calculating stresses is to make a

sketch showing the points of application, and the vertical and horizontal components, of the outer forces. Then indicate by arrows the assumed horizontal and vertical components of the reactions, using the suffixes *R* and *L* to indicate the direction or assumed direction of the horizontal components. Since the whole structure is in equilibrium under the influence of the various loads and reactions, it is merely necessary to see that the three following conditions are satisfied at any point considered:

(*a*) The sum of all vertical force components = zero.

(*b*) The sum of all horizontal force components = zero.

(*c*) The sum of all moments about any point = zero.

When taking moments in any particular plane, all those in a clockwise direction would be considered positive and those in a counter-clockwise direction negative. All joints are considered as frictionless pivots, which assumption is, of course, not strictly correct, especially in the case of riveted joints. It is usually an easy matter to choose a section through the structure in such a position that the stresses in a given bar can readily be calculated by applying one or more of the three equations of equilibrium.

The sketch, Fig. 85, will serve to illustrate the usual method of calculating the stresses in the main members of a tower structure such as the one shown in Fig. 84. The loading considered is that corresponding to the condition of test loads *I* and *V* applied simultaneously.

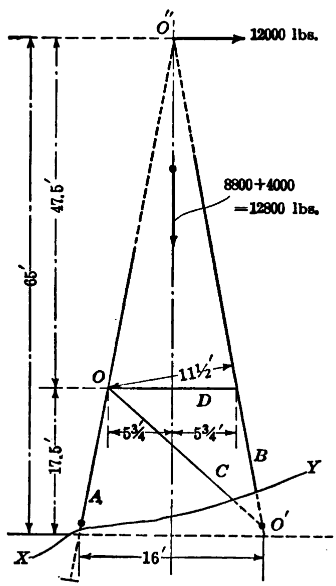

FIG. 85.—Sketch for calculation of stresses in tower members.

The point at which the horizontal breast pull of 12,000 lb. is applied corresponds approximately with the point 65 ft. above ground level where the corner legs would meet if produced beyond the points (13). The weight of the tower (which it is supposed has not yet been designed in detail) is taken at 4000 lb., and this, together with the test load *V*, gives a resultant vertical loading of 12,800 lb. applied somewhere on the center line of the tower.

Consider a section such as XY which cuts only three members, namely, the leg A at ground level, the leg B just above joint O', and the diagonal brace C.

Select a point O where the members A and C meet, and consider the moments, in the plane of the paper, which are produced round this point by the external forces and the reactions in the members severed by the imaginary section XY. It is obvious that the stresses in A and in C have no effect on the tendency of the part of the structure above the section line to rotate on the point O, and the whole of the externally applied turning moment must be resisted by the stress in the member B. Therefore

$$(12800 \times 5.75) + (12000 \times 47.5) - (x \times 11.5) = O$$

from which it is found that $x = 56{,}000$ lb.

Since there are two members B taking the whole crushing stress, the total load tending to crush the one member B is 28,000 lb. The length of the unsupported portion of this member is 5.5 ft. or 66 in. The cross-section of 4 in. $\times 4$ in. $\times 1/4$ in. angle is 1.93 sq. in.: and the least radius of gyration, $r = 0.79$. The test load should not strain the tower beyond the elastic limit.

Using the formula (3), the allowable stress is,

$$\begin{aligned} T_c &= 26{,}000 - 90 \times 66 \div 0.79 \\ &= 18{,}500 \end{aligned}$$

This corner member is therefore capable of supporting without permanent deformation a compressive load of $1.93 \times 18{,}500 =$ 35,700 lb. It should be of ample strength to resist the test load of 28,000 lb.

Turning now to the uplifting force acting in the member A and tending to pull up the foundation, the center from which the moments are calculated is shifted to the point O' where the members C and B meet. The equation of moments is now,

$$\begin{aligned} &(12{,}000 \times 65) - (12{,}800 \times 8) - (x \times 16) = O \\ &x = 42{,}300 \end{aligned}$$

and the tension in one corner angle A is 21,150 lb.

The above example briefly describes what is known as the method of moments. It has been assumed that the side of tower considered lies in the same plane as the external forces: but the error introduced is practically negligible. If desired, it is an easy matter to make the necessary correction.

In calculating the stresses in a diagonal member such as C of Fig. 85, the moments would be taken round the point O'', which is the junction of the members A and B; but in that case the actual loads on cross arms and the wind pressure on the side of the tower would have to be taken into account and substituted for the concentrated test load of 12,000 lb. at the point O'' which does not produce any stress in the brace C so long as the corner angle A remains truly straight and exerts no lateral pressure at the point O. The method of moments can usually be applied for all sections of a tower structure if the imaginary dividing planes are properly placed. The counter members or ties that are not in tension under the conditions of load considered are usually assumed to be non-existent, *i.e.*, to serve no useful purpose as compression members.

119. Foundations.—The upward pull on the tower legs, which was found in the above example to amount to 21,150 lb., has to

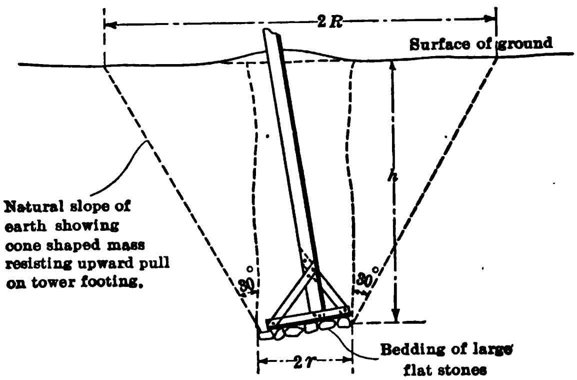

FIG. 86.—Foundation for steel tower anchor stub.

be resisted by the foundation. A factor of safety of two is usually allowed. The weight of concrete may be taken at 140 lb. per cubic foot, and of good earth at 100 lb., the volume of the earth to be lifted being calculated at the angle of repose, which may be about 30 or 33 degrees with the vertical, as indicated in Fig. 86. If the footing of a tower is in gravel, or a mixture of sand and loam tightly packed, there is actually a far greater resistance to the pulling up of the footings than that which is offered by the mere weight of the footings with prism of earth as calculated in the usual way.

When concrete has to be used, it is generally cheaper to reinforce it with steel of an inverted *T* form, as this makes a lighter construction than a solid block of concrete, and an equally good hold is obtained owing to the increased weight of the packed earth which has to be lifted. At the same time it must not be forgotten that the digging of a large hole 5 to 8 ft. deep is considerably more costly than the digging of a hole about 2 ft. square, and this extra cost in erection must be taken account of in designing the footings. In marshy or loose soil, or where the right of way is liable to be flooded, special attention should be paid to the design of durable foundations. Concrete footings with or without piles, or rock-filled crib work may be necessary; it is a matter requiring sound judgment and, preferably, previous experience on the part of the engineer in charge of construction. Crumbling hillsides are best avoided; it is extremely difficult to guard against damage by land slides or even snow slides when towers are erected on the steep slopes of hills.

The use of concrete adds considerably to the cost of foundations and it should be avoided if possible; on the other hand, it is not easy to design foundations to resist a given uplift without an exact knowledge of the soil conditions at the site of the tower. For the greatest economy of foundation, it is necessary that the designer obtain reliable information on this point.

Assuming an average angle of slope of 30 degrees, as indicated in Fig. 86, and a weight of soil of 100 lb. per cubic foot, the depth of foundation may be calculated as follows.

Let h = depth of footing below ground level,
r = equivalent radius of footing area,
R = radius at ground level of conical section of earth to be lifted.
θ = angle of natural slope of earth.

The volume of frustrum of cone to be lifted is,

$$V = \frac{\pi}{3}h \quad (r^2 + R^2 + rR) \tag{12}$$

or, if $r + h \tan\theta$ be put in the place of R,

$$V = \frac{\pi}{3}h \quad (3r^2 + h^2 \tan^2\theta + 3rh \tan\theta) \tag{13}$$

If $\theta = 30$ degrees, $\tan\theta = 0.5774$ and (approximately),

$$V = \pi h \quad (r^2 + 0.11h^2 + 0.58rh) \tag{14}$$

Try $r=1$ ft., and $h=7$ ft.; the volume of earth to be lifted, by formula (7) is then,

$V=230$, which gives,
$W=23{,}000$ lb.

As previously mentioned, if the soil is firm, this method of calculation usually gives results well below actual values of pull required to uplift the footing. Under the conditions upon which this example has been based, it is probable that the footing would not move with a pull appreciably smaller than 30,000 lb.; there would then be a packing of the soil immediately above the footing, and a final pull of about 40,000 lb. might be necessary to uproot the stub and footing.

If the footings are imbedded in concrete, it is well to let the ironwork project through the bottom of the concrete block, to ensure that the tower is properly grounded.

120. Stiffness of Steel Towers. Deflection Under Load.—The deflection of the top of a transmission-line tower of the ordinary light "windmill" type with wide square base, when bolted to rigid foundations and subjected to a horizontal load such as to stress the material to nearly the elastic limit, might be from 2 to 5 in. With regard to the two-legged or "flexible" type of tower, if this is of uniform cross-section, it can be treated as a beam fixed at one end and free at the other end. If the resultant pull can be considered as a single concentrated load of P lb. applied in a horizontal direction, at a point H inches above ground level, the deflection, in inches, will be,

$$\delta=\frac{1}{3}\frac{PH^3}{MI} \tag{15}$$

where M is the elastic modulus for steel (about 29,000,000; being the ratio of the stress in pounds per square inch to the extension per unit length), and I is the moment of inertia of the horizontal section of the structure.

121. Concluding Remarks Regarding Steel Tower Design.—Generally speaking, there is a tendency to economize in the cost of steel towers by using sections of structural steel in which stiffness is obtained by making the thickness of metal small in proportion to the other dimensions of the cross-section. It is true that light weight of parts and of the complete tower are important if the advantage of lightness can be obtained without sacrifice of other

advantages, the chief of which is durability. When a transmission line is not intended to last longer than 15 or 20 years, these light sections are allowable; but for the more important and costly lines, it is well to avoid the use of metal thinner than 1/4 in. for the main members, or than 3/16 in. for the secondary or bracing members. In the writer's opinion it is not wise to use 4″ by 4″ angles for the corner legs less than 5/16 in. thick, although a thickness of 1/4 in. is very common in towers actually in use at the present day. The ultimate life of such towers is, however, as yet unknown. Towers made of few pieces of comparatively heavy section steel will probably prove more durable than those built of a larger number of lighter parts.

The above remarks concerning thickness of metal are not intended to apply to structures of which the members are of tubular section. If carefully designed and painted when necessary, the metal in tubular members could be considerably thinner than would be permissible with the usual sections of structural steel. In fact it would appear that the slenderness ratio l/r which is used in determining the allowable stress in columns or struts, is not entirely reliable in practice. A symmetrical section such as a tube, or I-beam, would seem to be more satisfactory than the more common angle section as used in the corner legs of towers.

When considering designs of towers for a long transmission line, cases will occasionally arise, requiring special treatment. It is well to avoid if possible a number of different designs, and where the height does not require to be increased, it may sometimes be found more economical to use two standard towers close together for supporting special long spans, or for turning sharp corners, than to design special towers for the purpose. An angle not exceeding 7 degrees can usually be turned on a standard tower. This angle may even be as great as 10 degrees, especially if the length of the approach spans is decreased. In fact by reducing the length of approach spans, very much sharper angles can be turned; but it then becomes a question whether or not a special structure might not be the cheaper alternative.

There is an unexplained prejudice against the guying of steel towers where extra strength to resist lateral loads is required. By giving proper attention to the method of guying, and inspecting the line at regular intervals, there is no apparent reason why this fairly obvious device to save the extra cost of special structures should not prove entirely satisfactory: it cannot, however, be ex-

pected to find favor with those whose particular business it is to make and sell transmission-line towers.

A brief specification for a complete transmission line using steel towers is given in Appendix G. This line is generally similar to the one for which an estimate of cost was given in Chapter III.

122. Determining Position of Supports on Uneven Ground.—The lowest point of the span is not necessarily the point at which the wires come closest to the ground. When there is doubt as

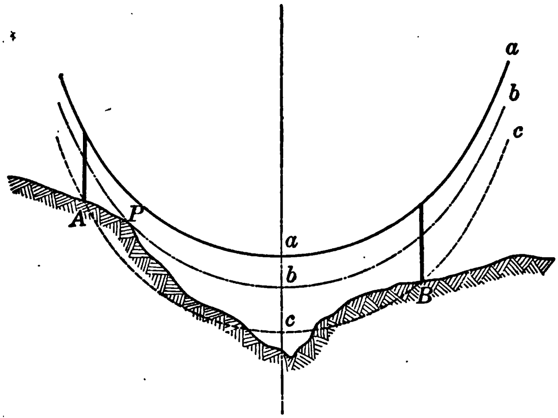

FIG. 87.—Method of locating position of towers in rough country.

to the proper location of the supports in rough country, the method illustrated in Fig. 87, and described by Mr. J. S. Viehe in the *Electrical World* of June 15, 1911, will be found very convenient. The curve a is the parabola corresponding to the required tension in the particular wire to be used. The ratio of the scale of feet for vertical measurements to the scale for horizontal measurements should be about 10 to 1. The dotted curves b and c are exactly similar to a, but the vertical distance $a\ b$ represents the minimum allowable clearance between conductor and ground, while the vertical distance $a\ c$ is the height above ground level of the point of attachment of the lowest wires to the standard transmission pole or tower. These curves should be drawn on transparent paper: they can then be moved about over a profile of the ground to be spanned, drawn to the same scale as the curves, until the best location for the supports is found. The point P where the curve

b touches the ground line is seen to be far removed from the lowest point of the parabola, in the example illustrated in Fig. 87. A little practice will make the finding of the points *A* and *B* an easy matter, even if the length of span, or distance between *A* and *B*, must be kept between close limits.

This method is particularly applicable to long-span lines carried over rough country.

APPENDIX A

INDUCTANCE OF ELECTRIC TRANSMISSION LINES WITH UNSYMMETRICALLY DISPOSED CONDUCTORS

The manner in which the inductance and the induced e.m.f. can be calculated when the conductors of a three-phase system occupy the vertices of an equilateral triangle, was explained in Chapter II; and it was also stated that a departure from the symmetrical arrangement of conductors does not modify the calculated results to any great extent. It will be interesting to study the problem in its broader aspect, with a view to ascertaining what is the nature and magnitude of the modifying factors. It is proposed to indicate a simple method of calculating the total induced e.m.f. in any conductor of an electric-energy transmission system, whatever may be the actual arrangement or relative positions of the conductors. It is assumed in all cases that the conductors are of circular section and that they remain parallel with each other throughout the whole distance of transmission.

1. Flux Surrounding a Straight Conductor.—The magnetic lines of induction surrounding a long straight wire carrying an electric current, of which the return path is at a considerable distance, will be in the form of circles concentric with the conductor. The path of the magnetic lines will, therefore, increase in length in direct proportion to the distance from the conductor; and the intensity of the magnetic field—usually denoted by the letter **H**—will vary inversely as the distance (from the center of the conductor) of the point considered. This relation was first experimentally proved by Biot and Savart, the formula being:

$$\mathbf{H}=\frac{2I}{l} \tag{1}$$

where l is the distance of the point considered from the center of the conductor, all quantities being in c.g.s. units. The total flux of self-induction surrounding unit length (1 cm.) of straight

conductor, carrying a current I, up to a limiting distance of D cm., when the diameter of the conductor is $2\ r$ (see Fig. 1) is

$$\mathsf{N} = \int_r^D \frac{2I}{l} dl$$
$$= 2I\ (log_e D - log_e r)$$
$$= 2I\ log_e \frac{D}{r} \qquad (2)$$

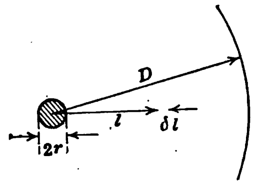

FIG. 1.—Magnetic flux round straight conductor.

2. Effect of Taking into Account the Return Conductor.—The effective flux surrounding any single conductor of a transmission system will depend upon the distance of the parallel return conductor or conductors.

Consider, first, the loop formed by two parallel conductors of circular cross-section, one carrying the outgoing current I and

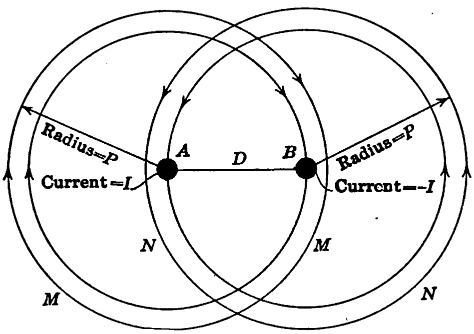

FIG. 2.—Magnetic lines of force round two parallel conductors.

the other carrying the return current $-I$ (see Fig. 2). The flux due to the current I in the conductor A may be considered as extending indefinitely throughout space, with ever-weakening intensity as the distance from the conductor increases, and the same argument applies to the flux surrounding the return con-

ductor B, the only difference being that, if the direction of the flux round A be considered positive, that which surrounds B will be in a negative direction. It follows that the whole of the magnetic flux due to the current I in A, which is situated at a distance greater than the distance D between centers of the outgoing and return conductors, is exactly neutralized by the flux due to the current $-I$ in B. Thus, in Fig. 2, it will be seen that the flux of induction surrounding A up to a distance D, is not neutralized by the current $-I$ in the conductor B; but any magnetic line, such as M, situated at a greater distance, P, from the center of the conductor A, is exactly neutralized by the magnetic line N, due to the return current in conductor B, since it also surrounds the conductor A, but in an opposite direction to the line M. It follows that the total *effective flux* surrounding A—that is, the resultant flux which will give rise to an induced e.m.f. in the conductor when carrying an alternating current—is merely that portion of the total self-produced flux included between the surface of the conductor[1] and the surface of an imaginary cylinder, concentric with the conductor, and of radius D, equal to the distance between the centers of the outgoing and return conductors. The formula (2) can, therefore, be used for calculating the flux which is effective in producing an e.m.f. of self-induction in a straight conductor when the whole of the return current is situated at a distance D from the center of the conductor.[2]

Calculation of the Total Flux When there are Several Return Conductors

In Fig. 3 the total outgoing current I is supposed to flow along one conductor, while the total return current is divided up between a number of conductors, the condition being that

$$I = -(I_1 + I_2 + I_3 \; . \; . \; . \; . \; . \; + I_n)$$

Let D_1, D_2, D_2, etc., represent the distances between centers of the corresponding conductors carrying the return currents and the conductor carrying the outgoing current, and note that the

[1] The induction *within* the material of the conductor will, for the sake of simplicity, be disregarded.

[2] The intensity of the magnetic field at any point outside a long, straight cylindrical conductor can always be calculated on the assumption that the whole of the current is concentrated along the axis of the conductor.

total flux surrounding the latter conductor may be considered as the algebraic sum of several separate fluxes, namely, the flux due to a current I_1 returning at a distance D_1; the flux due to a current I_2 returning at a distance D_2, and so on, for any number of components of the total current I. All these separate components of the total flux can readily be calculated by means of formula (2), and the expression for the total flux due to the current I returning along a number of separate conductors, as indicated in Fig. 3, becomes:

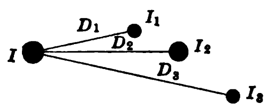

FIG. 3.—Section through four parallel conductors.

$$N = 2\times\left[-I_1\log_e\frac{D_1}{R} - I_2\log_e\frac{D_2}{R} - \ldots . I_n\log_e\frac{D_n}{R}\right] \qquad (3)$$

Where R stands for the radius of cross section of the outgoing conductor.

Practical Application—Polyphase Transmission

In the case of energy transmission by polyphase currents, with any number of conductors, the algebraic sum of the currents in the conductors must, at any given instant, be equal to zero. Any one conductor can be looked upon as carrying the outgoing current, while the remaining conductors together carry the return current. Formula (3) can, therefore, be used for calculating the effective flux of induction surrounding any one conductor in a polyphase transmission, whatever may be the arrangement of the conductors. The phase relations of the various component fluxes must, however, be taken into account, and for this reason the graphical addition of vector quantities with the help of a diagram will be found most convenient. Instead of drawing the vectors representing magnetic flux components—in phase with the current vectors—the component vectors of the resulting e.m.f. of self-induction may be drawn—in this case 90 time-degrees behind the corresponding current vectors.

In order to calculate the induced e.m.f. it will be advisable first to put equation (3) in a more practical form. The symbols I_1, I_2, etc., in equation (3), when the latter is to be used for calculating the maximum value of the induction due to an alternating current, must be considered as representing the maximum

value of the current wave expressed in absolute c.g.s. units. But

$$\text{C.g.s. unit of current} = \frac{\text{amperes}}{10}$$

$$\left.\begin{array}{l}\text{Maximum value of current on}\\ \text{the assumption of sine curve}\\ \text{wave form}\end{array}\right\} = \sqrt{2} \times \sqrt{\text{mean square}}\ \text{value.}$$

Hyperbolic logs = common logs × 2.3026, and the multiplier in front of the brackets in formula (3) becomes.

$$\frac{2 \times \sqrt{2} \times 2.3026}{10}$$
$$= 0.6512$$

If common logs are to be used, and the current (on the sine wave assumption) is to be expressed in amperes; then if N is to represent the total flux *per mile* of conductor instead of per centimeter:

$$\mathsf{N} = 104800 \times \left[-I_1 \log_{10} \frac{D_1}{R} - I_2 \log_{10} \frac{D_2}{R} \dots - I_n \log_{10} \frac{D_n}{R} \right] \qquad (4)$$

where the currents I_1, I_2, etc., in the various conductors are the amperes as indicated by an ammeter. In calculating the induced e.m.f. it must be remembered that the total effective magnetic flux surrounding the conductor is twice created and twice destroyed during one complete period. It follows that if N is the maximum value of the flux and f the frequency, the *mean* value of the induced volts will be

$$e_m = \frac{4\mathsf{N}f}{10^8}$$

and, on the sine wave assumption,

$$\sqrt{\textit{mean square value}} = \frac{\pi}{2\sqrt{2}}\ \textit{mean value, and,}$$

$$\text{Induced volts} = e = \frac{2\pi \mathsf{N} f}{\sqrt{2} \times 10^8} \qquad (5)$$

Inserting for N the value as given by formula (4), the formula for the induced volts per mile of conductor becomes:

$$e = 0.004656 \times f \times \left[-I_1 \log \frac{D_1}{R} - I_2 \log \frac{D_2}{R} \dots - I_n \log \frac{D_n}{R} \right] \qquad (6)$$

3. Numerical Example.—Consider the special case, which frequently arises in practice, of the conductors of a three-phase transmission being arranged as indicated in Fig. 4—that is, with the centers of the three conductors lying in the same plane, the minimum distance, D, between any two of the wires being approximately equal to the side of the equilateral triangle which would have been adopted had the triangular arrangement been decided upon.

In a three-phase transmission system the current flowing out through any one wire may, as previously mentioned, be considered as returning along the two remaining wires, and when the three conductors occupy the vertices of an equilateral triangle the whole of the return current is at a distance D from the outgoing current. This condition also applies to the middle conductor (No. 2) in the arrangement shown in Fig. 4; but it does not apply to either of the outside conductors, Nos. 1 and 3. In the case of conductor No. 1 a part of the outgoing current returns along conductor No. 2 at a distance D, while the remainder returns along conductor No. 3 at a distance $2\ D$; so that the total flux of induction surrounding conductor No. 1 must necessarily be greater than that surrounding conductor No. 2. The same argument applies to conductor No. 3.

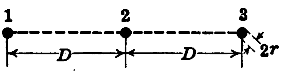

FIG. 4.—Three conductors in one plane.

Applying formula (6) to the arrangement of conductors, as shown in Fig. 1, the quantity between brackets in the case of conductor No. 1 becomes:

$$-I_2 \log\frac{D}{r} - I_3 \log\frac{2D}{r}$$

$$= -(I_2+I_3)\log\frac{D}{r} - I_3 \log 2$$

$$= I_1 \log\frac{D}{r} - I_3 \log 2$$

The total induced e.m.f. per mile of condutor No 1. will therefore be:

$$E_1 = 0.004656 \times f \times \left[I_1 \log\frac{D}{r} - I_3 \log 2 \right] \qquad (7)$$

Similarly, for conductor No. 3:

$$E_3 = 0.004656 \times f \times \left[I_3 \log\frac{D}{r} - I_1 \log 2 \right] \qquad (8)$$

while the volts induced in the middle conductor (No. 2) will be simply:

$$E_2 = 0.004656 \times f \times I_2 \log \frac{D}{r} \tag{9}$$

It is interesting to note that what may be referred to as the disturbing element in the case of the two outside wires (the quantities $I_3 \log 2$ and $I_1 \log 2$ respectively) is not dependent upon the actual diameter or distance apart of the conductors. It consists of an e.m.f. component either 30 time-degrees or 150 time-degrees behind the phase of the line current, depending upon the order of the phase rotation; and the magnitude of this e.m.f. component relatively to the total e.m.f. of self-induction will depend upon the value of the ratio $\frac{D}{r}$. If D is large and r relatively small, as in the case of a high-pressure overhead transmission system, then the first quantity between brackets, in equations (7) and (8), is relatively large, and the disturbing element ($I_3 \log 2$ or $I_1 \log 2$) is usually negligible. On the other hand, if the conductors consist of three separate single cables, laid side by side in a trench, with the distance, D, between them small in comparison with the diameter, $2r$, of the cables, then the "disturbing element" becomes of greater importance relatively to the total self-induction or induced e.m.f.

In order to form some idea as to the magnitude of this out-of-balance component of the induction, it will be well to work out two numerical examples, one for a high-tension overhead scheme and the other for a low-tension transmission system with the three conductors in comparatively close proximity.

Example 1.—Assumed data: Three-phase power transmitted $= 20{,}000$ kw.; e.m.f. $= 110{,}000$ volts; power-factor $= 0.8$; frequency $(f) = 25$ cycles per second; length of line $= 200$ miles. Conductors of aluminum; diameter, $2r = 0.6$ in. Minimum distance between wires, $D = 10$ ft. $= 120$ in. On the above data the current per conductor is about 130 amp. With the aid of formulas (7), (8) and (9) it is an easy matter to determine the induced e.m.fs. in the several conductors, and since the quantity, $\log \frac{D}{r} = \log \frac{120}{0.3} = 2.6021$, while $\log 2 = 0.3010$, it will at once be seen that the "disturbing element" is relatively small.

The e.m.fs. induced in each conductor 200 miles long, in round figures, are as follows: In the middle conductor (No. 2), 800

volts, the time-phase of which is exactly one-quarter cycle behind the time-phase of the current I_2.

In conductor No. 1, an e.m.f. component of 800 volts, exactly a quarter cycle behind the current I_1 *less* another component (referred to as the disturbing element) equal to about 90 volts, the phase of which is exactly one-quarter cycle behind the current I_3. The resultant is the difference between two *vector* quantities separated by a time-phase angle of 120 deg., so that this resultant is actually *greater* than either of the two components, as will be shown hereafter.

In conductor No. 3 there will be an e.m.f. component of 800 volts, one-quarter cycle behind the current I_3, and a component of 90 volts, one-quarter cycle behind I_1.

Example 2.—Assumed data: Three-phase power transmitted $=20$ kw.; e.m.f. $=110$ volts; power-factor $=0.8$; frequency $(f)=$ 60 cycles per second; current per wire $=130$ amp; distance of transmission $=1/2$ mile; three single cables in trench, lying in the same plane with a distance between centers $D=3$ in.; diameter over copper $=2r=0.5$ in.

In this example the quantity $\log \frac{D}{r}$

$$= \log \frac{3}{0.25}$$
$$= 1.0792.$$

The ratio between log 2 and this number is $\frac{0.3010}{1.0792}=0.28$. That is to say, the component of the total induced e.m.f., which appears only in the two outside conductors, as indicated by formulas (7) and (8) is, in this example, numerically greater than a quarter of the more important component; while in the previous example of a high-tension overhead transmission system the ratio was $\frac{0.3010}{2.6021}=0.115$, being considerably smaller because of the greater distance between the wires.

Vector Diagram Illustrating Example 2.—The vectors I_1, I_2 and I_3 in Fig. 5 represent the currents in the three conductors, the time-phase angle between them being 120 deg. The rotation of the phases is assumed to be in the order I_1, I_2, I_3; in other words, I_2 lags behind I_1 by one-third of a cycle, and I_3 lags behind I_2 also by one-third of a cycle. The lengths of these vectors are such as to represent the line current of 130 amp.; but,

as the diagram has been drawn to illustrate the phase angles and magnitudes of the various components of the induced e.m.fs, the magnitude of the current vectors need not be considered. If the numerical values of the induced volts are determined with the aid of formulas (7), (8) and (9), it will be found that the component common to all three conductors amounts to approximately 19.5 volts, while the "disturbing element"—that is, the component appearing in the two outer conductors only—amounts to 5.5 volts.

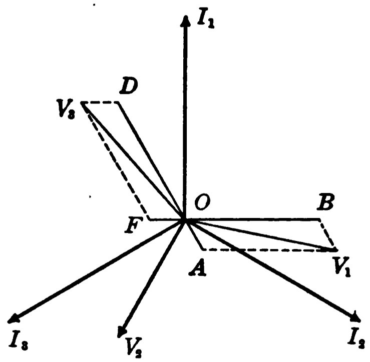

FIG. 5.—Vector diagram. Three conductors in same plane.

The vectors $O\ B$, $O\ V_2$ and $O\ D$ must, therefore, be drawn of such a length as to represent 19.5 volts in a direction exactly 90 time-degrees behind the corresponding current vectors; and, so far as the middle conductor is concerned, the vector $O\ V_2$ will represent the whole of the induced e.m.f.; but in the case of conductor No. 1 (carrying current I_1), $O\ A$ must be drawn exactly 90 time-degrees *in advance* of $O\ I_3$—that is, exactly *opposite* to $O\ D$, because of the negative sign in equation (7)—and of such a length as to represent 5.5 volts. By combining $O\ A$ with $O\ B$ in the usual way, $O\ V_1$ is obtained as representing the total e.m.f. induced in conductor No. 1. In a similar manner $O\ V_3$ is obtained for the total induced e.m.f. in conductor No. 3. It is interesting to note that $O\ V_1$ lags behind the current I_1 by a time interval *greater* than a quarter period, while the lag of the induced volts V_3 behind the current I_3 is *less* than a quarter period.

In the particular example under consideration the numerical

value of V_1 or V_3 as scaled off from the diagram is 22.75 volts. V_2 being 19.5 volts.

It is not difficult to understand why the magnitude and phase relations of the induced e.m.fs. in the various conductors of a polyphase transmission are not the same for an unsymmetrical arrangement of conductors as for an arrangement in which each conductor is similarly placed in relation to all the other conductors. With an unsymmetrical arrangement, the unbalancing effect may be said to be due to the mutual induction between the loops formed by different pairs of wires; there may, in fact, be a transfer of energy between one loop and another just as in the case of the primary and secondary windings of a transformer.

Effect of Transposing the Conductors.—If each conductor of the arrangement referred to in the above example is made to

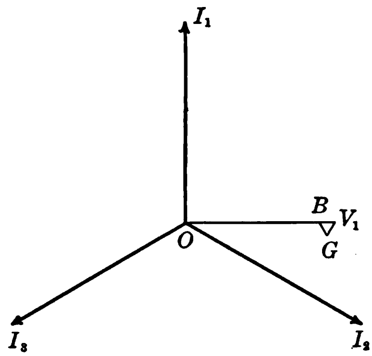

FIG. 6.—Vector diagram illustrating effect of transposing unequally spaced conductors.

occupy, in turn, the position midway between the remaining two conductors for a distance equal to one-third of the total distance of transmission, it is obvious that the out-of-balance effect will be corrected. It will, however, be of interest to ascertain what will be the numerical value of the (equal) voltages induced in the three conductors if transposed in the manner suggested. It is not necessary to consider more than one of the conductors, and, in Fig. 6, OB represents (as in Fig. 5) that portion of the e.m.f. induced in conductor No. 1 which remains unaltered whether the conductor be midway between the other two, or be itself one of the outer conductors. The length of this vector will, therefore, be such as to represent 19.5 volts. Now, when the

arrangement of the conductors is in the order 1, 2, 3, (as in Fig. 4), the "disturbing element" will be $B\ G$, drawn 90 degrees in *advance* of $O\ I_3$, exactly as $O\ A$ (or $B\ V_1$) in Fig. 5; but the *length* of this vector, instead of being equivalent to 5.5. volts, will be only one-third of this value, or 1.83 volts, because conductor No. 1 occupies this position over only one-third of the total distance of transmission. When the arrangement of the conductors is 1, 3, 2, the "disturbing element" will be $G\ V_1$ (Fig. 6), drawn 90 degrees in advance of $O\ I_2$. Clearly $B\ G\ V_1$ is an equilateral triangle, and the resultant of the induced e.m.f. in conductor No. 1 is $O\ V_1$, drawn 90 time-degrees behind the current vector $O\ I_1$ and equal in magnitude to the algebraic sum of $O\ B = 19.5$ volts and $B\ V_1$ = one-third of 5.5. volts.

If, therefore, the wires of a transmission line are disposed in one plane, as indicated in Fig. 4, but transposed at intervals so that each wire shall occupy the middle position over a space equal to one-third of the distance of transmission, then the resultant induced e.m.f. per conductor will, so far as phase is concerned, lag behind the current by a quarter period, exactly as if the wires occupied the vertices of an equilateral triangle; but the amount of the induced volts will be somewhat greater than in the latter caes under otherwise similar conditions.

The numerical value of the induced volts per conductor—that is, the length of the vector $O\ V_1$ in Fig. 6—can be calculated by the formula:

$$E = 0.004656 \times f \times \left(I \log \frac{D}{r} + \frac{I \log 2}{3} \right) \qquad (5)$$

where I is the current in any one conductor, and the two quantities between brackets have merely to be added algebraically. If preferred the quantity between brackets can be written:

$$I \log \left(\frac{D}{r} \times \sqrt[3]{2} \right)$$

or $I \log \left(1.26\ \frac{D}{r} \right)$ so that formula (5) appears in the form:

$$E = 0.004656 \times f \times I \times \log \frac{1.26 D}{r} \qquad (6)$$

APPENDIX B

Inductance of Electric Transmission Lines as Affected by the Subdivision of the Circuits and the Arrangement of the Conductors

There are reasons in favor of transmitting large amounts of electric power through two or more sets of wires, quite distinct from mechanical considerations or the increased security against a total shut-down in the event of accidents. The inductive drop of pressure may be reduced by substituting, for a single set of transmission lines, two or more sets of suitably arranged lines of a correspondingly reduced cross-sectional area. As to whether or not the subdivision of a transmission line into two or more parallel circuits would be justifiable in practice will depend upon economic and other considerations which it is not proposed to touch upon here.

Single-phase Systems

In Fig. 1 the two conductors of a single-phase transmission are shown, with distance D between centers of wires. The current may be considered as going out through the conductor 1 and returning through conductor 2. The diameter of the wire is assumed to be $2r$ and the current 1 amp.

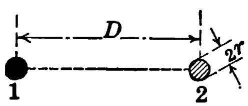

Fig. 1.—Two parallel conductors.

The formula which gives the induced volts per mile of single conductor when the whole of the current may be considered as returning at a distance D from the center of the outgoing conductor is

$$e = 0.004656 \times f \times I \times \log \frac{D}{r} \tag{1}$$

on the sine wave assumption.

For the purpose of comparing different arrangements of circuits, the frequency may be assumed constant in all cases, and if m be put for the quantity $0.004656f$, the formula can be written,

$$e = m\, I \log \frac{D}{r} \tag{2}$$

This formula alone is sufficient to indicate that an improvement in the matter of inductive voltage drop is to be expected if, instead of transmitting the total current I through one pair of conductors, there be provided two or more pairs of conductors spaced sufficiently far apart to prevent mutual inductive effects, each pair being of sufficient cross-section to carry one-half or one-third of the total current, as the case may be; because, although the quantity $\log \frac{D}{r}$ will increase slightly on account of the reduction in the dimension r, this increase will not be of nearly so much importance as the reduction of I.

Numerical Example.—In order to illustrate the above point a few examples will be worked out based on the following assumed data:

Total current, $I=100$ amp.

Diameter of single conductor to transmit the total current, $2r=0.5$ in.

Frequency, $f=60$ cycles, from which $m=0.2793$.

Distance between centers of wires (corresponding to a pressure of about 50,000 volts), $D=70$ in.

If the transmission line is divided into two equal sections, the current in each section will be 50 amp., and for equal total weight of copper (leading to the same ohmic drop of pressure), the radius of each conductor will be $r\div\sqrt{2}$. Similarly, if there are three equal sections, the current will be 33.33 amp., and the radius of the conductors $r\div\sqrt{3}$.

The induced volts as given by formula (2) work out as folows for the three conditions:

Single pair of lines

$$e=68.34 \text{ volts} \tag{3}$$

Two pair of lines of equal total cross-section,

$$e=36.25 \tag{4}$$

Three pair of lines of equal total cross-section,

$$e=25.00 \tag{5}$$

These figures show that the inductive drop of pressure on a single-phase transmission may be reduced by splitting up the current and transmitting along two or more pairs of lines spaced sufficiently far apart to prevent appreciable magnetic interference between the sets of lines; and the reduction of the inductive

drop is very nearly in proportion to the number of subdivisions of the single line.

Although electric transmission systems have been arranged with two distinct sets of conductors run upon separate pole lines spaced sufficiently far apart to avoid magnetic interference, such an arrangement is necessarily costly. Consider, therefore, two alternative arrangements, shown in Figs. 2 and 3, by which a single circuit can be split up into two parallel circuits, the four wires being carried on the one set of poles with the spacing between the individual wires as small as possible—that is, such that in no case shall the distance D between outgoing and return conductors be less than the minimum determined by the voltage of the supply.

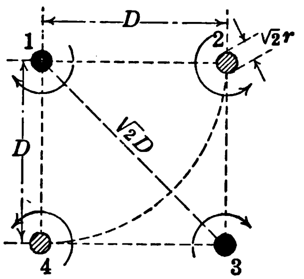

FIG. 2.

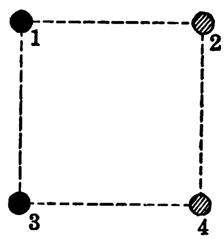

FIG. 3.

FIGS. 2 and 3.—Alternative arrangement of conductors. Single-phase transmission.

In Fig. 2 is shown a symmetrical arrangement with the four conductors of equal cross-section occupying the corners of a square; the outgoing conductors are marked 1 and 3, and the return conductors, 2 and 4. Even if the two circuits 1–2 and 3–4 are connected in parallel at both ends of the line, the symmetry of the arrangement will insure that the total current will divide itself equally between the two sets of conductors. The effective or resultant magnetic flux surrounding any one conductor will, for the same reason, be equal to that which surrounds any one of the remaining three conductors. It will, therefore, suffice to calculate the e.m.f. of self-induction generated in any one conductor.

Consider the conductor 1, in which there is the current $\frac{I}{2}$. If the other outgoing conductor, 3, were situated anywhere on

the dotted circle of radius D, passing through 2 and 4, then the magnetic effect of the current in 3—so far as conductor 1 is concerned—would counteract the effect of the return current in either 2 or 4. On the basis of the data previously assumed, the flux round 1 would generate an e.m.f. of 36.25 volts, as in equation (4). If, on the other hand, conductor 3 were coincident with 1, there would be the condition of the full current I in the conductor 1, the whole of which would be returning at a distance D, and the induced volts would be 68.34, as given in equation (3). With the conductor 3 situated at a distance $\sqrt{2}D$ from conductor 1, as shown in Fig. 2, the resultant effective flux surrounding conductor 1 may be considered as the difference between the flux due to a current I up to a distance D less the flux due to a current $I/2$ up to a distance $\sqrt{2}D$; and this resultant flux would produce a back e.m.f.

$$e = m\,I \log \frac{D}{r \div \sqrt{2}} - m\frac{I}{2} \log \frac{\sqrt{2}D}{r \div \sqrt{2}} \tag{6}$$

On the data previously assumed, the e.m.f. is

$$e = 72.5 - 38.39 = 34.11 \text{ volts.} \tag{7}$$

Thus, by arranging the conductors of the divided circuit in the manner shown in Fig. 2, which permits of the four wires being supported on the one set of poles, a better result is obtained in regard to inductive voltage drop than if the two circuits had been run entirely separately; the voltage drop in this latter case being 36.25, as in equation (4).

If, on the other hand, the position of one pair of conductors be assumed to be reversed, as indicated in Fig. 3, then the magnetic flux in the loop formed by the outgoing and return conductors 2 and 3 has no effect on the conductors 1 and 4, and the effective flux surrounding any one conductor is clearly that due to a current $\frac{I}{2}$ returning at a distance $\sqrt{2}D$: the induced volts per conductor will be 38.39, this being the value of the second term in formula (6). With an arrangement of conductors, such as in Fig. 3, it is obvious that the conditions are worse than if the two circuits are quite distinct, because a portion of the flux produced by one pair of conductors, such as 3 and 4, passes also through the loop 1–2, thereby increasing the inductive drop in these wires.

Polyphase Systems

The satisfactory results obtained in regard to inductive drop when a single-phase circuit is split up into two circuits arranged as indicated in Fig. 2, suggest that a somewhat similar arrangement might be adopted with advantage in the case of polyphase transmissions. An arrangement of wires suitable for three-phase transmission is shown in Fig. 4. Here the three-phase line is supposed to be split up into two parallel three-phase circuits, 1, 2, 3 and 1′, 2′, 3′. The arrangement being symmetrical and all conductors being assumed to be of equal size, the same amount of current will be carried by each of the six conductors, provided the load is a balanced one, such as is usual in the case of a three-wire, three-phase scheme.

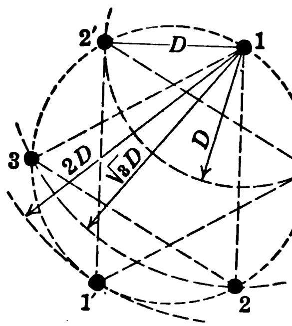

Fig. 4.—Arrangement of conductors—three-phase transmission.

With the arrangement of wires as in Fig. 4 the minimum distance D is maintained between all wires at different potentials, and the currents in conductors, such as 1 and 1′, placed at opposite ends of a diameter, will be of the same time-phase and equal in magnitude.

It will be interesting to work out a numerical example based on data already assumed in connection with the single-phase transmission, namely, a current of 100 amp. per phase and a minimum distance, D, of 70 in. between conductors at different potentials. The points to bear in mind are:

(1) That owing to the symmetrical arrangement of the conductors, with the rotation of the phases always in the same direction, the total effective magnetic flux round any one conductor is the same (except in regard to phase) as that which surrounds any

one of the other five conductors. The calculations can therefore be made for any one conductor, such as No. 1.

(2) That the current in any outgoing conductor, such as 1, may be considered as returning through the five remaining conductors, due attention being paid to phase relations.

(3) That the resultant of the currents in conductors 2′ and 3′, or the resultant of the currents in conductors 2 and 3, is equivalent to a current equal to that in conductor 1, but exactly opposite as regards phase. The total effective flux round conductor 1 may, therefore, be considered as the resultant of three component fluxes:

(a) A flux due to a current $\frac{I}{2}$ returning (through 2′–3′) at a distance D; *plus* (b) a flux due to a current $\frac{I}{2}$ returning (through 2–3) at a distance $\sqrt{3}D$; *less* (c) a flux due to a current $\frac{I}{2}$ returning (through 1′) at a distance $2D$.

The numerical values for the induced volts are found to be: $(a)=36.25$ [being the same as in equation (4)]; $(b)=39.47$; $(c)=40.49$; and $(a)+(b)-(c)=35.23$.

If two separate three-phase lines spaced a considerable distance apart were substituted for the arrangement in Fig. 4, the induced volts per mile per conductor would be as given in equation (4), namely, 36.25, assuming the triangular arrangement of wires, with distance D between them. The arrangement shown in Fig. 4 is therefore slightly better from the point of view of inductive drop, notwithstanding that both sets of wires can be run on the same pole line with no greater spacing between wires than the minimum distance D determined by the voltage between phases. The figure 35.23 volts for the split three-phase system may be compared with 34.11 volts as given in equation (7) relating to the single-phase transmission with two circuits. It is clear that in either example, the drop in volts per conductor in the undivided circuit, with each conductor of sufficient section to carry the total current of 100 amp., would be 68.34, as given by equation(3).

APPENDIX C

Two-phase Transmission with Three Conductors

When a common return conductor is used in connection with the transmission of two-phase currents, the phase relations at the distant end of the transmission are not the same as at the generating end. It is the resistance of the common conductor which is largely responsible for this upsetting of the phase angle, but owing to the calculations for a three-wire two-phase transmission being somewhat difficult, the manner in which the resistance and reactance (or inductance) of the lines affect the currents in the two phases is not very easily explained.

The reason why four wires are generally adopted for the transmission of two-phase currents will perhaps be better understood after an examination of the vector diagrams relating to a three-wire two-phase transmission. Such vector diagrams are by no

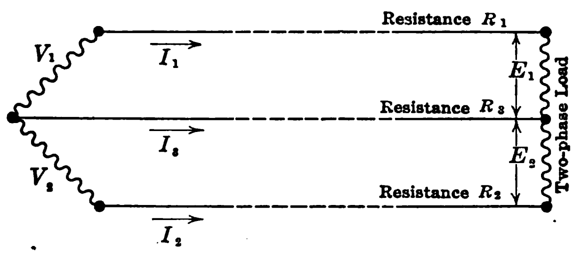

Fig. 1.—Two-phase transmission with three conductors.

means easy to draw if it is attempted to derive the current and voltage conditions at the receiving end from assumed known conditions at the generating end. But by assuming the load conditions at the receiving end of the transmission, it is a comparatively simple matter to calculate the necessary magnitude and phase relations of the impressed voltages at the generating end which will counteract the effects due to the ohmic resistance and self-induction of the transmission lines.

In Fig. 1 the pressures available at the receiving end of the line are E_1 and E_2 respectively, and in the following examples these

pressures will be assumed equal, but separated by a phase angle of 90 degrees, *i.e.*, the pressure E_2 will be considered as lagging behind E_1 by exactly one-quarter of a complete period. The corresponding currents I_2 and I_1 return to the source of supply through the common conductor; and, assuming the direction of all currents to be positive when flowing away from the source of supply, we have the condition:

$$I_1 + I_2 + I_3 = 0 \tag{1}$$

and the current I_3 in the third wire is readily calculated in the usual way by combining the vectors representing the currents I_1 and I_2 flowing in the outer wires. The e.m.fs., V_1 and V_2, at the generating end are the quantities which it is proposed to determine by means of a series of simple vector diagrams.

1. Load Non-inductive: Resistance of Common Conductor Assumed Negligible.—In Fig. 2, let OE_1 and OE_2 be the vectors representing the pressures at the receiving end of the line; they

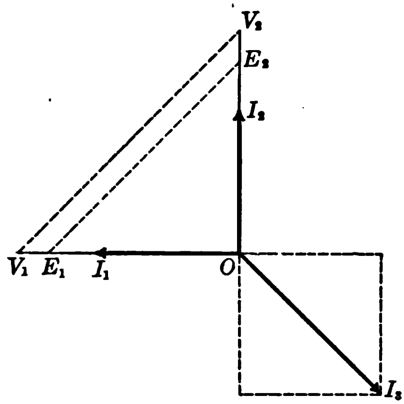

FIG. 2.—Vector diagram—return conductor of negligible resistance.

are drawn at right angles and of equal length, because the pressures are equal and the phase angle is 90 degrees. The load being non-inductive, the current vectors I_1 and I_2 will be in phase with the corresponding pressure vectors, and if, as we may assume, the load is balanced, *i.e.*, if the resistance of the load on the two phases is the same, then the lengths OI_1 and OI_2, representing the respective current magnitudes in the two phases, will be equal. The vector for the resultant current I_3, in the common conductor, is obtained by combining the two other current vectors in the usual

way. It is clearly 135 degrees in advance of I_1 and 135 degrees behind I_2, its numerical value being $\sqrt{I_1^2 + I_2^2}$. If, now, the common conductor is supposed to be without resistance, and if there is no back e.m.f. due to the self-induction of the lines, then it is merely necessary to add to OE_1 a length E_1V_1 equal to the product $I_1 \times R_1$, and to OE_2, a length E_2V_2 equal to $I_2 \times R_2$, in order to obtain the requisite impressed e.m.fs. OV_1 and OV_2 at the generator end; these pressures being such as will cause the currents I_1 and I_2 to flow in the two-phase circuit. It is evident that, under these ideal conditions, the phase angle at the receiving end would be the same as at the generating end, and the transmission of two-phase currents by three wires would offer no difficulties.

2. Load Non-inductive: Resistance of Common Conductor taken into Account.—The vector diagram, Fig. 3, is drawn on the assumption that the load conditions are exactly the same as in the

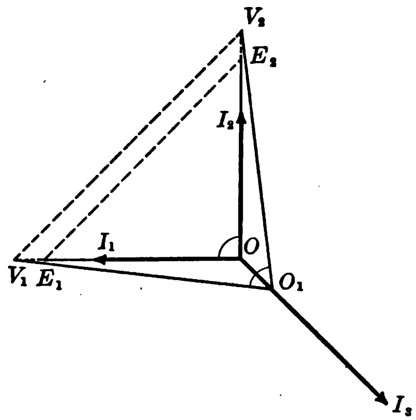

FIG. 3.—Vector diagram—return conductor of appreciable resistance.

case of Fig. 2, and the current vectors I_1, I_2, and I_3, together with the vectors E_1 and E_2, representing the pressures at the receiving end of the line, will all be of exactly the same lengths, and will all have exactly the same phase relations as the corresponding vectors in Fig. 2. But since the resistance of the third conductor is now assumed to have an appreciable value, R_3, it follows that there must be a component of the applied e.m.f. at the generator end equal in value to $I_3 \times R_3$ of which the phase is the same as that of the resultant current I_3; this component of the applied e.m.f. being necessary to overcome the resistance of the common

conductor in order that the required pressure at the receiving end shall remain unaltered. The vector OO_1 represents this required e.m.f. component; and since the e.m.f. components E_1V_1 and E_2V_2, required to overcome the resistance of the two other conductors, remain as before, the vectors O_1V_1 and O_1V_2 indicate by their length and direction the required impressed e.m.fs. at the generator end: the phase angle between them is clearly less than 90 degrees, and it therefore follows that if the phase angle of the two-phase supply at the generator terminals were exactly 90 degrees, the phase angle at the distant end of the line would be *greater* than 90 degrees, that is to say, the current I_2 would lag behind I_1 by more than a quarter period. It is this effect which constitutes the chief disadvantage of a two-phase transmission with three wires only.

3. Load Non-inductive: Self-induction and Resistance of Lines taken into Account.—If the frequency, f, is known, and the sine-

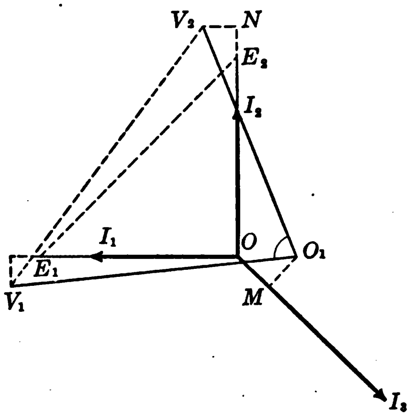

FIG. 4.—Vector diagram, which takes account of the inductance of the transmission line.

wave law is assumed, the formula for calculating the induced volts may be written:

$$e = 0.004656 \times f \times I \ \log\frac{D}{r} \qquad (2)$$

where e stands for the volts induced per mile of the conductor. The *phase* of this induced pressure is exactly 90 degrees behind the current which produces it, hence it must be balanced by a component of the impressed e.m.f. exactly 90 degrees *in advance* of the current.

In the diagram, Fig. 4, the current and pressure vectors at the receiving end of the line remain as in previous diagrams. The vectors representing ohmic drop of pressure are also the same as in Fig. 3; but in the case of the common conductor there is a new component MO_1 (drawn 90 degrees in advance of the current vector), the magnitude of which can be calculated by means of formula (2). In the case of the conductor carrying the current I_2, we have E_2N as the volts to overcome ohmic resistance exactly equal to E_2V_2 in Fig. 3, together with a new component NV_2, at right angles to OI_2, the length of which bears the same relation to MO_1 as the current I_2 bears to I_3. The construction in the case of the remaining conductor is similar; and the result is a new pressure triangle $O_1V_2V_1$, in which the phase angle $V_1O_1V_2$

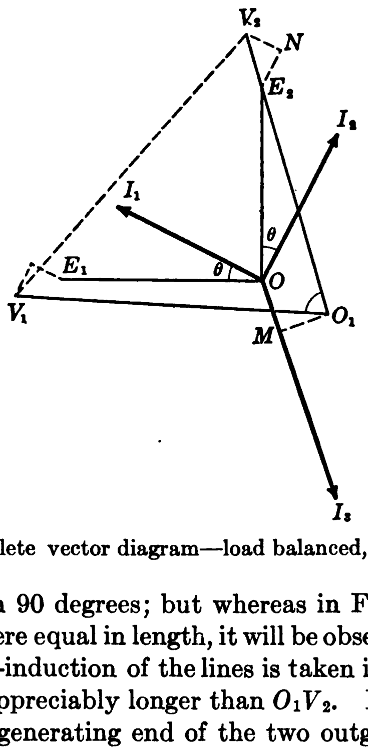

FIG. 5.—Complete vector diagram—load balanced, but inductive.

is again less than 90 degrees; but whereas in Fig. 3 the vectors O_1V_1 and O_1V_2 were equal in length, it will be observed that in Fig. 4, where the self-induction of the lines is taken into account, the vector O_1V_1 is appreciably longer than O_1V_2. If, therefore, the pressures at the generating end of the two outgoing conductors are equal, and differ in phase by exactly 90 degrees, the combined effect of the resistance and self-induction of the lines, even when the *load* is balanced, will be not only to increase the phase angle at the receiving end, but also to produce inequality of pressure on

the two phases; the pressure available for the load on the leading phase being lower than that available on the lagging phase.

4. Load Balanced, but Inductive.—In practice the load on a two-phase system generally consists partly of induction motors, and it is therefore inductive. For this reason Fig. 5 has been drawn representing the conditions for a balanced load of power factor cos θ. The construction and lettering of the diagram are generally as in Fig. 4, the difference being that the current vectors I_1 and I_2 are no longer in phase with the vectors representing pressures at the receiving end of the line, but lag behind these vectors; the angle θ being such that cos θ is the power factor of the load. Moreover, all the current vectors have been drawn of a greater length; the increase being such that the actual *power* available at the receiving end is the same as in the previous examples. By comparing this diagram with Fig. 4, it will be found that the effect of the load being partly inductive instead of being entirely non-inductive is:

(*a*) To increase the total loss of pressure in the lines.

(*b*) To cause the increase in the phase angle at the receiving end to be somewhat greater.

(*c*) To reduce the difference between the pressures on the two phases at the receiving end. In fact, so far as item (*c*) is concerned, the difference in the pressures available on the two phases might, in many a practical scheme, be negligible.

APPENDIX D

Approximate Method of Determining Deflections and Stresses in Flexible Tower Lines

Consider a series of poles as in Fig. 1, the end one being rigid while all the others are flexible and of equal height and stiffness. It is assumed that all spans were originally of equal length l, and that there were b wires in each span, strung to a tension of T pounds per square inch and having a corresponding sag S. In span No. 1, terminating at the rigid support, some of the wires

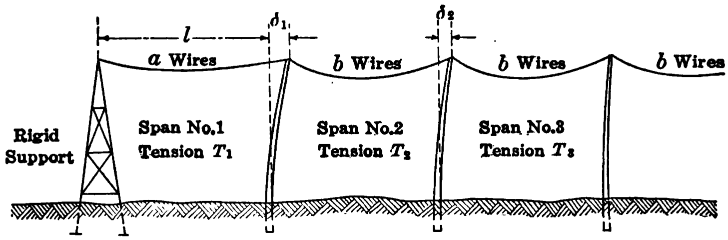

Fig. 1.—Flexible pole line.

have been severed, leaving only a wires in this span. It is assumed also that there is no slipping of the wires in the ties and no yielding of pole foundations.

The elastic deflection of a pole or tower considered as a beam fixed at one end and loaded at the other is

$$\delta = \frac{1}{3}\frac{PH^3}{MI}$$

where P is the load, H the height, M the elastic modulus and I the moment of inertia of the cross-section.

In the special case considered, the value of P, which produces the deflection δ_1 of the first flexible pole, is

$$P = A(bT_2 - aT_1)$$

where A is the cross-section of one conductor and T_1 and T_2 are the stresses in the conductors of spans No. 1 and No. 2 respec-

tively. It is assumed that all the wires are attached to the pole tops at a point H in. above ground level.

By putting $K=\frac{H^3}{3MI}$, the successive deflections may be written:

$$\delta_1 = KA(bT_2 - aT_1) \tag{1}$$
$$\delta_2 = KAb(T_3 - T_2) \tag{2}$$

and the sum of the deflections of a series of flexible poles of the same height and stiffness is

$$\Delta = KA(bT_n - aT_1) \tag{3}$$

where n is the number of the last span. It is usually safe to assume that T_n is equal to the initial tension T in the fourth to sixth span from the break.

Fig. 2 shows the conductors in the first span with a sag S under normal conditions with b wires in the span, and a smaller sag S_1 after some of the wires have been cut, leaving only a wires in the span. For simplicity in calculating the movement of the point

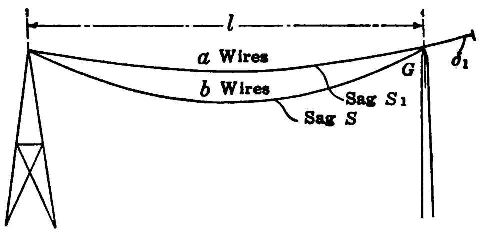

FIG. 2.—Elongation of wire in span due to deflection of pole top.

of attachment of the wires on the flexible pole, instead of considering the span as increasing in length from l to $(l+\delta)$, the span l may be supposed to remain unaltered while the length of the conductor is reduced by pulling it through the tie of the insulator (G) on the flexible pole until the sag is reduced from S to S_1. The length of wire pulled through in this manner may, for all practical purposes, be considered equal to the actual pole-top deflection, δ. This assumption is justifiable since the deflection δ is always small relatively to the span l.

The length of the (parabolic) arc with sag S is

$$L = l + \frac{8S^2}{3l}$$

and with sag S_1

$$L_1 = l + \frac{8S_1^2}{3l}$$

The difference is

$$L - L_1 = \frac{8(S^2 - S_1^2)}{3l}$$

to which must be added the elongation due to the stretch of the wire under increased tension; this is

$$\frac{L_1(T_1 - T)}{M},$$

or, with quite sufficient closeness,

$$\frac{l(T_1 - T)}{M}$$

Hence the deflection of the first flexible pole expressed in terms of the sag and tension of the conductors in the first span is:

$$\delta_1 = \frac{8(S^2 - S_1^2)}{3l} + \frac{l(T_1 - T)}{M} \tag{4}$$

Returning again to the arrangement of line depicted in Fig. 1, it is necessary to consider (1) the total pull of all the wires in span No. 2 and the effect of this pull on pole No. 1 if all the wires are broken in span No. 1, and (2) the effect on the first flexible pole and the stresses in the remaining wires in No. 1 span on the assumption that all the wires in the faulty span are not broken.

If the particulars of the poles are known so that the factor K in the formulas for deflection can be determined, it is desired to calculate the stresses in poles and wires corresponding to the new conditions of equilibrium; or, if the poles have yet to be designed, the factor K must be determined, in order that the stiffness of the poles shall satisfy certain necessary or assumed conditions, such as the maximum deflection of pole top which will not stress the remaining wires in span No. 1 beyond the elastic limit of the conductor material. (A factor of safety must be used to allow of momentary increased stresses due to probable mechanical surges.)

No attempt will be made to obtain an exact mathematical solution of these problems, but close approximations can be obtained with sufficient accuracy for practical purposes, especially when

it is considered that many possible influencing factors, such as the yielding of foundations and the slipping of wires in the ties, cannot be taken into account even in the most complete mathematical treatment of the subject.

It is assumed that the poles are equidistant and in a straight line, and that the first support is rigid, all as indicated in Fig. 1. Four separate limiting conditions will be considered:

(A) All wires are severed in the first span, and the second pole beyond the break is considered to be rigid.

(B) All wires are severed in the first span, but the second pole beyond the break, and all subsequent poles, are considered to be infinitely flexible.

(C) There are a wires remaining in span No. 1 and b wires in all other spans. The second pole beyond the break is considered to be rigid.

(D) There are a wires in the first span, but the second and all subsequent poles beyond the break are considered to be infinitely flexible.

In order to illustrate the calculations by means of numerical examples, the transmission line will be supposed to have the following characteristics:

Six No. 2-0 aluminum conductors.

Cross-section of conductor, $A = 0.1046$ sq. in.

Length of span, $l = 400$ ft.

Normal sag $= 9.76$ ft., which corresponds to

Stress $T = 2400$ lb. per square inch.

It is assumed that there is no grounded guard wire above the conductors, and that the average height of the point of attachment of the wires above ground level is $H = 45$ ft.

The modulus of elasticity of aluminum cables for the purpose of these calculations is assumed to be $M = 7{,}500{,}000$. The flexible towers are in the form of braced A-frames, each vertical limb consisting of one 7-in. steel channel of light section (9 3/4 lb. per foot). The moment of inertia of the section of such a channel is 21.1, and since there are two channels, the value of I is $21.1 \times 2 = 42.2$ and the section modulus $\frac{42.2}{3.5}$ = (say) $12 = Z$. The elastic modulus for steel is $M = 29 \times 10^6$. The factor for use in pole deflection formulas as previously given is

$$K = \frac{(45 \times 12)^3}{3 \times 29 \times 10^6 \times 42.2} = 0.0428$$

The maximum deflection of this particular structure before permanent deformation would take place will occur when the difference of pull due to the wires is such as to stress the metal to (say) 30,000 lb. per square inch. The resisting moment is $T \times Z = 30{,}000 \times 12$ and the resultant pull at the pole top will be

$$\frac{30{,}000 \times 12}{45 \times 12} = 667 \text{ lb.}$$

The maximum allowable deflection is, therefore,

$$\begin{aligned}\delta &= K \times 667\\ &= 0.0428 \times 667\\ &= 28.5 \text{ inches.}\end{aligned}$$

Case (A). All wires broken in span No. 1; second pole beyond break considered rigid.

Since all the wires are severed in span No. 1 ($a = 0$) it is not possible to make use of formula (4), but a similar formula can be used by expressing the deflection in terms of the constants for span No. 2. This formula is

$$\delta_1 = \frac{8}{3l}(S_2{}^2 - S^2) + (T - T_2)\frac{l}{M} \qquad (5)$$

By calculating δ_1 for various arbitrary values of T_2 smaller than T, curve No. 1 of Fig. 3 can readily be drawn. This gives the relation between the stress T_2 in the wires of the second span and the pole-top deflection δ_1 on the assumption that the second pole beyond the break is rigid. On the same diagram draw the straight line marked curve No. 2, which gives the relation between pole deflection and the stress T_2, as given by formula (1) when the tension T_1 in wires of the first span is equal to zero. The point of crossing of curves No. 1 and No. 2 evidently indicates the deflection corresponding to the condition of equilibrium. This deflection is $\delta_1 = 29.5$ in. and stress $T_2 = 1100$.

It will be noted that in this particular example the deflection is about the same as the maximum allowable deflection (28.5) previously calculated; but even if allowance is made for shocks and mechanical surges, it is probable that the pole would not suffer serious injury, because some of the wires would be liable to slip in the ties and so relieve the tension. If wind pressures acting on snow or ice deposits are added to the stresses due to weight of conductor material only, the strain will be greater, but on the

other hand, much sleet deposit is liable to be shaken off the wires in the event of a sudden severing of all the wires in the first span.

The above results are, however, based on the assumption that the second pole beyond the break is rigid, which may not be in accordance with practical conditions.

Case (*B*). Conditions as above; but the second and subsequent poles beyond the break are supposed to be infinitely flexible ($K = \infty$).

In this case the tension T_2 will not depend upon the deflection of the first flexible pole; it will be equal to the original tension T

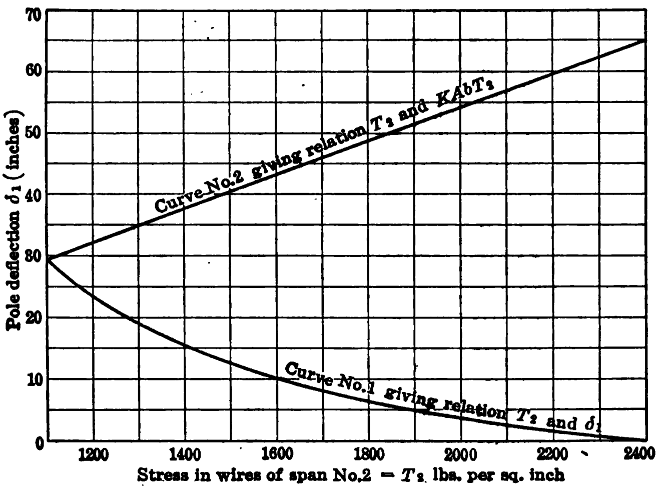

FIG. 3.—Graphic Solution of Problems (A) and (B).

$=2400$ for all values of the deflection δ_1. The deflection obtained when $T_2=2400$ is of course readily calculated by means of formula (1), or it can be read off Fig. 3, since it is the deflection indicated at the point where curve No. 2 meets the vertical ordinate for $T_2=2400$. This value of δ_1 is 64.5 in., which would lead to permanent deformation of the flexible structure. The actual deflection of the first pole in a series of flexible poles of equal stiffness would lie somewhere between these limiting values of 29.5 in. and 64.5 in. if the law of elasticity may be considered to apply in the case of the higher deflections. As a general rule the breaking of all wires in one span will lead to the wrecking or permanent deflection or uprooting of the first pole, which cannot be at the same time flexible enough greatly to reduce the combined pull

of all wires in span No. 2, and yet strong enough to resist the ultimate combined pull of these wires. There would be an exception in the case of short spans with tall flexible poles; and in any case it is probable that only the first pole would be damaged or moved in its foundations.

It is rare that all the wires in one span are broken simultaneously unless the design of the line is such that the severing of one or more wires leads necessarily to the rupture of the remaining wires owing to the excessive stresses imposed on them. The calculation of stresses and deflections when a certain number of wires remain in the faulty span is more difficult than in the cases already considered, but the solution is of greater practical value.

Case (*C*). There are a wires in faulty span and b wires in sound spans. The second pole beyond break is considered rigid. (For the purpose of working out numerical examples it will be assumed that only one wire remains in faulty span; thus $a=1$ and $b=6$.)

Instead of only two equations, there are now three equations to be satisfied simultaneously; these are:

(a) Formula (1):

$$\begin{aligned}\delta_1 &= KA(bT_2 - aT_1)\\ &= 0.0269\ T_2 - 0.00448\ T_1\end{aligned}$$

(b) Formula (4), giving deflection in terms of elongation of remaining wires in span No. 1:

$$\delta_1 = \frac{1}{150}(95.3 - S_1{}^2) + \frac{T_1 - 2400}{18{,}750}$$

(c) Formula (5), giving deflection in terms of the shortening of the wires in span No. 2. (This relation is given by curve No. 1 already plotted in Fig. 3.)

It should be mentioned in connection with formulas (4) and (5) that by assuming a constant length of span the sag S is always inversely proportional to the stress T. The assumption of a constant length of span for the purpose of simplifying the relation between sag and tension introduces no appreciable error in practical calculations. In the particular example from which the curves are plotted and the numerical results obtained the relation is $S = \dfrac{23{,}420}{T}$.

Proceed, now, to plot curve No. 3 in Fig. 4 from formula (4) by assuming various arbitrary values of T_1 from the lowest possible limit of $T_1 = T = 2400$ up to the elastic limit of about 13,000. For a reason to be made clear hereafter this curve should be drawn on transparent paper; the horizontal scale used for the values of T_1 may be arbitrarily chosen, but the scale of ordinates giving the deflections δ_1 must be exactly the same as used for Fig. 3. On the same diagram (Fig. 4) draw also the straight line marked curve No. 4, giving the relation between T_1 and the quantity $KAaT_1$. This latter quantity when subtracted from

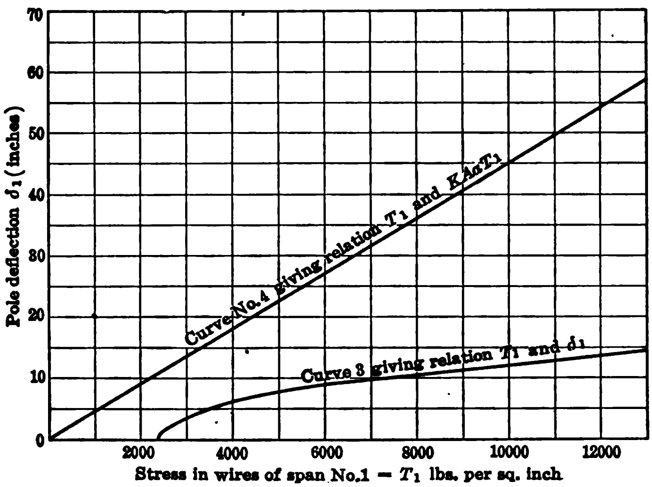

FIG. 4.—Curves to be drawn on tracing paper for solution of problems (C) and (D).

the quantity $KAbT_2$ will give the pole deflection to fulfil the condition of formula (1). The reason for drawing the curves of Fig. 4 on transparent paper will now be clear.

The transparent paper with the curves of Fig. 4 is placed over Fig. 3, with the horizontal datum lines of zero deflection coinciding as shown in Fig. 5. The point of intersection of curves No. 1 and No. 3 will give the corresponding values of the stresses T_1 and T_2,[1]

[1] There is a definite value of T_1 for any given value of T_2 independent of all considerations of pole stiffness and size of wire and number of wires in adjoining spans. This is the relation which will satisfy formulas (4) and (5) simultaneously; it is expressed by the equation

$$\frac{8}{3l}(2S^2 - S^2_1 = S_2^2) = \frac{l}{M}(2T - T_1 - T_2)$$

but with a pole having definite elastic properties there is only one value of the deflection which will satisfy the three conditions previously referred to. The deflection as a function of the pole stiffness is the distance $E F$ (Fig. 5), being the difference between the corresponding ordinates of curves No. 2 and No. 4. By moving the tracing paper with the curves No. 3 and No. 4 over the other curves until the distances $H G$ and $F E$ on the same vertical ordinate are equal, the deflection corresponding to the condition of equilibrium is readily obtained. If preferred, the curve $O P R E$,

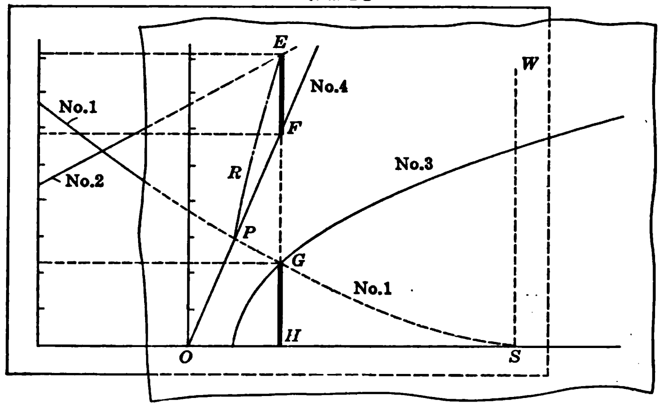

FIG. 5.—Graphic solution of problems (C) and (D).

representing the *sum* of the quantities of curves No. 3 and No. 4. may be drawn on the tracing paper instead of the curve 4, and when the point of intersection (E) of this new curve with curve No. 2 on the lower sheet lies on the same vertical ordinate as the junction (G) of the curves No. 1 and No. 3, the distance $H G$ will be the required deflection.

The solution of the numerical example worked out in this manner is

$$\delta_1 = 10.2 \text{ in.}$$
$$T_1 = 7400 \text{ lb. per square inch.}$$
$$T_2 = 1500 \text{ lb. per square inch.}$$

Case (D). Same conditions, with the exception that the second pole beyond the break, instead of being rigid, is assumed to be infinitely flexible. This assumption is made also in the case of all

subsequent poles. This means that $T_2 = T = 2400$ whatever may be the amount of deflection of the first flexible pole, and the problem can be solved graphically as indicated above, the only difference being that curve No. 1 giving the relation between δ_1 and T_2 when the second pole beyond the break is rigid must be replaced by the vertical line $S\,W$ (Fig. 5), being the ordinate corresponding to a tension $T_2 = 2400$.

The numerical solution in this case is:

$$\delta_1 = 13.2 \text{ in.}$$
$$T_1 = 11{,}450$$

It is interesting to note that there is little difference between the deflections for the two extreme cases (C) and (D); the average value for δ_1 is 11.7 in., corresponding to a stress $T_1 = 9400$ in the remaining wire of the faulty span. This is well below the elastic limit, and it is probable that this wire would not break even if the five other wires were severed. The figures chosen for illustrating the calculations relate to a practical transmission line, and it will be seen that the stresses and deflections corresponding to the state of equilibrium after the severing of some or all of the wires in one span can, by use of simple diagrams, be predetermined within reasonably narrow limits.

APPENDIX E

Graphical Statics Applied to Transmission-line Calculations with Special Reference to Steep Grades

Many students and practical engineers appear to have trouble in understanding the distribution of forces in overhead lines carried up a steep grade. An unsigned article appeared in the London Electrical Review of August 18, 1911, in which an attempt was made to show that, if the supports are of the so-called flexible type, or if the suspension type of insulator is used, there is a transference of the weight of the wires from the poles at the bottom of an incline to the poles on the higher ground. In other words, the supports on the hill-side at the lower levels are said to take less than their proper share of the weight of the conductors. The faulty reasoning in the article referred to is due to the assumption that the tension in the wire at the point of support is the same on the up-hill side as on the down-hill side of the pole. This is incorrect, and the conclusions arrived at are therefore valueless. It is because trouble has been experienced on lines carried up steep inclines that theories of this kind are advanced; and the idea that the poles on a steep grade carry increased loads as they occupy positions higher up the hill-side, appears to be shared by many. As a matter of fact the troubles referred to have probably been due to want of care in erecting the line, or to want of knowledge on the part of the designer, with the result that the maximum tensions in the conductors have been greater than where the line runs over level country. In this Appendix particular attention will be given to the calculation of stresses in lines carried up steep grades.

General Problem.—Consider a mass of any irregular shape, the weight of which may be represented by the vertical vector OP_G passing through its center of gravity O' as shown in Fig. 1. This mass is suspended by two perfectly flexible ties from the fixed points A and B. Except for the special case of parallel forces, the resultant P_G and the components P_A and P_B acting in the direction of the suspension cords, AC and BD, will pass

through the common point O. The system of forces is in equilibrium and the conditions to be fulfilled are therefore,

(1) that the vectorial sum of all forces and reactions shall be zero, and

(2) that the sum of all moments taken about any point shall be zero.

The following is an explanation of the manner in which a force diagram similar to Fig. 1 may be constructed.

Known Data.—(1) The position of suspension points A and B.

(2) The magnitude and position relatively to points A and B

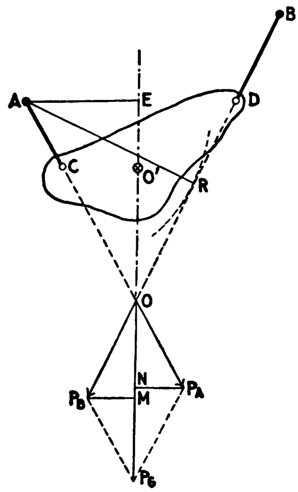

Fig. 1.

of the force of gravity (*i.e.*, the length of the vector OP_G and the distance AE of this vector from the point A.)

(3) The magnitude, but not the direction, of the force P_{B1} acting through the suspension point B.

Required.—(1) The direction of the force P_B.

(2) The magnitude and direction of the force P_A.

(3) The horizontal and vertical components of the forces at the points of support.

Taking moments about the point A gives the equation

$$P_B \times AR = P_G \times AE$$

thus $$AR = AE \times \frac{P_G}{P_B}$$

From the point A as a center, use this radius to draw an arc of circle the tangent to which, passing through the point B, will locate the point O. Draw $O\ P_G$ to the proper scale to represent the force of gravity and complete the parallelogram of forces. The vertical component of the reaction at point A is $N\ O$, and at the point B it is $M\ O$. The horizontal reactions at the points of support are $P\ N$ and $P_B M$ respectively. These are obviously equal, but opposite in direction, in every conceivable case of a body in equilibrium subject only to the force of gravity acting, as it always does, vertically downward.

Stretched Wire. Supports on Same Level.—In Fig. 2, a wire weighing W lb. per foot is stretched between the supports A and B lying in the same horizontal plane. The maximum tension in the wire is P_B lb. This is the tension at the points of

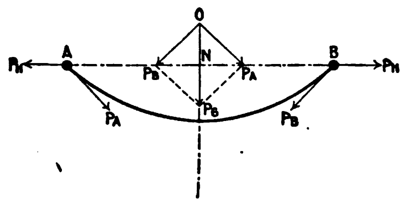

FIG. 2.

support, and, owing to the symmetry of the figure, it is the same in amount at A as at B. The assumption is now made that the total weight of wire is equal to the weight per foot multiplied by the straight line distance between the points A and B. Thus

$$P_G = W \times l$$

where l is the distance in feet between A and B. This assumption is allowable on all except spans of extraordinary length, because the actual length of the wire differs only by a very small amount from the shortest distance between the points of suspension.

Draw the vector $O\ P_G$ to represent the total downward force $W\ l$. It will lie on a vertical line midway between A and B. Let it be bisected by any horizontal line such as $A\ B$. From O lay off $O\ P_B$ at such an angle that the head P_B of the vector lies on the horizontal line bisecting $O\ P_G$. Complete the parallelogram of forces. Then $O\ N$ represents the vertical component at each point of support, and $N\ P_B$ or $N\ P_A$ is the horizontal force

P_H acting at each support; it is also the total tension in the wire at the center or lowest point of the span.

Calculation of Sag.—Referring to Fig. 3, which shows a span of length l, it is required to calculate the sag s at the center. Con-

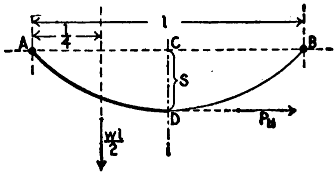

FIG. 3.

sider the moments about the point A due to the half span $A\ D$. These must balance, and the equation is

$$s \times P_H = \frac{l}{4} \times \frac{W\,l}{2}$$

$$\text{or } s = \frac{W\,l^2}{8\,P_H} \tag{1}$$

which is the well-known formula for sag calculations when the parabolic assumption is made.

Supports at Different Elevations.—In Fig. 4, the difference in elevation between supports is h feet. The span measured hori-

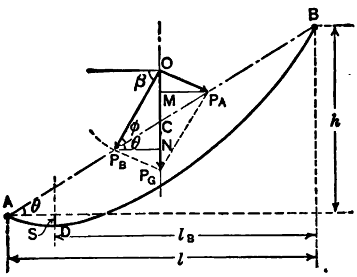

FIG. 4.

zontally is l ft., and the angle θ which the straight line $A\ B$ makes with the horizontal is

$$\tan^{-1}\left(\frac{h}{l}\right)$$

The actual length of the direct line between points of support is

$$l' = \frac{l}{\cos\theta}$$

The weight of wire per foot is W lb., and it is assumed that the total weight is $W\,l'$ lb. This force acts through the point C midway between A and B. The other known quantity is the magnitude (but not the direction) of the maximum tension in the wire: this is the force P_B acting through the highest point of support (B).

Draw the vertical line $O\,P_G$ to represent the total force of gravity ($W\,l'$). Through its center C draw the line $A\,B$ (or a line parallel to $A\,B$) making an angle θ with the horizontal. Then from O as a center describe an arc of radius $O\,P_B$ equal to the known force P_B. Its intersection with $A\,C$ locates the head of the vector $O\,P_B$. Complete the parallelogram of forces, and drop the perpendiculars $P_A\,M$ and $P_B\,N$ on to the vertical $O\,P_G$. These last are a measure of the (equal) horizontal components of the reactions at the two points of support, and also of the total tension in the wire at the lowest point D. The length $M\,O$ is the vertical component of the reaction at the lower support A; while $N\,O$ is the vertical reaction at B. Their sum is, of course, equal to $O\,P_G$. The angle β which the vector P_B makes with the horizontal is the slope of the wire where it leaves the point B.

The construction as above described satisfies the fundamental conditions of equilibrium, namely, that all vertical forces shall balance; that all horizontal forces shall balance, and that the sum of all moments taken about any point shall be equal to zero.

In practice it is not convenient to solve such problems by actual measurement of quantities plotted to scale on drawing paper, because the horizontal components of the forces are usually very much greater than the vertical components. A trigonometrical solution is therefore desirable.

In Fig. 4, the known quantities are the two sides $O\,P_B$ and $O\,C$ of the triangle $O\,C\,P_B$ of which the angle $O\,C\,P_B$ is also known, being equal to $\theta+90$ degrees.

The angle $O\,P_B\,C$ or φ may be calculated from the relation

$$\sin\varphi=\frac{OC}{P_B}\sin\,(90+\theta)=\frac{Wl'}{2P_B}\cos\theta$$
$$=\frac{Wl}{2P_B}$$

and the angle β which the wire at the upper support makes with the horizontal is therefore

$$\beta=\theta+\sin^{-1}\left(\frac{Wl}{2P_B}\right) \tag{2}$$

With the aid of trigonometrical tables or a slide rule, the values of sin β and cos β can be obtained, and the other components of the force diagram readily calculated. Thus, the horizontal component at either point of support, which is also the total tension in the wire at the point (if any) where the slope is zero, is

$$P_H = P_B \cos \beta \tag{3}$$

The vertical component of the reaction at the highest point B, where this reaction is greatest is,

$$P_{BV} = N\ O = P_B \sin \beta \tag{4}$$

The weight supported by the lower pole is,

$$P_{AV} = M\ O = W\ l' - P_{BV} \tag{5}$$

This may, obviously, be a negative quantity, *i.e.*, the wire may exert an upward pull on the lower support. In that case the sag s below the point A, will be zero, and this would correspond to the usual condition on a steep incline, or on a moderate incline if the spans are short.

Position of Lowest Point of Span. Supports at Different Elevations.—The position of the lowest point D in the span will be determined by the vertical weights carried by each of the two supports. Thus, referring to Fig. 4, the vertical component of the forces acting at B is simply the weight of that portion of the wire comprised between the support B and the point D where the tension in the wire has no vertical component. The horizontal distance of the point D from B (in feet) is,

$$l_B = l \times \frac{ON}{OP_G}$$

$$= \frac{l\ P_{BV}}{Wl'} \qquad = \frac{P_{BV}\ \cos \theta}{W} \tag{6}$$

This may give a distance for l_B which is greater than l. In that case the support A would be the lowest point in the span. It may be observed that if the formula (4) gives a value for P_{BV} greater than the total weight of the span (Wl') it is unnecessary to proceed with the location of the point D, since the resultant force at A must be in an upward direction.

Calculation of Sag with Supports on an Incline.—The formula (1) as calculated for spans with supports on the same level may be used for spans on an incline provided the distance l_B of Fig.

4 is considered as half of a level span of which the sag is $(h+s)$. Thus,

$$(s+h) = \frac{W}{\cos\theta} \times \frac{4l^2_B}{8P_H}$$
$$= \frac{Wl^2_B}{2\,P_H\cos\theta} \qquad (7)$$

On a steep incline, where A is the lowest point of the span, it is more useful to know the maximum deflection of the wire from the straight line AB as observed by sighting between the points A and B. This maximum deflection will occur at the center of the span. A careful study of the force diagram in Fig. 4 will make it clear that, just as $O\,P_B$ is the slope of the tangent to the wire at the point B, and $P_B\,N$ is the slope of the tangent to the wire at the lowest point D, so $P_B\,C$ is the slope of the tangent to the curve at the middle of the span. This is therefore the point of maximum deflection from the straight line $A\,B$.

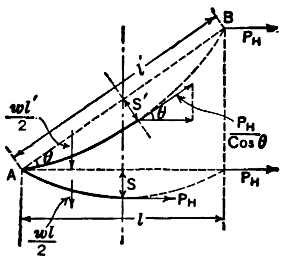

FIG. 5.

Consider now Fig. 5. The maximum deflection of the line on the slope is s', and, by taking the sum of all the moments about A and putting this sum equal to zero, the value of this deflection is found to be,

$$s' = \left(\frac{Wl'}{2} \times \frac{l}{4}\right) \times \frac{\cos\theta}{P_H}$$
$$= \frac{Wl^2}{8\,P_H} \qquad (8)$$

which indicates that the maximum deflection from the straight line, of a conductor strung between supports on a slope, as measured at the centre of the span, is exactly the same as the maximum sag s of the same conductor strung between points on the same level; provided the span measured horizontally

and the horizontal component (P_H) of the tension are the same in both cases.

Example Illustrating Use of Formulas.—Consider a span of 485 ft. measured horizontally, with a difference of level of 40 ft. between points of support. The wire is No. 2-0 B. &. S. gage, aluminum, weighing 647 lb. per mile. Calculate sag and vertical and horizontal components of forces if the maximum tension in the wire is 185 lb.

The known quantities are:

$$l = 485$$
$$h = 40$$
$$W = 0.1226$$
$$P_B = 185$$

The unknown quantities are calculated as follows:

$$\tan\theta = \frac{40}{485} = 0.0825$$
$$\theta = 4° \ 43'$$
$$\cos\theta = .9966$$
$$\sin\varphi = \frac{485 \times .1226}{2 \times 185} = .161$$
$$\varphi = 9° \ 15'$$

By formula (2) $\beta = \theta + \varphi = 13° \ 58'$

$$\cos\beta = .971$$
$$\sin\beta = .2413$$

By (3) $P_H = 185 \times .971 = 180$ lb.

By (4) $P_{BV} = 185 \times .2413 = 44.6$ lb.

By (6) $l_B = \dfrac{44.6 \times .9966}{.1226} = 364$ ft.

By (7) $(s+h) = \dfrac{.1226 \times (364)^2}{2 \times 180 \times .9966} = 45.2$ ft.

Thus the sag below the bottom support will be 5.2 ft. when the maximum tension is 185 lb.

Conclusions. Overhead Lines on Steep Grade.—Except for the fact that the pole foundations must be wisely chosen, and special attention paid to the setting of the poles, there are no engineering difficulties in the way of running an over-head line up a steep incline. It may be well to reduce the length of span

measured horizontally so that this is equal to about $l' \cos \theta$, where l' is the distance between supports on level ground, and θ is the angle which the average slope of the line makes with the horizontal. This will ensure that the vertical load to be supported by the insulators does not exceed the vertical load on the level runs; but no difficulty need be experienced in providing that each pole shall take its proper share of the weight of the wire (with or without ice loading).

The stringing of the wires may be done with the aid of a dynamometer; or the sag, as calculated by formula (8), can be measured by sighting from pole to pole. If the poles or suspension insulators of a line carried on an incline are deflected from the vertical owing to the unbalancing of horizontal forces, at the time when the wires are strung, the effect must be attributed to incompetence on the part of the engineer in charge of construction, and not to any unexplained flaw in the known laws of equilibrium. If the conductor is tied to the insulator at the proper point, the horizontal components of the forces at this point will balance.

APPENDIX F

Specification for Overhead Electric Power Transmission Line from Generating Station at...... to Substation at......

1. General Description of Transmission Line.—The total distance of transmission is 25 miles. The system will be three phase with a pressure of 22,000 volts between wires supported on wooden poles. The conductors will be No. 2/0 stranded aluminum. The average span will be 150 ft., and the separation between wires will be 3 ft., the three conductors being arranged in the form of an equilateral triangle with one conductor at the top and the remaining two conductors below, at the ends of a wooden cross-arm, all as shown on drawing No. ......

In cases where long spans are necessary, a double-pole arrangement, as shown on drawing No. ......, will be adopted. Particulars relating to special precautions and methods of procedure in the case of exceptionally long spans, will be found in clause (8) under the heading "Spans."

There will be no telephone wires supported on the transmission-line poles.

There will be no grounded guard wire above the conductors, but galvanized iron lightning rods, as shown on drawing No. ...., will be fitted to every third pole on the average. Further particulars relating to protection against lightning are given in clause (7) under heading "Grounding." For particulars of sags and tensions, refer to clause (12) under heading "Stringing of Wires."

2. Clearing.—The width of the right-of-way shall be 100 ft., and all lumber, brush and other growth of every description must be cut and cleared so that, in no portion of the right-of-way, shall the tops of tree stumps, undergrowth or bush be higher than 18 inches above ground level.

At all points where a space of 50 ft. on each side of the pole line is insufficient to prevent possible damage to wires by falling trees, the normal width of the clearing may be exceeded.

All useful material shall be separated and suitably stacked at a safe distance from waste material piled for burning.

All cedar suitable for electric light or telephone poles shall be reserved for this purpose. Cedar unsuitable for poles, also jack pine, shall be cut to standard 8-ft. railway ties. Other merchantable soft woods, such as pine, balsam and spruce, shall be cut to 16-ft., 12-ft. and 8-ft. lengths. Shorter pieces, also birch, etc., suitable for cordwood shall be cut in 4-ft. lengths, split and corded ready for removal.

All tops, limbs, brush and other waste shall be burned, great care being taken to prevent spread of fire beyond the limits of clearing. Suitable fire-fighting appliances shall be kept at hand while burning is proceeding.

Poles. *Dimensions.*—The greater number of the poles required will be 35 ft. long: they shall be sawn square at both ends. These poles shall measure not less than 24 in. in circumference at the top under bark, and not less than 38 in. under bark 6 ft. from butt. The approximate number required will be 840. In addition to these, about 100 poles 45 ft. long will be required, and these shall measure not less than 24 in. circumference at top and 42 in. 6 ft. from butt.

Quality.—All poles to be cut of best quality live green timber, well proportioned from butt to top and well seasoned; the bark to be peeled, and all knots and limbs closely trimmed. The poles shall be reasonably straight, and no poles having short crooks or a reverse curve will be accepted. The amount of "sweep" measured between six foot mark and top of pole shall not exceed 10 1/2 in. in the 35-ft. poles, or 13 1/2 in. in the 45-ft. poles.

Twisted Poles.—No poles having more than two complete twists in the total length, and no cracked poles will be accepted.

Dead Poles.—No dead poles or poles having dead streaks covering more than one-quarter of their surface will be accepted.

Butt Rot.—This must not exceed 10 per cent. of the cross-section of the pole, and the diameters of poles with butt rot or hollow hearts must be substantially greater than the corresponding diameters of sound poles. Poles with hollow hearts exceeding 8 in. in diameter will not be accepted. If average diameter of rot does not exceed 6 in., the butt measurement must be 2 in. greater than in the case of sound poles. If the average diameter of rot is 7 in., the butt measurement must be 4 in. greater.

The treatment of all poles before erection shall be as follows: The gains shall be sawn square with the axis of the pole and in

such a position that, when erected, the curvature of the pole (if any) shall be in the direction of the line. The position of the gains is indicated on the accompanying drawings Nos. .... showing the standard pole construction. The gains shall be not less than 5/8 in. and not more than 7/8 in. deep; they shall be accurately cut so that the cross-arms will have a driving fit, and the holes for the 5/8-in. bolts securing cross-arm to pole shall be bored *after* the cross-arm has been fitted in position. These holes, together with all other necessary holes, as indicated on drawings, shall be bored clean and true without splintering. The holes for lag screws securing braces to poles shall be bored after braces have been fitted to cross-arms; they must be small enough in diameter to ensure that the threads of the lag-screw shall engage properly in the wood.

The butts of all poles, together with the gains and tops, shall be treated with two coats of coal-tar-creosote oil, heated to about 220° F. and applied with a brush. At least 24 hours must be allowed to elapse between applications. The painting of the butts shall be carried at least 18 inches above ground level.

4. **Cross-Arms.**—The cross-arms shall be of yellow birch, Oregon fir, or long-leaf yellow pine, well seasoned, close grained, and free from knots or sap wood. They must be dressed on all sides. They must measure 4 1/2 in. deep by 3 1/2 in. wide, and be bored, as indicated on drawing No. ...., with templet, true and symmetrical: the holes to be bored clean and without splintering. After having been bored, the cross-arms shall be painted with two coats of good asphaltum paint. In cases where double cross-arms are required, it will be necessary to bore the standard cross-arms with additional holes for the 3/4-in. spacing bolts, the position of which is shown on the pole drawings previously referred to.

5. **Grading.**—An effort should be made to maintain as far as practicable an even grade. When the average run is level, a change of level exceeding 5 ft. between consecutive poles should be avoided. By carefully choosing the location of each pole so as to avoid the highest points and greatest depressions when passing over uneven ground, it may be possible to avoid the use of poles differing in length to any great extent. Should it be necessary to shorten a pole, this must be done by sawing a piece off the butt end; but unless this is done before the treatment with preservative liquid, the butt must receive a further treatment

with the creosote oil before erection of the pole. In some cases where the ground is favorable, the shortening of poles may be avoided by digging the hole one or two feet deeper than would otherwise be necessary. When using shortened poles, and when passing over uneven ground, it is important to bear in mind that under no condition shall the bottom conductors hang lower than 18 ft. above the ground, and when crossing tote roads or public footpaths, the minimum distance between wire and ground shall be 22 ft.

6. Pole Setting.—Where poles are set in good solid ground, the depth of holes shall be as follows:

35-ft. poles on straight runs	5 1/2 ft.
45-ft. poles on straight runs	6 1/4 ft.
35-ft. poles at corners or where stresses are excessive	6 ft.
45-ft. poles at corners or where stresses are excessive	6 3/4 ft.

If the ground is soft, the depth of setting shall be 6 in. greater than when setting in solid ground. If the soil is very soft, but not such as would be described as swampy, one or more transverse logs may be bolted to the butt of the pole in order to obtain additional bearing area.

When erected in solid rock, the depth of hole shall not be less than 4 ft.

In loose or sandy soil, the sand barrel or its equivalent should be used. This must be filled with a firm soil which may contain stone or rock.

In swampy ground the base of the pole must be provided with an arrangement of transverse timbers securely braced to the pole, in addition to which the hole shall, if necessary, be lined with sheet piling and filled with good soil which may contain stones or rock. As an alternative, a stone- or rock-filled crib may be built round the butt of the pole above ground level. In some cases concrete may be used with advantage in the pole foundation, but it will generally be found that the use of concrete can be avoided.

Drawings Nos. ..... show several pole foundations suitable under various conditions. The nature of the ground may necessitate slight departures from the methods there shown, but as the provision of good and enduring foundations is of great importance, the setting of poles in difficult situations must always receive careful attention.

Poles must not be set along the edge of cuts or embankments

or where the soil is liable to be washed out, unless special precautions are taken to ensure durable foundations.

When setting the poles in good ground, the holes shall be dug of ample size to allow of easy entrance of the butts, and the size at bottom must be large enough to admit of the proper use of tampers. When back-filling holes, there should be three or four tampers to one shoveler, in order to ensure that the dirt shall be packed tight. In no case must the earth be thrown in to a greater depth than 6 inches without being tamped hard before the next layer is thrown in. The proper filling of holes is a matter of great importance. When the filling is properly done, it should not be necessary to remove any excess soil; this should be packed firmly around the pole, the object being to raise the level of the ground near the pole and so cause water to drain away from, rather than toward, the butt.

When setting poles on a straight run, the lining up should be done with a transit, and the poles placed with cross-arms at right angles to the direction of the line. Where the direction of the line alters, the poles at the angles must be set so that the cross-arm halves the angle. If the deviation exceeds 5 degrees, the corner poles shall be provided with double cross-arms and fixtures. When possible, the cross-arms, braces, and other fixtures (but not the insulators) should be mounted on the poles before erection.

7. Grounding.—The proper grounding of lightning rods on the pole line is a matter of importance. Judgment must be used in determining when and how to ground the poles; but either of the following alternative methods will be considered satisfactory, provided the soil is reasonably moist:

(1) A piece of galvanized iron pipe 1 1/2 in. in diameter and 8 or 9 ft. long shall be buried in the hole alongside the butt or driven into soft soil, the ground wire being attached thereto in such a manner as to ensure a good and enduring electrical contact.

(2) The ground wire, consisting of 5/16 in. galvanized stranded steel cable, after being carried straight down the side of the pole and secured with cleats, shall be wound spirally round the butt and carried right down to the bottom of the pole. Not more than 15 ft. of wire should be buried in the ground.

It is of little use to ground a pole in solid rock, but where a pole is set in rock, it may be found that the ground wire can be carried down the face of the rock, or in a crevice, to a point where a good ground can be obtained. Where grass is growing, the soil

will usually contain sufficient moisture to afford a reasonably good ground. When the ground wire does not enter the ground alongside the pole, sudden bends or turns should be avoided in the wire connecting the lightning rod with ground plate or pipe.

It is not intended to provide all poles with lightning rods; but, except when the soil is clearly unsuitable for a ground connection, the poles in the positions described below shall be grounded:

Both poles supporting extra long spans requiring the double pole arrangement as shown on drawing No. ..... previously referred to.

The poles on each side of railway crossings.

All guyed poles.

The six poles nearest to generating station.

The six poles nearest to substation.

In addition to the above-mentioned poles, one pole out of every three poles shall be grounded. It is not necessary that every third pole be grounded: judgment must be used in determining the location of the poles to be grounded. As a general rule, it is more important to ground poles on heights and in exposed positions than those on the lower ground; but, on the other hand, it is of little advantage to ground where the soil is dry or otherwise unsuitable. In exposed positions it may be wise to ground two or more consecutive poles, while in unexposed positions four or five consecutive poles may be left without lightning rods.

8. Spans.—The standard length of span shall be 150 ft. Shorter spans must be used at angles and on curves, as mentioned in clause 9. If the span exceeds 160 ft. the poles must be specially selected for strength. No span greater than 180 ft. shall be carried on single poles. For longer spans, the double-pole arrangement as shown on drawing No. ..... previously referred to, shall be adopted, with a horizontal spacing of 5 ft. between wires for spans up to 600 ft.; but spans exceeding 500 ft. shall be avoided if possible.

Railroad Crossings.—(*a*) The span where line crosses railroad shall be kept as short as possible; but in no case must a pole be placed a smaller distance than 12 ft. from the rail, except in the case of sidings, where the distance may be reduced to 6 ft. At loading sidings sufficient space must be allowed for a driveway between rail and pole. When possible the distance between rail

and pole should not be less than the height of the pole, but if this spacing requires a span greater than 110 ft., it will be preferable to place the pole nearer to the rail provided the ground is suitable. If it is necessary to cross the railroad with a span greater than 140 ft., the double-pole arrangement as used for extra-long spans, and as shown on drawing No. ..... shall be adopted.

(*b*) In all cases the cross-arms and insulators shall be doubled on the poles nearest the rail.

(*c*) The poles at railroad crossings must be set not less than 6 ft. in solid ground and 4 ft. 6 in. in rock.

(*d*) If the crossing is located at a spot where grass or other fires might cause injury to the poles, these shall be provided with a casing of concrete at least 2 in. thick, to a height of 5 ft. above ground level.

(*e*) The clearance between rail and high-tension conductor shall not be less than 30 ft., and the poles should be specially selected for strength and straightness.

(*f*) When crossing over telephone wires, the clearance should be not less than 10 ft.

(*g*) The poles at railway crossings must be securely guyed, whether or not there is a bend in the line. If a departure from the straight run is necessary, special attention should be paid to the method of guying.

(*h*) The poles on each side of the rail shall be provided with lightning rods, and well grounded. Bent iron lightning guards, as shown on drawing No. ..... shall be fixed at each end of cross-arm and connected to the ground wire; these will also serve the purpose of hook guards, to engage the conductor if it should become detached from the insulator. If the nature of the soil is quite unsuitable for the purpose of grounding, the lightning rod may be omitted; but if the pole is not grounded, two strain insulators must be placed in each guy wire securing poles nearest to rail, the lowest of these insulators being not less than 8 ft. above ground level.

(*i*) Special attention shall be paid to the tying of the conductors to the double insulators on the poles at each side of the rail. As a protection against damage by arcs over insulators, the serving of No. 2 aluminum tie wire shall be carried far enough to ensure that the conductor is protected by the serving or tie to a distance of not less than 12 in. from the center of insulator.

(*j*) In addition to the pole number, the poles on each side of the crossing shall bear a label with the Company's name and the voltage (22,000 volts) painted thereon in easily distinguishable characters.

9. Angles and Curves.—Whenever there is a change in the direction of the line, a sufficient number of poles must be provided to prevent the angle of deviation on any one pole exceeding 15 degrees. If the deflection from the straight run does not exceed 5 degrees it is not necessary to use a pole with double fixtures. When

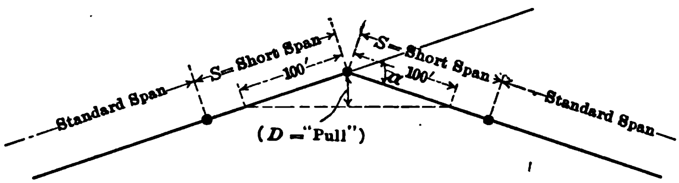

FIG. 1.

Limit of length of spans on each side of angle pole (standard span = 150 ft.).

"Pull" D feet	Deflection α degrees	Short span, S, not to exceed: feet	Remarks
2	2–18′	145	
3	3°–26′	143	Double fixtures not necessary.
4	4°–36′	140	
5	5–44′	138	
6	6°–52′	135	
7	8°–00′	133	Use double fixtures on
8	9°–10′	130	poles.
9	10–20′	128	Side-guy.
10	11°–28′	125	
11	12°–38′	123	Use judgment as to
12	13°–47′	120	whether or not head-guys
13	14°–56′	118	shall be used.
			Double Fixtures.

the deflection exceeds 5 degrees, poles with double fixtures shall be used, and these must be side guyed. When the "pull" at corner pole exceeds 2 ft. the span on each side of pole shall be less than 150 ft.; the reduction in the length of span being at the rate of about 2 1/2 ft. per foot of "pull," all as indicated in the accompanying table. Should it be necessary to turn the line at a point where space is limited, through an angle greater than 15 degrees, two or more poles with double fixtures may be set close together,

each pole being side guyed, or securely braced. In all caset where there is a departure from the straight line, the poles muss be set so that the cross-arms will bisect the angle.

10. Guying.—The material to be used throughout for guys is 5/16-in. galvanized seven-strand steel cable. Where the wire is wrapped round the pole, a protecting strip of No. 24 galvanized sheet iron shall be put under the wire. The wire shall make two complete turns about the pole.

The anchoring shall generally be done by burying an anchor log from 4 ft. to 6 ft. long, and bolting thereto a 5/8-in. guy rod, all as shown on drawing No...... Other methods must be adopted to suit the varying nature of the ground, but in all cases it is important to ensure a good hold and to see that the guy rod is in line with the guy wire. The angle of the guy wire when anchored in the ground shall be approximately 45 degrees where circumstances permit.

No strain insulators shall be used on guy wires, except as called for at railway crossings; but all guyed poles shall be provided with lightning rod and be well grounded. It is not intended that work be done on live wires on guyed or other grounded poles. As a general rule all poles shall be guyed before the conductors are strung. Poles must be guyed at all points as mentioned below:

(*a*) At angles exceeding 5 degrees.

(*b*) Where the line goes up a 15 per cent. or steeper grade (head guys every fifth or sixth pole, or only at top of hill on short lengths).

(*c*) On hillsides where the footing may be good, but where there is danger of slipping stones or soil producing side pressures on the pole (side guy).

(*d*) At each end of exceptionally long spans, where double poles are used.

(*e*) All poles with double fixtures.

11. Insulators.—The line insulators will be supplied by Messrs. .... They will be of the pin type, the pins having porcelain bases with wood thimbles and $\frac{5}{8}$-in. galvanized iron bolts for fixing to cross-arms. The pole-top insulators will be supported on maleable-iron pole-top pins, and the separable thimbles of these pole-top pins will be cemented into the insulators at the makers' works. The insulators shall be mounted on the cross-arms after the poles have been erected. The pole-top insulator pins may be bolted in position before erection of

pole. Double fixtures and insulators shall be used on all poles liable to abnormal stresses, but the location of poles should be so chosen as to avoid, if possible, the necessity for double fixtures.

12. Stringing of Wires.—No. 2/0 seven-strand bare aluminum cable will be used throughout. Care must be used in handling the conductors, to guard against cuts or scratches or kinks. The conductor must not be drawn over rough or rocky ground where it is liable to be injured by stones, etc.

It is important that the cables be pulled up to the proper tension so that the sag will be in accordance with the particulars given on the accompanying curves. These curves give not only the correct sag at center of span, but also the required tension in the cable at the time of stringing. The curves are calculated for wires subject only to their own weight and hanging in still air. A light wind blowing across wires while stringing will make no appreciable difference to the tension required, and in a very strong wind, the work would not be proceeded with. With a moderately strong wind blowing across the wires, the tension required at time of stringing is greater than as calculated for still air. As an example of the amount of correction required when stringing in a moderately stiff wind, a wind of 28 miles per hour blowing across the line, would necessitate increasing the tension on the No. 2/0 stranded aluminum conductor until it should be equal to the tension required on a span 15 per cent. longer than the actual span.

In the case of extra long spans, and where the grade is not constant, it will generally be found more convenient and quicker to adjust the tension by means of a spring dynamometer than by measuring the sag.

The cables must not be pulled round insulator pins on angle poles.

The tie wire shall be No. 2 B. & S. solid, soft aluminum wire. The tie shall be a modification of the type used by the Niagara, Lockport and Ontario Power Co., between Niagara Falls, Buffalo, Rochester and Syracuse. The serving of No. 2 tie wire on the conductor is for the purpose of preserving the latter from abrasion and from damage due to possible electric discharges over the insulator. Shields of sheet metal will not be used between the conductor and the top groove of the insulator. The use of pliers should be avoided in making the ties, except for the final clinching, when they must be used with care to avoid cutting or otherwise injuring the conductor.

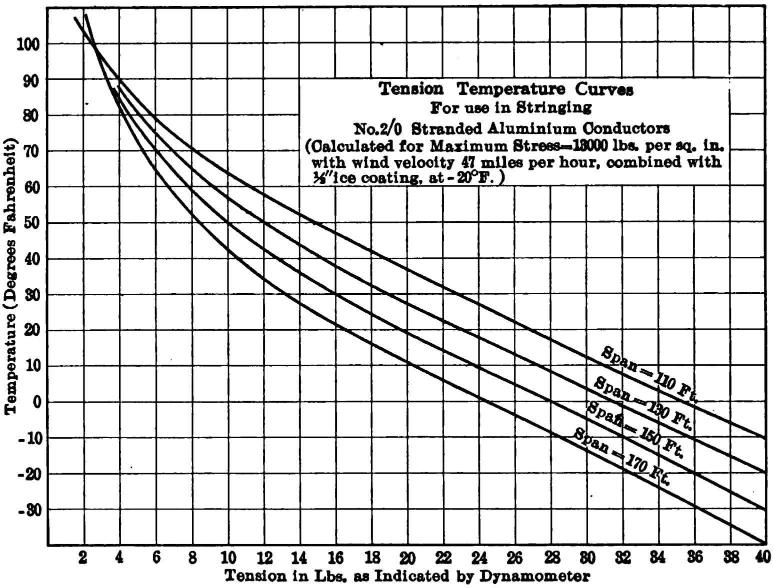
Tension Temperature Curves
For use in Stringing
No.2/0 Stranded Aluminium Conductors
(Calculated for Maximum Stress=13000 lbs. per sq. in.
with wind velocity 47 miles per hour, combined with
½"ice coating, at -20°F.)
Span=110 Ft.
Span=130 Ft.
Span=150 Ft.
Span=170 Ft.
Temperature (Degrees Fahrenheit)
100
90
80
70
60
50
40
30
20
10
0
-10
-20
-30
2 4 6 8 10 12 14 16 18 20 22 24 26 28 30 32 34 36 38 40
Tension in Lbs. as Indicated by Dynamometer

FIG. 2.

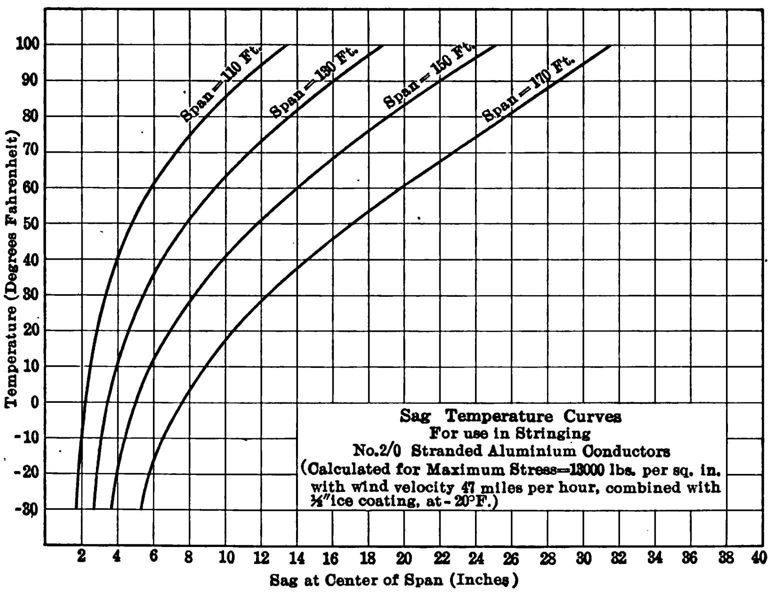
Span=110 Ft.
Span=130 Ft.
Span=150 Ft.
Span=170 Ft.
Sag Temperature Curves
For use in Stringing
No.2/0 Stranded Aluminium Conductors
(Calculated for Maximum Stress=13000 lbs. per sq. in.
with wind velocity 47 miles per hour, combined with
½"ice coating, at -20°F.)
Temperature (Degrees Fahrenheit)
100
90
80
70
60
50
40
30
20
10
0
-10
-20
-30
2 4 6 8 10 12 14 16 18 20 22 24 26 28 30 32 34 36 38 40
Sag at Center of Span (Inches)

FIG. 3.

The pole against which the wires are pulled shall always be head guyed, even if this guying is only temporary; and in no case shall the wires be allowed to slack back against the tie wire except where permanently secured to poles with double fixtures.

When joints are required in the conductors, they shall be made with MacIntyre tubes which shall be given two twists with the splicing clamps provided for the purpose.

13. Locating and Numbering Poles.—All poles shall bear a distinguishing number in clear bold figures about 10 ft. above ground level. These numbers will correspond with the numbers on the plans which will be prepared as soon as possible after the poles have been erected in position. The plans will be drawn to a scale sufficiently large to show the location of each pole.

APPENDIX G

Sample Specification for Steel Tower Transmission Line

This specification, which is a preliminary specification subject to revision in minor details after bids for the various materials have been received and considered, covers the construction of an overhead transmission line connecting the generating station at .... with the distributing station at ...., a distance of approximately 60 miles as measured along the right-of-way of the transmission line.

Methods of construction are not dealt with in detail, because the work in the field will be in the hands of a competent and experienced construction engineer who will be allowed considerable latitude in regard to the actual handling of materials and in deciding upon the best methods to be adopted in the erection of towers, stringing of conductors, and other details of practical line construction.

General Description of Line.—Two three-phase circuits of No. 2/0 equivalent copper cable will be run in parallel on one set of steel towers spaced approximately eleven to the mile. The towers will be of the semi-flexible type, with rigid strain towers at intervals of about a mile, or more frequently where corners or extra long spans render their use necessary. A 7/16-in. galvanized Siemens-Martin steel-strand cable will be carried the full length of the line and be firmly secured to the top of each tower.

The pressure between conductors will be 80,000 volts, and the suspension type of insulator will be used throughout.

Details of entering bushings and methods of connecting lightning arresters at generating and receiving stations are not dealt with in this specification as they come under another contract.

The proposed line has been staked out by the stadia survey party, and the right-of-way secured where necessary. Stakes have been driven to indicate proposed location of towers, but these positions are subject to modification.

The line passes through country that is for the greater part uncultivated; the ground is undulating and in some parts wooded.

A considerable amount of clearing has yet to be done. Roads are bad; but the transmission line is within 2 to 3 miles of the railway at all points.

Duties of Engineer in Charge of Construction.—Before the work of construction is begun, the construction engineer will go over the line as staked out by the preliminary survey party and as shown on plan No. ...... herewith. He will take with him an engineer equipped with a light transit, and an assistant capable of acting as axeman or rodman as circumstances may require. The construction engineer will decide in the field the position of each tower, making changes in the preliminary plan in the matter of tower locations and even to a small extent in the route to be followed, if in his opinion such changes will result in a better and more economical line. Hub-stakes shall be driven to mark the center-point of each tower, and a second stake shall be driven about 12 ft. ahead or in the rear of the hub-stake in the direction of the line. This is for reference when setting the anchor stubs.

The construction engineer must check clearances between conductor and ground, and on long spans, especially if there is doubt as to position and amount of minimum clearance, he should take the necessary particulars to allow of the matter being settled in the office. After agreeing and checking the alterations to plan in the office, the construction engineer will assist in making out the shipping schedules for delivery of materials at the most suitable points.

In regard to the work of erection proper, the construction engineer will attend to all details of organization of the parties in the field, and will study the best means of distribution of materials along the line; all with the view of avoiding unnecessary expenditure, and of carrying out the work expenditiously and in a workman-like manner. Such details as the actual methods to be adopted in the erection of towers and stringing of wires will be decided upon after discussion with the chief engineer, and after due weight has been given to manufacturers' suggestions.

Clearing.—On those parts of the line on which clearing is required, it is proposed that this work be done immediately after the line has been finally staked out. This clearing will extend 60 ft. on each side of the center line of the right-of-way, and it will be carried out under a separate contract.

Towers.—Two standard types of steel tower will be used: these will be referred to as the strain type and flexible type respec-

tively. The strain towers shall be designed with four corner legs and square bases, generally as indicated on plan No. ...... herewith. An effort will be made to avoid the use of special structures, and where extra long spans have to be carried, two standard strain towers may be placed close together. In one or two places it may be necessary to use extra high towers, and it is proposed to use the standard tower mounted on a special base, generally as shown on plan No. ......, designed to raise the tower 18 ft., or such other amount approximating to this dimension as may best suit manufacturers' designs.

The intermediate or flexible type of support will be of the "A" frame design, generally as shown on plan No. .... Preference will be given to a design consisting of few parts, provided this will not add appreciably to the cost of transporting the towers over rough roads to the point of erection. The parts of all towers shall be galvanized when ready for assembling; but, in the case of the flexible type of structure, an alternative offer for painted steel work will be considered, provided the number of parts is small and the section of metal reasonably large.

The plans referred to, which accompany this specification, give all necessary leading dimensions; but the cross-section of the various members and the details of design are left to the manufacturer, who is also at liberty to submit alternative proposals. In no case must the distance between conductors be less than 8 1/2 ft. or the height above ground of the point of attachment of insulators on lowest cross-arm less than 40 ft. The sections of structural steel used for the main corner members of the strain towers or for the main members of the flexible towers shall not be less than 1/4 in. thick, and no metal less than 3/16 in. thick shall be used in the construction of these towers.

Number of Towers Required.—Offers shall be based on the following quantities, which are subject to slight modification.

Flexible towers (plan No. ......)	592
Rigid towers (plan No. ......)	65
Extension bases (plan No. ......)........................	4

Working and Test Loads for Towers.—The normal length of span is 480 ft. and the total vertical load per tower, consisting of six conductors and one guard wire together with estimated possible ice loading and the weight of the six insulators, is 3100 lb.; but the spans will in many cases exceed the average length. The maxi-

mum total overturning pressure in a direction at right angles to the line, due to wind blowing across the wires, is estimated at 3300 lb.; this may be considered as distributed equally between the points of attachment of the seven wires. The manufacturer should estimate the pressure of wind on the tower structure itself by allowing a maximum pressure of 13 lb. per square foot of tower surface. A factor of safety of 2 1/2 shall be used in making stress calculations; that is to say, the unit stress in the steel under working conditions shall not exceed 25,000 lb. in tension.

One tower of each type shall be tested in the presence of purchaser's representative, and must withstand without exceeding the elastic limit of the steel, or suffering appreciable permanent deformation, the following test loads applied at the points indicated on the plans previously referred to.

Strain Tower Test Loads.—(1) A breast pull of 15,000 lb. applied in the direction of the line at the point of attachment of the middle cross-arm.

(2) A vertical load of 1000 lb. applied at the end of any cross-arm.

(3) A torsional load of 3500 lb. applied in a direction parallel to the line at the end of any cross-arm.

Flexible Tower Test Loads.—(1) A transverse pull of 4500 lb. applied in a direction at right angles to the line at the point of attachment of the middle cross-arm.

(2) A vertical load of 800 lb. applied at the end of any cross-arm.

Metal steps shall be provided on all towers within 8 ft. of ground level for the use of linemen.

It is requested that manufacturers tendering for steel towers call attention to any features of the particular design proposed which may tend to reduce cost of transport and erection on site, as these are matters which will receive consideration when placing the contract.

Galvanizing Test.—The purchaser reserves the right to reject all towers of which the galvanizing is not of the best quality. Tests will be made before erection as follows:

Samples of steel work will be immersed in a solution of sulphate of copper (specific gravity about 1.185) maintained at a temperature of 60 to 70° F. After remaining in the solution 1 minute, the sample will be removed, thoroughly washed in water, and wiped dry. This process will be repeated four times, after

which there must be no appearance of red spots indicating copper deposit.

Foundations for Towers.—The use of concrete is to be avoided, but in marsh land or loose soil concrete footings may be necessary. The decision as to where concrete is to be used will rest largely with the engineer in charge of construction in the field. A safety factor of two shall be allowed in foundations; that is to say, at least double the test load of 15,000 lb. in the case of the rigid towers, and 4500 lb. in the case of the semi-flexible towers, shall be necessary to move the anchor legs.

When the tower stands on solid rock—which may occur in a few instances—the standard footings will not be used; but a special wedge bolt, shaped at the top to take the standard tower, will be grouted in with sulphur or other approved cement. In leveling up on rock foundations, it may sometimes be cheaper to build up one or two piers of concrete, securely tied down to the rock, rather than level off the rock on the high side.

In selecting sites for towers, the construction engineer shall pay attention to the matter of foundations, and endeavor to secure sites where the foundations are good. Hillsides are to be avoided, especially where the soil is liable to crumble or slide. The matter of grading should also be considered when finally selecting sites: much may be accomplished in the judicious selection of tower sites by slightly adjusting the length of span to obtain sites which will tend to equalize the grade.

To facilitate the work of erection, a wooden digging templet will be provided, together with a rigid but light weight angle steel templet to ensure the correct placing of the anchor stubs; the latter being bolted to the templet before the work of back-filling the holes is commenced.

The second stake which, as previously mentioned, will be driven truly in line with the hub stake, will be used for the correct setting of these templets. The steel templet must be carefully leveled up in order that the center line of the tower shall be vertical.

Grounding.—In cases where the iron work of the foundations is completely encased in concrete, the tower shall be well grounded by means of a 10-ft. length of 1-in. galvanized-iron pipe driven or buried in the ground, and electrically connected to one of the tower legs. When the tower stands on rock, an effort should be made to obtain a good ground by carrying a length of the galvan-

ized guy wire from the tower leg to a rod driven in damp soil at a short distance from the tower if a suitable spot can be found.

Guying.—Where guy wire is required, the 7/16-in. Siemens-Martin steel ground wire shall be used. When the distance between strain towers exceeds 3/4 mile, one flexible tower situated about midway between the strain towers shall be head-guyed in both directions. Flexible towers used at corners where the deviation lies between 5 and 8 degrees, and the approach spans are of normal length, shall be guyed with two guy wires so placed as to take the corner strain and resist overturning of the tower owing to the resultant pull of the wires.

Angles.—The semi-flexible support is designed for use on straight runs only; but if the deviation from the straight line does not exceed 5 degrees, these intermediate supports may be equipped with strain insulators and used at corners. For angles greater than 5 degrees, but not exceeding a limit of 8 degrees, these supports may be used with two guy wires to take the transverse stress due to the resultant pull of the wires. If the approach spans are reduced to 240 ft., an 8-degree curve may be turned on a semi-flexible structure without guy wires.

Strain towers shall be used for turning corners up to 30 degrees; but when the total deviation exceeds this amount, two towers must be used.

Erection of Towers.—The actual organization of the various crews for distributing material, setting anchor legs, assembling and erecting the towers, will be left to the engineer in charge of construction, who will so conduct operations as to carry out the work efficiently at the lowest possible cost.

Insulators.—The insulators shall be of the suspension type and of approved design. This specification has been drawn up on the assumption that porcelain insulators will be used; but alternative quotations for composition insulators, if accompanied by full particulars and manufacturers' specification, will be considered.

Number of Insulators Required.—The approximate quantities required, as based on preliminary estimates are:

Suspension type..	4000
Strain type..	880

Climatic Conditions.—The transmission line on which the insulators will be used is located in the...........district, where severe thunder storms and heavy rain may be expected during

the summer months, and where sleet storms and low temperatures are prevalent in the winter.

Working Voltage.—The transmission is three-phase off delta connected transformers, at a frequency of 60 and a maximum working pressure of 84,000 volts between wires.

Design of Insulators.—The design of the suspension type and strain insulators is left to the manufacturer, who must submit dimensioned drawings or samples with his offer. The units making up the strain insulators need not necessarily differ in design from the units of the suspension insulators, provided the latter are capable of withstanding the mechanical tests required for the strain insulators. The towers have been designed on the assumption that the weight of one complete string of unit insulators shall not exceed 70 lb. and that the distance between point of suspension and conductor shall not exceed 40 in. If possible these limits should not be exceeded. It is preferred that the number of units in the complete string be not less than four nor more than six.

Glaze.—The surface of the porcelain shall be uniformly coated with a brown glaze, free from grit.

Cement.—Pure Portland cement only shall be used in assembling the parts of the unit insulator.

Mechanical Tests.—An inspection will be made of all insulators with the object of rejecting those containing open cracks in glaze or porcelain. The inspector, by using a light weight mallet, may detect by the ring whether cracks or air cells are present.

One complete suspension insulator, selected at random, and consisting of the requisite number of units, shall withstand a load of 5000 lb. without rupture or sign of yielding in any part.

At least three units of which the strain insulators are built up shall be tested to the breaking limit, and must withstand an ultimate load of not less than 12,000 lb.

Electrical Tests.—The complete suspension insulator shall withstand without flash over a "wet" test of 150,000 volts. The string of units forming the strain insulator shall withstand a "wet" test of 175,000 volts without flash over. In both cases the electrical stress shall be applied for 1 minute, and the spray shall be directed upon the insulator at an angle of 45 degrees under a pressure of 40 lb. per square inch at the nozzles, the precipitation being at the rate of 1 in. in 5 minutes. The suspension insulators shall be hung vertically, and the strain insulators horizontally.

The connection of the test wires shall be so made as to reproduce as nearly as possible the working conditions.

The manufacturer shall satisfy himself by his standard factory tests that each unit is sound electrically; and if all units in the complete insulator are of the same size and design, the dry test voltage must not be less than the following figures:

On each unit, if string consists of four units.	75,000
On each unit, if string consists of five units.	60,000
On each unit, if string consists of six units.	50,000

The transformer used for the electrical tests must be capable of a reasonably large kilowatt output; the e.m.f. wave shall be as nearly as possible sinusoidal, and the frequency shall be within the limits of 25 and 60 cycles.

Packing of Insulators.—It is desirable that the parts for one insulator, or at most for two insulators, be packed complete in a separate barrel or crate, and that the contents be clearly described on attached label.

Wire Clamps.—A suggested clamp for use with suspension insulators is shown on drawing No. . . . . herewith; and drawing No. . . . . shows a proposed strain insulater clamp. Makers are asked to submit samples or drawings of their standard types, preferably of the general design indicated by the above-mentioned drawings. The conductor to be carried is an equivalent No. 2-0 gage (B. & S.) stranded copper cable; the groove for the wire should be slightly curved and flared at the ends. Preference will be given to a design provided with spring washers to clamping bolts or similar means of providing for the slipping of the conductor in the clamps before the pull exceeds a specified limit. This feature is not required in the design of the strain insulator clamps.

Conductors.—The conductors shall be 19 strand hard-drawn copper cables equivalent in section to 00 B. & S. gage. The tensile strength of the finished cable shall not be less than 90 per cent. of the strength of the individual wires forming the cable; and these shall satisfy the strength requirements of the standard specification drawn up by the American Society for Testing Materials.

The electrical conductivity shall not be less than 97 per cent. by Matthiessen's standard.

The total weight of copper conductor required is estimated at

800,000 lb. It shall be delivered on drums or reels each containing 1 mile of cable.

Joints in Cables.—The splices shall be made with copper sleeves of the "MacIntyre" or similar approved type. The finished joint shall consist of 3 turns. The tools provided for the purpose shall be used in making the joints.

Spans and Wire Stringing.—The actual method of stringing the wires will be left to the judgment and experience of the construction engineer. It is, however, suggested that three conductors be drawn up at a time, using the arrangement of sheave blocks known as an "equalizer."

The average span shall be approximately 480 ft. This may be increased to a limit of 500 ft. between flexible supports, and to a limit of 1200 ft. between two strain towers without intermediate supports. It is thought that three or four points on the line may advantageously be spanned between two strain towers placed from 1000 to 1200 ft. apart. In the case of abnormally long spans, it is important to see that the contour of the ground is such as to allow of maximum sag while maintaining the specified minimum clearance between H. T. conductors and ground.

The clearance between lowest wire and ground shall in no case be less than 28 ft.

The charts Nos. . . . . . . and. . . . . . . give all necessary particulars for the stringing of conductors and guard wire at various temperatures. The guard wire connecting the tops of all towers shall be strung and securely clamped to the steel structure before the conductors are drawn up. Dynamometers will be provided, and their use is recommended, especially when spans are unequal in length, and on extra long spans between two strain towers. The sag in conductors can, however, readily be measured by means of targets hung from the cross-arms. If an equalizer is used, it is not necessary to insert a dynamometer in more than one leg. Special attention shall be paid to the drawing up of cables to the proper tension or sag. Too great a sag is almost as objectionable as too great a tension; but it must be remembered that where a dip occurs in the line of supports there is sometimes a possibility, in very cold weather, of the conductor being drawn up (by contraction) above the proper level of the lowest insulator. The construction engineer should watch for this possibility with a view to guarding against it.

When the conductors have been drawn up and transferred

from snatch block to insulator clamp, it is important to see that the suspension insulator hangs truly vertical before finally tightening up the clamp.

There will be no transpositions on the H. T. conductors. The telephone wires are run on a separate set of wooden poles and they will be transposed at every support.

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SUPPLEMENTARY INDEX

Giving references to practical formulas and other information that may be used or consulted when actually engaged on transmission line calculations.

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